A 3D DYNAMIC TRAIN-TRACK INTERACTION MODEL TO STUDY TRACK PERFORMANCE UNDER TRAINS RUNNING AT CRITICAL SPEED

A Thesis in
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by
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ABSTRACT

In this thesis, the ground-borne vibrations generated by high-speed trains are investigated, by implementing a three-dimensional dynamic train-track interaction model. Previous research has shown that high speed trains on soft ground can induce a significant increase in vibration level when the trains move at a “critical speed.” The critical speed is explained as that at which resonance occurs between the moving train and the Rayleigh wave of the subgrade soil.

Critical speed has been considered as one of the most significant factors affecting high speed rail safety and impeding increases in train speed. In this research, a 3D dynamic track model is introduced to determine the ground response generated by high-speed trains. It is based on a sophisticated train model, a theoretical model for the track and layered subgrade soil. In order to investigate the performance of this model, sensitivity analysis is conducted to explore the influence of each parameter. Simulation by commercial finite element software (ABAQUS) is also used for separate verification of the model. After model verification, the effects of critical speed on ground-borne vibrations are further discussed.
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Chapter 1

Introduction

1.1 Background

For more than 150 years, railways have been a major form of public transportation. High speed rail (HSR) has been growing rapidly, due to its environmentally friendly technologies and time saving in short-distance travel. From the world’s first HSR system (Japan, in 1964), over ten thousand miles of HSR rail have been built. TGV, the current world speed record holder on conventional rails, can reach speeds as high as 574.8 km/h (160m/s).

Increasing speed brings new challenges to conventional railway engineering, which include rail system control, passenger safety and comfort, and noise and vibration hazards. Ground-borne vibrations induced by HSR cannot only bring damage to adjacent buildings, but can also affect the operation of high-precision devices. In addition, it is a complex problem to measure the effect of vibrations on human comfort. Not only the magnitude of monitored vibrations, but also the noise pollution to tenants along the railways can be a noteworthy issue. Critical speed has been considered as one of the most significant factors affecting high speed rail safety. According to previous research, high speed trains on soft ground can induce a significant increase in vibration level when trains move at the “critical speed.” The critical speed is explained as that at which resonance occurs between the moving train and the Rayleigh wave of subgrade soil.

Many researchers have studied the phenomenon of trains running at critical speed. In 1927, theoretical modeling of a rail as a beam supported by track structure and ground revealed that the dynamic amplification will occur when train reaches a critical speed (Timoshenko, 1927).
However, under the knowledge of normally assumed soil properties, this critical speed is around 500 m/s, which is highly above the realistic HSR speed, because only pressure wave velocity is considered in Timoshenko’s theory. Hence, for a long period, the train loads have been assumed as quasi-static moving loads.

Krylov (1994) considered that a train will encounter the ‘sound barrier’ when reaching the velocity of Rayleigh surface waves propagating in the ground. This phenomenon can be compared with the Mach effect by supersonic jets and Cherenkov radiation of light. The velocity of Rayleigh waves travelling in soft sandy soils is 90 to 130 m/s (Krylov, 1994), which is already reachable by today’s high speed trains. Furthermore, for peat, marine clays, and other soft clays, the characteristic wave velocities could be reduced to as low as 30-40 m/s (Kaynia, et al. 2000). Rayleigh waves have a speed slightly less than shear waves, depending on the soil properties. The ground medium which has low shear-wave velocity needs much attention because it is susceptible to the increasing vibrations generated by high speed trains. These vibrations could induce the deterioration of rail track structure, and even result in derailment. Not only the stability of the rail track structure, but also, more importantly, passenger safety, should be major concerns for HSR systems.

However, it is expensive and perilous to run a train at critical speed to test the vibration of the subgrade soil. Therefore, significant previous research has been focused on modeling critical speed effects. How to simulate the vehicle, track and ground system is the core research in this field, as well as the methods to couple the vehicle, track and ground models. The models, according to the literature reviewed, can be divided into three categories. One is the ground vibration generated by moving rail loads, which is the simplest. The moving rail load usually is modeled as a moving point load. The rail, modeled as a beam, just lies on the ground surface
without track. A second category of model, which is extensively used, couples a track with the previous model. The models coupled with track model can simulate the realistic rail/wheel dynamic effect and are always more accurate than the first category of models. Recently, many researchers focus on coupling the vehicle, track structure and ground together to form an integrated system (Zhai et al. 2010). The vehicle model is a complicated system with car mass, bogie mass and wheel, all coupled by a spring/dashpot system. For the track system, the rail is usually considered as an Euler-Bernoulli beam. The sleepers can be modeled as discretely supported or as a continuously supported system. Green’s function is widely used to calculate the ground surface vertical displacement. The contact between wheel and rail can be assumed using Hertzian nonlinear elastic contact theory or other contact theories. For track models, ballast is usually modeled as another spring/dashpot system; sometimes the ballast mass is also considered in calculation. In this thesis, a train-track interaction model referred to as the Sandwich Dynamic Track Model is introduced. As later described, 2.5D FEM is used to simulate the behavior of the ground. The whole system is coded in MATLAB. Sensitivity analysis for this proposed model, considering factors such as domain size, element size, soil properties effects, and track parameters effects, is conducted. Also, to verify the proposed 3D dynamic track model, commercial software (ABAQUS) is introduced here. The purpose of the verification is to increase the confidence in the proposed model. Train load in ABAQUS is modeled as a sequence of point loads running on the rail at a constant speed. In addition, the location and influence depth of critical speed effects are also studied in this thesis.
1.2 Definitions

1.2.1 Rail

Rail is an important longitudinal steel track component put on the top of the rail track and is used to support and guide the vehicle by providing smooth running surfaces. It can accommodate the wheel loads and distributes these loads over the sleepers or supports. The horizontal transverse forces on the rail head can be transferred to sleepers and lower track components. Also, the rail enables the vehicle to move in a stable direction.

There are many types of rail with regards to its profile, including flat-bottom rail, non-standard profile, grooved rail, block rail and crane rail, which are shown in Figure 1. Flat-bottom rail is the standard profile used as a general rule in conventional track, and it is also introduced in this thesis. Also, in the United States, the flat-bottom rail can be divided into several categories according to different self-weights per linear yard. In addition, each rail is connected together by joint bars.

![Figure 1. Rail profile types [Coenraad, 2001]](image)

1.2.2 Railpad

The function of railpad is to provide an absorbing component between the steel rail and sleepers through transferring the rail load to sleepers and screening out the high frequency force. Also, railpad can make a more stable track and significantly lengthen the life of wood sleepers.
In addition, railpads are embedded under rails acting as electrical insulation. The railpad is shown in Figure 2.

![Figure 2. Railpad [Unitrac Railroad Materials, Inc.]](image)

1.2.3 Sleepers

The sleepers are rested on the transverse direction of the track, which is vertical to the moving direction of the vehicle. The function is to maintain track gauge and fasten the rails to be aligned. Also, it can be considered as electrical insulation for the rails. Sleepers transmit the train loading to the lower track structure. The available materials for sleepers can be wood, steel and concrete. Timber and concrete ties are widely used and steel ties are limitedly used.

1.2.4 Ballast

Ballast is a layer that is formed by crushed granular material and placed on the top of the subground. Ballast bed can absorb considerable compressive stresses, but not tensile stresses. Also, it has a large bearing strength in the vertical direction, but it is reduced in the lateral direction. The thickness of the ballast bed is typically about 50 cm from the top of the ballast or 25-30 cm from the lower side of the sleeper.

The main functions of ballast are to: 1) distribute the stresses transmitted by sleepers; 2) drain rainwater; 3) resist transverse and longitudinal shifting of track; 4) attenuate train vibration significantly.
1.2.5 Train and Track Model

Typical train model is illustrated in Figure 3. It has a secondary suspension and primary suspension which both are modeled as spring/dashpot system. Bogie is located between the secondary suspension and primary suspension.

Figure 3. Train model [Coenraad, 2001]

An integrated track model is depicted in Figure 4, which includes all the track components introduced previously, such as rail, railpad, tie, ballast.

Figure 4. Principle of track structure [Coenraad, 2001]
1.2.6 HSR Speed

The definition of high speed rail varies between different countries and different institutions. The various speeds that can be attributed to HSR are listed in Table 1.

<table>
<thead>
<tr>
<th>Institution</th>
<th>Standard for High Speed Rail (Minimum speed)</th>
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<tr>
<td>European Union</td>
<td>Upgraded track: 200 km/h (124 mph)</td>
</tr>
<tr>
<td></td>
<td>New track: 250 km/h (155 mph)</td>
</tr>
<tr>
<td>Federal Railroad Administration</td>
<td>177 km/h (110 mph)</td>
</tr>
<tr>
<td>U.S. Department of Transportation</td>
<td>201 km/h (125 mph)</td>
</tr>
<tr>
<td>Congressional Research Service</td>
<td>240 km/h (150 mph)</td>
</tr>
</tbody>
</table>

1.3 Current Status

This thesis is a preliminary summary of the ongoing HSR research project funded by Federal Rail Administration (FRA). The focus of this thesis is on formulating the model and conducting elementary verification. Future study may concentrate on improving the model by further verification and results from field test, strain/stress calculation of the ground, and then mitigation methods.

1.4 Problem Statement

High speed trains could induce a remarkable increase in ground vibration level when moving at or over a critical speed. That critical condition can be defined by the resonance between the moving train and the Rayleigh wave of subgrade. It can significantly increase the risk of operation of high speed train. Therefore, it is important to predict this phenomenon and implement further methods of control. Field measurements are not feasible at this stage of the research, nor over the broad range of potential conditions, due to the considerations of long time
and big amount of funds. In order to address the issue, an integrated model that could better predict the train-induced vibrations is greatly needed.

Chapter 2
Goals and Overview of Approach

The goals of this research are twofold. First, to develop a 3D dynamic train-track interaction model that is able to predict the critical effect induced by high-speed rail. Second, to use the model to accurately predict the ground-borne vibrations generated by high speed rail.

In order to obtain an improved model, the prediction model integrates a train model, a track model, and ground soil (subgrade). A 2.5D finite element method is also introduced to simulate the subgrade soil. Then, a sensitivity analysis is conducted to investigate the effect of each parameter in the model. With the purpose of reducing the risk of investing in further development and application of this model, verification for this model is performed with the commercial software ABAQUS. The flow chart for the first goal is shown in Figure 5.

![Flow Chart](image.png)

Figure 5. Technical flow chart for first goal
As shown in Figure 6, for the second goal, the critical speed effect can be predicted if the first goal is achieved. The location and the depth of the critical layer, which is defined as the major cause for critical effect, are explored. The conditions of occurrence of critical speed effect need to be further investigated.

![Critical Speed Effect Prediction](chart.png)

Figure 6. Technical flow chart for second goal
Chapter 3

Literature Review

3.1 Rail Track Model

The function of rail track models is to interrelate each component in the track structure in order to simulate the integrated properties in determining the reactions of the moving train load. Rail track models which include all the track components enable us to predict the track performance more effectively and precisely.

As an important track component, rail is used to support and guide the vehicle by providing smooth running surfaces. The forces of a train on the top of the rail are distributed over sleepers. Lateral loading, from train wheels, uniformly distributed on rails, and longitudinal loading, which is generated by braking and acceleration, are also passed on to the track structure. In simulation, the rails are usually simplified as two mathematical models: Euler-Bernoulli Beam (E-B beam) and Rayleigh-Timoshenko Beam (R-T beam). E-B beam only considers bending behavior of rails. R-T beam theory includes not only bending, but also shear deformation of the beam. Train-induced vibrations with frequency less than 500 Hz carry higher energy. As a result, the vibration dissipates at a lower rate and its impact is felt farther from the source. Also, it was found that when the frequencies of train loading are less than 500 Hz, shear deformation of the rail can be neglected (Dahlberg, 2003). Hence, E-B beam theory is sufficient to simulate the ground vibration induced by HSR.

Railpad is an absorbing component between the steel rail and sleepers. The functions of railpad are transferring the rail load to sleepers, and screening out the high frequency force. Usually, railpads are embedded under rails acting as electrical insulation and as a protective layer.
for sleepers. The railpads also affect the dynamic behavior of the track. Pairs of springs and dashpots are introduced to simulate the effect of railpads.

The sleepers are placed in the transverse direction to the track, or, in other words, perpendicular to the movement direction of the vehicle. Their function is to maintain track gauge and fasten the rails to be aligned for both construction and operation of rail track. Sleepers also transmit the train loading to the lower track structure. The available material for sleepers can be wood, steel and concrete. For the FRA project, the track structure is conventional and just needed to be upgraded. So the wooden sleepers are simulated here.

Ballast denotes a layer of crushed stone of uniform size, on which the sleepers are resting. It is granular material used to provide support for sleepers and fill the spacing between sleepers. The granular material is hard to simulate. The objective for this research focuses on overall track performance, not each track component. So, the ballast is also simulated as a spring/dashpot system without its mass.

Many researchers have been working on track models. Two-dimensional models are suitable for study of vertical track performance, but they ignore the transverse cross-section of the track. Three-dimensional models are rapidly being developed because they can provide more detailed performance of the track and responses from all directions. However, 3D models are usually very time-consuming. As a potential compromise, a 2.5D method is very promising, in that it has three-dimensional motions but only two-dimensional elements. This feature makes it very time-saving. The summary of the different types of track models are listed in Table 2.
### Table 2. Summary of Key Research on Track Models

<table>
<thead>
<tr>
<th>Track Models (selected)</th>
<th>2D models</th>
<th>3D models</th>
<th>2.5D models</th>
</tr>
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</table>

#### 3.1.1 Two-Dimensional Track Models

Many early models are based on a beam-on-elastic-foundation (BOEF) formulation. In this system, rails, simulated as continuous Euler-Bernoulli beams, are placed on elastic spring supports. Thus, the rail reactions in the longitudinal direction are proportional to their deflections. This model which is shown in Figure 7 has long been accepted and used in modeling rail track, and is the backbone of the subsequent models.

![Diagram of a beam on an elastic foundation](image)

**Figure 7.** Beam (bending stiffness EI) on elastic foundation (bed modulus k) [Coenraad, 2001]

Kaynia et al (2000) developed a 2D model, which evolved from the basic BOEF. Besides the bending rigidity and mass per unit length of embankment, hysteretic damping ratio is also added. The whole track system is bonded to the half-space at discrete points (nodes) along the embankment. The train loads that applied to the nodes with time shifts are combined into one concentrated load at the centerline of the bogie. The track model is illustrated in Figure 8.
Madshus and Kaynia (1999) simulated and analyzed the response of the track-ground system from high-speed train passage by using a computer program, Vibtrain. The ground is modeled as layered half-space by Green’s functions. And the rail/ballast system is taken as a beam by finite elements. Track and ground are related by enforcing vertical displacement and stress. Train loads are applied to the system through delaying the loads point by point according to different train speeds.

J.J. Kalker (1996) introduced a discretely supported beam model, which is shown in Figure 9. In his research, the rail is modeled as an Euler beam. The most salient feature of the paper is the irregular discrete support of the sleepers. The vertical displacement of a railway rail due to a travelling vertical point load of variable intensity is calculated.
In order to obtain the ballast effect, Huang, et al. (2009) introduced a 2D track model, called the Sandwich track model, which is shown in Figure 10. The ballast is modeled as discrete masses that are connected to sleepers and the ground with spring/dashpot. Rail, in this Sandwich model, is also modeled as an Euler beam. The rail pad, tie and ballast are all represented by mass and spring/dashpot in this research.

However, 2D models are insufficient to simulate the ground vibration vertical to the track, thus the Mach radiation effect of soil cannot be detected. Also, most early researchers’ work on
ground-borne vibrations was based on plane strain assumptions, and the beam on elastic foundation system. Hence, strictly speaking, only a more sophisticated 3D model is appropriate to investigate the wave propagation in the ground.

### 3.1.2 Three-Dimensional Track Models

Due to the limitations of 2D models, many researchers proposed 3D models to simulate the integrated track system to study the ground vibration generated by moving trains.

The track system of Takemiya (2003) shown in Figure 11, including rails, sleepers and ballast bed, is also modeled as an Euler-Bernoulli beam on elastic foundation. The Fourier Transform that changes the time-domain problem to a frequency domain problem is applied to obtain the solution for a moving load. The railway track is modeled as an Euler beam resting on ballast, then on the ground. However, the effect of sleepers is not considered in this model.

![Figure 11. Track ground interaction [Takemiya, 2003]](image-url)
Cai (1994) introduced a 3D model with coupling spring/damper elements, representing the mechanism of railpads, sleepers and ballast bed. The rail can be either modeled as R-T beam or E-B beam to describe the rails which are discretely coupled with sleepers. Another beam element with mass is placed to model the sleepers. The sleeper rests on another spring-damper system, as shown in Figure 12.

![Figure 12. Rail on discrete support [Cai, 1994]](image)

The previous types of rail track models do not include the effect of real ballast behavior, such as ballast deformation. Ballast is formed by granular material, which cannot be modeled the same as subgrade soil. To fulfill this task, discrete ballast masses need to be used to mimic the mechanical responses of a granular material. Furthermore, when more and more details of the track such as geometry and/or material properties have to be considered, numerical solving techniques are employed. 3D Finite Element Model is one of the most widely used models as it is able to cover almost all geometry considerations and is commercially available. In order to investigate the train speed effect, XiTRACK Limited proposed a 3D FEM model incorporating
all track components, including rail, sleepers, ballast, and subgrade. Train load is modeled as a sequence of constant load running at a constant speed over the rails. Detailed information about the model is given by Woodward et al. [Woodward, 2005].

3.1.3 2.5-Dimensional Track Models

Although 3D FEM can always serve as a benchmark program to calibrate different track models when field measured data is not available, it is time consuming and is usually utilized only for particular cases. Hence, realizing the limitations of two-dimensional models and the unfavorable time efficiency associated with three-dimensional models, researchers [Bian, 2008; Yang & Hung, 2001, 2003] proposed an innovative model called 2.5D FEM by assuming the track property remains uniform along the direction of train movement; only a profile of half-space vertical to the direction of load movement is considered. Also, the 2.5D approach is suitable for tunnels due to its assumption of uniform material properties along the movement direction. The accuracy of this approach is verified via comparison of results obtained from analytical solutions [Yang & Hung, 2009].

Bian (2008) modeled track-ground interaction by moving load using 2.5D FEM. The material properties and geometry are assumed consistent along the movement direction. The ground is modeled by isoparametric quadrilateral elements to condense the 3D issue to a plane strain problem. The details of this modeling method are further described in the Methodology of this thesis, in Chapter 5. The 2.5D BOEF model is shown in Figure 13.
Figure 13. 2.5D track-ground interaction model with a moving load [Bian, 2008]

For 3D soil, the 2.5D FEM technique is employed. Fourier Transform was only performed in the direction of train movement, and the transverse and vertical directions were discretized by plane stress quadrilateral finite elements. A typical 2.5D element is illustrated in Figure 14.

Figure 14. 2.5D element (2D element, 3D motion)
3.2 Train-Track Interaction Models

The vehicle-track dynamic has consistently not been valued in previous research. However, due to the track irregularities and vehicle dynamic properties, a train-track interaction model is essential. A 3D vehicle-track system allows dynamic loads to transmit to the track structure and subgrade through the wheel-rail contact. Over the past decade, research on train-track interaction models has been rapidly developing, with computer technology facilitating the possibility of analyzing large and sophisticated dynamic train-track coupled models (Shen, 2004; Zhai, 2010).

In Shen’s model (2004), two rails are taken as a single Euler beam with mass per unit length and bending rigidity. The lower beam, also with a mass per unit length but no bending rigidity, represents sleepers. The continuous springs between two beams have a complex stiffness by considering spring stiffness and damping loss factor. The ballast has an infinite length and mass; vertical complex stiffness is modeled as a viscoelastic layer. The ground is simply modeled as a layered elastic medium of infinite extent. This model is shown in Figure 15.
However, the validity of the assumption for the soil part (i.e. half infinite space) is still limited. Also, this model becomes insufficient in cases that tracks are laid on slopes, or inside tunnels, where cross-section geometry characteristics of the track and foundation are of great importance.

In order to account for the continuity and coupling effect of the interlocking ballast, Zhai (2003) recommended shear stiffness and shear damping between adjacent ballast masses. The model is shown in Figure 16.
3.3 Train Speed Effect

Vibration would not have been an unfamiliar occurrence in the past. Even during the age of horse-driven carriages on cobblestone streets, strong complaints of vibrations arose from occupants of buildings along the route. However, no convincing explanations were made in that period. Therefore, for a long period, train loads have been believed to be reasonably assumed as quasi-static moving loads. Krylov (1994) considered that a train will encounter the ‘sound barrier’ (critical speed) when reaching the velocity of Rayleigh surface waves propagating in the ground. The critical condition is explained as resonance between the moving train and the Rayleigh wave of the subgrade soil. This phenomenon can be compared with the Mach effect by supersonic jets and the Cherenkov radiation of light. Depending on whether the speed of a moving train is less than, great than or close to the velocity of Rayleigh wave, the train speed effect can be categorized as subsonic, supersonic and transonic. (Kaynia, 2000) When a train runs at a speed less than the Rayleigh wave velocity, the ground vibrations behave and represent a quasi-static condition. The increase in vibration magnitude is slow relative to increase in train speed. However, under transonic and supersonic cases, the dynamic effects of ground vibrations perform like the development of Mach lines and Mach surfaces. Vibration magnitude increases...
exponentially with train speed. Critical speed, the focus of this research, is defined as the speed at which moving trains resonate with waves traveling in the track and produce excessive ground vibrations. The effect of train speed is illustrated in Figure 17 and Figure 18.

Figure 17. Ground surface deflection contour plots for trains running at different speeds: (a) c=100 km/h; (b) c=200 km/h; (c) c=300 km/h [Bian, 2008]
As can be observed, both the displacement field and vibration level increase significantly when trains run at critical speed. The reason may be the surface irregularity of wheels and rails, and the rise and fall of axles over sleepers. In practice, the rails were found not to be straight, and the wheels were found not to be round. The vibrations can readily arise when the mass of the vehicle is being made to follow the alignment of the track. Eventually, this would result in dynamic forces. Similarly, the wheels would traverse the joints on the rails, leading to dynamic forces. In addition, it can be understood that the actual support stiffness under the rails is not uniform. Ties with intervals between each other are laid vertical to the movement direction, acting as periodic supports for the upper structure. The rails right under the ties offer higher support stiffness when the wheels are right over, but in between, the ties offer smaller stiffness, which makes the rail stiffness takes over the role to support the train predominantly. Furthermore, even when trains are moving on smooth rails, vibrations can still be induced by the regular repetitive action of the moving train loads [Yang & Hung, 2009].
This explanation for critical speed is cited from CRITICAL SPEED OF SHAFTS, written by Krueger (Krueger, September 2012):

In solid mechanics, in the field of rotordynamics, the critical speed is the theoretical angular velocity that excites the natural frequency of a rotating object, such as a shaft, propeller, leadscrew, or gear. As the speed of rotation approaches the object's natural frequency, the object begins to resonate, which dramatically increases system vibration. The resulting resonance occurs regardless of orientation. When the rotational speed is equal to the numerical value of the natural vibration, the speed is referred to as critical speed.

In rail engineering, the theory is similar. If train speeds reach critical velocity and the vibrations resonate with the natural frequency of the subgrade soil or the building nearby, it can cause increasing ground vibration, internal noise in building, and even structural damage in buildings up to 250m from the track. This critical velocity is relative to the speed of propagation of Rayleigh waves on the ground, which depends on geotechnical properties.

Rayleigh waves are surface waves, commonly produced by earthquakes but also caused by heavy, fast-moving loads such as high speed trains. Rayleigh waves typically take two-thirds of the energy in a vibrating soil system, and also attenuate at a much slower rate than P- and S-waves. Consequently, Rayleigh waves are so important because they are able to cause considerable structural vibrations when trains reach critical speed. As can be seen in Figure 19, the velocity of a Rayleigh wave is slightly less than the velocity of shear wave, particularly for higher Poisson’s ratio. Also how the Rayleigh wave propagates on the surface of the ground is shown in Figure 20. The relationship between the velocities of the two waves is [Suiker, 2002]:
\[ V_R \approx \frac{0.87 + 1.12\nu}{1 + \nu} V_S \] (3.1)

Where, \( \nu \) is the Poisson’s ratio; \( V_R \) is the velocity of Rayleigh wave; \( V_S \) is the velocity of shear wave.

Figure 19. Variations of Rayleigh wave and Body wave propagation velocities with Poisson’s ratio

[Kramer, 1996]

Figure 20. Motion of Rayleigh wave

(http://www.geo.mtu.edu/UPSeis/images/Rayleigh_animation.gif)

In the past decade, critical speed effect has been considered as one of the most important factors affecting high speed rail safety and precluding higher speed operations. The critical speed
effect is the combination of train suspension system, train speed, rail surface smoothness, track structure, and most importantly the ground soil or subgrade. It has been usually investigated case-by-case and there is a lack of general solutions. One possible solution is to develop a sophisticated computer model to identify any potentially critical train-track combinations. Through the computer model proposed in this research, the critical speed effect can be predicted which will greatly decrease the risk of the operation of railway traffic.
Chapter 4
Methodology

4.1 Sandwich Dynamic Train-Track Coupled Model

Previous track models are insufficient to investigate the critical speed effect for U.S. high speed rail for several reasons: 1) three-dimensional numerical solutions are time consuming and computing-power demanding, which usually becomes cumbersome for research investigations that require hundreds or even thousands of test runs; 2) two-dimensional analytic solutions are effective for simulating track vertical responses, however, they completely ignore the characteristics of track cross-section; and 3) simply using a stiffness, k, to describe the subgrade is not adequate to study the critical speed effect.

A 3D train-track interaction model (called the sandwich dynamic track model), which couples vehicles to track structure by Hertzian Contact theory, is introduced here. The system is a ‘rail-tie-ballast’ upper structural form which contains rail on discrete ballast support on trackbed. To simplify, just a ‘beam on discrete support on beam’ combination is presented here. The rail is modeled as an Euler beam. The rail pad, tie, and ballast layer are represented by means of a system of mass, spring, and damper with designated spacing. The structure under the ballast is then modeled as another Euler beam on Winkler foundation, representing the subgrade soil. The whole system is shown in Figure 21.

However, the model in this thesis is still not a comprehensive model, because the effect of ballast mass is not considered. The focus of this thesis is on the overall performance of the track, not on each track component. For this reason, the proposed model is capable of simulating the ground-borne vibration generated by high speed trains.
Figure 21. 3D train-track interaction model for US high-speed rail [Huang, 2013]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car Body Mass (kg)</td>
<td>10,700</td>
</tr>
<tr>
<td>Car Body Inertia of Rotation (kg*m$^2$)</td>
<td>400,000</td>
</tr>
<tr>
<td>Mass of Boggy (kg)</td>
<td>1,300</td>
</tr>
<tr>
<td>Bogie Inertia of Rotation  (kg*m2)</td>
<td>1,000</td>
</tr>
<tr>
<td>Mass of Wheel (kg)</td>
<td>700</td>
</tr>
<tr>
<td>Contact Stiffness between Wheel and Rail (GN/m)</td>
<td>1</td>
</tr>
<tr>
<td>Stiffness of Primary Suspension (N/m)</td>
<td>170,000</td>
</tr>
<tr>
<td>Damping of Primary Suspension (N*sec/m)</td>
<td>4,200</td>
</tr>
<tr>
<td>Stiffness of Secondary Suspension (N/m)</td>
<td>400,000</td>
</tr>
<tr>
<td>Damping of Secondary Suspension (N*sec/m)</td>
<td>37,000</td>
</tr>
<tr>
<td>Distance between the Center of the Wheels (m)</td>
<td>16</td>
</tr>
<tr>
<td>Distance between the Center of the Bogies (m)</td>
<td>2.5</td>
</tr>
</tbody>
</table>
4.1.1 Train

A simplified vehicle model with both primary and secondary suspensions shown in Figure 22 was employed. It has 10 degrees of freedom and the displacements of the vehicle can be expressed in the frequency domain, as shown in the following steps.

Figure 22. Vehicle model

The governing equation for the train can be expressed as:

\[
[K - \omega^2 M]\{dV(\omega)\} = \{f(\omega)\} \quad (4.1)
\]

Where: “K” and “M” are the stiffness matrix and mass (including mass moment of inertia) and matrix of the car, respectively. “\{dV(\omega)\}” is the nodal displacement vector. “\{f(\omega)\}” is the nodal external force vector.

Since \{f(\omega)\} = \begin{bmatrix} 0 \\ I \end{bmatrix}\{P(\omega)\} where “\{P(\omega)\}” is the nodal wheel force vector and \{dW(\omega)\} = \begin{bmatrix} 0 & I \end{bmatrix}\{dV(\omega)\}, Equation 4.2 can be rewritten as:

\[
\{dW(\omega)\} = [GV]\{P(\omega)\} \quad (4.2)
\]

Where: \([GV] = [OI][K - \omega^2 M]^{-1}[OI]^T\) is called “Green Function of the Vehicle.”
Equation 4.2 depicts the relationship between the wheel displacements and the wheel rail contact forces in the frequency domain. It can be explained that as excitation forces \( \{P\} \) are applied on those wheels at frequency “\( \omega \),” those wheels will vibrate with those magnitudes of “\( dW \).”

### 4.1.2 Train Track Coupling Interface

Figure 23 shows the wheel rail coupling scheme. Equation 4.3 is the mathematical expression of this contact coupling scheme:

\[
-dW(\omega) - dR(\omega) - ds(\omega) = [P(\omega)]/[HK] \tag{4.3}
\]

Where: “\( dR(\omega) \)” is the downwards rail displacement in frequency domain; “\( ds(\omega) \)” is the soil surface displacement caused by combination of rail surface roughness and train speed, which is usually obtained by field measurements and is the presumed cause of the vehicle vibration.

![Figure 23. Wheel-rail coupling scheme](image)

Similarly to Equation 4.3, track deflection and wheel rail contact force can also be expressed as: \( dR(\omega) = [GT][P(\omega)] \) where “[GT]” is called “Green Function of the Track” in
this paper and will be derived later numerically. Therefore, the wheel-rail contact force can be expressed by using rail surface roughness and train speed:

\[
\{P(\omega)\} = -([HK][GV] + [HK][GT] + [I])^{-1}\{ds(\omega)\} \quad (4.4)
\]

### 4.1.3 Numerical Green Function of the Track under Moving Load

In this section, the construction of “[GT],” i.e. the Green Function of the track, is explained. It starts from the track responses under single-frequency unit moving load, which is depicted in Figure 24. The rail-tie-ballast structure is kept the same as most 2D models; however, the soil part is a three-dimensional plane-stress 3D finite element mesh.

![Figure 24. Model to obtain the numerical Green Function of the track](image)

Mathematical representations of this Sandwich track are listed below:

\[
EI\dddot{R} + m_r\dddot{R} + K_p(dR - dT) + D_p(\dddot{R} - \dddot{T}) = P_0 e^{iot} \delta(x - vt) \quad (4.5)
\]

\[
K_p(dR - dT) + D_p(\dddot{R} - \dddot{T}) - F = m_r\dddot{T} \quad (4.6)
\]

\[
K_b(dT - dS) + D_b(\dddot{T} - \dddot{S}) = F \quad (4.7)
\]

Where, “EI” is the bending rigidity of the rail beam; “dT” is the displacement of the tie; “dS” is the soil surface deflection; “Kp” and “Dp” are stiffness and damping of the rail pad respectively; and “Kb” and “Db” are stiffness and damping of the ballast respectively. “F” is the force on the top of the soil surface.
Applying Fourier Transformation from "\(x\)" - "\(\lambda\)" on Equation 4.5 yields:

\[
EI\lambda^4 dR(t, \lambda) + m_r d\dot{R}(t, \lambda) + K_p (dR(t, \lambda) - dT(t, \lambda)) + D_p (dR(t, \lambda) - dT(t, \lambda)) = P_0 e^{i(\omega - \lambda v)t} \tag{4.8}
\]

From Equation 4.8, it can be seen that rail and tie deflections have the form of:

\[
dR(t, \lambda) = \bar{dR} e^{i(\omega - \lambda v)t} \tag{4.9}
\]

\[
dT(t, \lambda) = \bar{dT} e^{i(\omega - \lambda v)t} \tag{4.10}
\]

Therefore, Equations 4.5 to 4.7 can be rewritten as:

\[
[EI\lambda^4 - m_r(\omega - \lambda v)^2 + KP] * \bar{dR} - KP * \bar{dT} = P_0 \tag{4.11}
\]

\[
KP * \bar{dR} + [m_t(\omega - \lambda v)^2 - KP] * \bar{dT} = \bar{F} \tag{4.12}
\]

\[
KB * (\bar{dT} - \bar{dS}) = \bar{F} \tag{4.13}
\]

Where, \(KP = Ko + i(\omega - \lambda v)Dp\) and \(KB = Kb + i(\omega - \lambda v)Db\).

Solving Equations 4.11 to 4.13 yields a very important relationship, which will be used in the later soil computations:

\[
\bar{dS} = \bar{F} \left(\frac{A}{B} - \frac{1}{KB}\right) - KP * \frac{P_0}{B} \tag{4.14}
\]

Where: \(A = [EI\lambda^4 - m_r(\omega - \lambda v)^2 + KP]\) and

\[
B = EI\lambda^4 * m_t * (\omega - \lambda v)^2 - m_r m_t(\omega - \lambda v)^4 - EI\lambda^4 * KP + (m_r + m_t)(\omega - \lambda v)^2 * KP
\]

\[\]

4.2 2.5D Finite Element Expression of Ground Motion

For three-dimensional soil, the 2.5D FEM technique is employed. Fourier Transform was only performed in the direction of train movement, and the transverse and vertical directions were discretized by Plane Stress Quadrilateral Finite Elements.
According to the stain-displacement relationship, the strain matrix is:

\[
[B] = \begin{bmatrix}
-i\lambda & 0 & 0 \\
0 & \frac{\partial}{\partial \xi} & 0 \\
0 & 0 & \frac{\partial}{\partial \eta} \\
\frac{\partial}{\partial \xi} & -i\lambda & 0 \\
0 & \frac{\partial}{\partial \eta} & \frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta} & 0 & -i\lambda
\end{bmatrix}
\]

In addition, according to the stress-displacement relationship, the elastic matrix is:

\[
C = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{22} & C_{23} & 0 & 0 & 0 \\
C_{33} & 0 & 0 & 0 \\
C_{44} & 0 & 0 \\
sym & C_{55} & 0 \\
C_{66}
\end{bmatrix}
\]

Where,

\[
C_{11} = E_1 (1 - v_{23} v_{32}) \gamma \\
C_{22} = E_2 (1 - v_{13} v_{31}) \gamma \\
C_{33} = E_3 (1 - v_{12} v_{21}) \gamma \\
C_{12} = E_1 (v_{21} - v_{31} v_{23}) \gamma = E_2 (v_{12} - v_{32} v_{13}) \gamma \\
C_{13} = E_1 (v_{31} - v_{21} v_{32}) \gamma = E_3 (v_{13} - v_{23} v_{12}) \gamma \\
C_{23} = E_2 (v_{32} - v_{12} v_{31}) \gamma = E_3 (v_{23} - v_{13} v_{21}) \gamma \\
C_{44} = \mu_{23} \\
C_{55} = \mu_{13} \\
C_{66} = \mu_{12}
\]

\[
\gamma = \frac{1}{1 - v_{12} v_{21} - v_{23} v_{32} - v_{31} v_{13} - 2 v_{21} v_{32} v_{13}}
\]
and where $E$ is the modulus of elasticity and $\nu$ is the Poisson’s ratio.

The adopted shape function is shown here.

\[
N = \begin{bmatrix}
N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\
0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\
0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4
\end{bmatrix}
\]

where $N_1, N_2, N_3, N_4$ are:

A quadrilateral element $N_i(\xi, \eta) = \frac{1}{4}(1 + \xi \xi_i)(1 + \eta \eta_i)$ ($i=1,2,3,4$)

\[
N_1 = \frac{1}{4}(1-\xi)(1-\eta) \quad N_2 = \frac{1}{4}(1+\xi)(1-\eta) \quad N_3 = \frac{1}{4}(1+\xi)(1+\eta) \quad N_4 = \frac{1}{4}(1-\xi)(1+\eta)
\]

Using the equations 4.15 and 4.16, the mass matrix ($M$) and stiffness matrix ($K$) can be obtained, as follows.

Mass matrix:

\[
M = \sum_\varepsilon \rho \int_{-1}^{1} \int_{-1}^{1} N^T N \left| J \right| d\xi d\eta \quad (4.15)
\]

Stiffness matrix:

\[
K = \sum_\varepsilon \int_{-1}^{1} \int_{-1}^{1} (B^T N)^T C(BN) \left| J \right| d\xi d\eta \quad (4.16)
\]

Where, 'e' represents element-wise integration.

The equivalent nodal force vector is:

\[
F^{\varepsilon} = \sum_\varepsilon \int_{-1}^{1} \int_{-1}^{1} N^T f \left| J \right| d\eta d\xi \quad (4.17)
\]

And $J$ is the Jacobi matrix:
\[ J = \begin{bmatrix}
\sum_{i=1}^{4} \frac{\partial N_i}{\partial \xi} y_i & \sum_{i=1}^{4} \frac{\partial N_i}{\partial \xi} z_i \\
\sum_{i=1}^{4} \frac{\partial N_i}{\partial \eta} y_i & \sum_{i=1}^{4} \frac{\partial N_i}{\partial \eta} z_i
\end{bmatrix} \]

\(|J|\) is the corresponding determinant, \(|J|=\text{det}J\)

Therefore, the following FEM equation is obtained:

\[
([K] - (\omega - \lambda \nu)^2 \times [M]) \times [\delta S] = [F] \tag{4.18}
\]

Combining Equations 4.14 and 4.18, one can solve the track and soil deflections under unit moving point load with any excitation frequencies, which is the numerical track Green Function, i.e. the matrix \([GT]\). By inserting this Green Function back into Equation 4.4, a fully coupled train-track dynamic interaction model is constructed. The complete construction and solving process were coded in MATLAB.
Chapter 5

Sensitivity Analysis

In this chapter, sensitivity analysis of the proposed 3D dynamic train-track interaction model is implemented. The effect of domain size and element size is studied. Then, the effects of the parameters of track and soil in this model, such as rail pad stiffness and damping, ballast stiffness and damping, soil properties (Poisson’s ratio, Young’s modulus), influence the train speed effect is examined.

5.1 Domain Size Effect

Although an absorbing boundary is applied along the sides of the domain in order to minimize the boundary reflection, it is still necessary to make sure the domain size is large enough to detect the critical speed effect. If small domain size is employed, the critical effect cannot be fully detected. Therefore, three potential domain sizes are chosen in order to obtain the best domain size for this research. As shown in Figure 25, the depth of ground and the depth of each layer remain unchanged in the three domains. The only difference is the width, which has three values: 60m, 100m, and 160m.

The domain size should be adequate to study the HSR-induced ground vibration, such as the critical speed effect. This means that the domain size is able to detect the critical effect, and that the value of critical speed generated from the best domain fits the theoretical value of critical speed. Also, it means that the size of domain is acceptable for running the computational program within a reasonable time period. For purposes of seeking the pure domain effect, the effect of element needs to be eliminated by keeping element size same for all conditions.
Meanwhile, the domain size effect is obtained under the model with the same parameters for all the track components. For the soil properties applied in this domain size study, the shear wave velocity of the subgrade soil is 83 m/s (shear modulus is about 10 MPa). As found in the references, Rayleigh waves have a speed slightly less than shear waves depending on the elastic constants of the material. The typical speed is on the order of 2 to 5 km/s. According to the relationship between shear wave velocity ($V_s$) and the Raleigh wave velocity ($V_R$) of ground (equation 3.1), $V_R$ is 78 to 81 m/s.

![Figure 25. Dimensions of half domain (the other side of domain is symmetric)](image)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Width (m)</th>
<th>Element Size (cm)</th>
<th>Initial Critical Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: 60</td>
<td>32.5</td>
<td></td>
<td>70</td>
</tr>
<tr>
<td>2: 100</td>
<td>32.5</td>
<td></td>
<td>75</td>
</tr>
<tr>
<td>3: 160</td>
<td>32.5</td>
<td></td>
<td>80</td>
</tr>
</tbody>
</table>

*Element size means the length of side of quadrilateral elements.

From the results generated by the proposed prediction model, subgrade with a width of 160m generates the best results. The three conditions in Table 4 are described for the situations
when the velocities of trains just reach the critical speed. The critical speed effect is the moment that the speed of trains reaches the ground Raleigh wave velocity, which is estimated with a value of 78-81 m/s. As can be observed in Table 4 Summary of the Domain Size Effect, only the domain with a width of 160m can reach the theoretical value of critical speed.

In addition, when train speed fully reaches the critical speed, 160m width can clearly display the dynamic response (cone-shaped displacement) without disturbance under the speed of 83m/s. The results are shown in Figure 26. Even though the other two smaller domain sizes are also able to detect the critical effect of trains, the domain with a width of 160m can best generate critical speed effect, as plotted in the figures. The cone-shaped soil displacement contours are well developed in this situation.
Figure 26. Contour of soil surface displacement under $V=83$ m/s
5.2 Element Size Effect

To study the element size effect, three different mesh conditions are applied. The previously-determined best domain size is used in this study. The ideal element size is also employed. It is common sense that the finer the mesh, the more accurate the results will be. However, finer mesh results in larger computational time. The general idea of ideal element size is when the displacement of rail surface approaches a constant value. For this purpose, three different sizes of elements are generated (51.6cm, 32.5cm, 28cm) and listed in Table 5, and the results are all obtained from the same train speed (v=5m/s).

<table>
<thead>
<tr>
<th>Condition</th>
<th>Number of Elements</th>
<th>Element Size (cm)</th>
<th>Rail Surface Displacement (mm)</th>
<th>Speed (m/s)</th>
<th>MATLAB Running Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition 1: 6000</td>
<td>51.6</td>
<td>5.6</td>
<td>5</td>
<td>5400</td>
<td></td>
</tr>
<tr>
<td>Condition 2: 15000</td>
<td>32.5</td>
<td>14.0</td>
<td>5</td>
<td>30000</td>
<td></td>
</tr>
<tr>
<td>Condition 3: 20000</td>
<td>28</td>
<td>14.1</td>
<td>5</td>
<td>54000</td>
<td></td>
</tr>
</tbody>
</table>

Fewer elements make the subgrade soil too stiff to deform; the elements with 51.6cm length of side only have 0.0056m rail surface displacement. However, 15000 elements and 20000 elements have almost the same displacement. When using 15000 elements or more in this study, the results (14mm for rail surface displacement) are almost the same. Therefore, the ideal element number is taken as 15000 for this domain size; namely, the size of element (the length of side of quadrilateral elements) is 32.5cm. The results are plotted in Figure 27.
Figure 27. Contour of soil surface displacement under $v=5$ m/s
5.3 Material Property Effects

After determination of the ideal domain size and element size for the prediction model, the parametric study is able to proceed. The approach for obtaining the effect of each material property is calculating changes in displacements of rail surface by changing the property of each rail track component and subgrade soil, including elastic modulus and Poisson’s ratio of soil, ballast stiffness and damping, railpad stiffness and damping. Table 6 below lists the parameters with original values for the track model.

Table 6 Original Values of Tested Parameters of Track Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail Pad Stiffness (MN/m)</td>
<td>90</td>
</tr>
<tr>
<td>Rail Pad Damping (MN*sec/m)</td>
<td>1</td>
</tr>
<tr>
<td>Ballast Stiffness (MN/m)</td>
<td>160</td>
</tr>
<tr>
<td>Ballast Damping (MN*sec/m)</td>
<td>30</td>
</tr>
<tr>
<td>Soil Modulus (MN/m²)</td>
<td>30</td>
</tr>
<tr>
<td>Soil Poisson's Ratio</td>
<td>0.45</td>
</tr>
</tbody>
</table>

5.3.1 Soil Properties

In order to investigate the effects of soil properties, three different values for each soil property, $E_s = 20$, $30$, $40$ MPa, $\nu = 0.25$, $0.35$, $0.45$, are adopted. For every single run, only one value is changed so that others do not take effect. The results for rail surface vertical displacements are plotted in Figure 28.
Figure 28. Effect of elastic modulus and Poisson’s ratio of soil on the rail surface displacement induced by a moving train with v=5m/s (low speed condition)

From the data plotted in Figure 28, it is observed that an increase in elastic modulus of soil can reduce the displacement of rail surface, or generally speaking, reduce the vibration level of the system. The results are reasonable, and can be explained simply that the stiffer the material, the smaller the displacement. In addition, a decrease in Poisson’s ratio results in an increase in the rail surface displacement. The explanation for this could be that lower Poisson’s ratio yields lower impact of radial stresses on vertical deformation.
In Yang & Hung’s study [22], the results of influence of Poisson’s ratio on the response fit the results from the model used in this research. Furthermore, the same conclusion can also be applied to high speed conditions. In the meantime, from Yang & Hung’s research, this conclusion also can be applied to all the positions along the cross-section of the rail track (vertical to the direction of train movement), not only to the position under the wheels.

5.3.2 Track Component Properties

For the effects of track components, different values in typical ranges for ballast stiffness and damping, and railpad stiffness and damping, are listed in Table 7. The rail vertical displacements, as well as the maximum wheel force on ballast top, are calculated and shown in Table 7.

Table 7 Changed Values of Ballast Stiffness and Damping, Railpad Stiffness and Damping

<table>
<thead>
<tr>
<th>Material Properties*</th>
<th>Ballast</th>
<th>Railpad</th>
<th>Results by Original Properties**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stiffness</td>
<td>Damping</td>
<td>Stiffness</td>
</tr>
<tr>
<td>Changed Values</td>
<td>80 MN/m</td>
<td>0.3 MN*sec/m</td>
<td>60 MN/m</td>
</tr>
<tr>
<td>Rail Surface Displacement (low speed)</td>
<td>14.0 mm</td>
<td>14.0 mm</td>
<td>14.1 mm</td>
</tr>
<tr>
<td>Wheel Force on Ballast Top (low speed)</td>
<td>10250 N/m</td>
<td>10250 N/m</td>
<td>10250 N/m</td>
</tr>
<tr>
<td>Rail Surface Displacement (critical speed)</td>
<td>25.8 mm</td>
<td>25.8 mm</td>
<td>25.8 mm</td>
</tr>
<tr>
<td>Wheel Force on Ballast Top (critical speed)</td>
<td>13000 N/m</td>
<td>13000 N/m</td>
<td>13000 N/m</td>
</tr>
</tbody>
</table>

*Units of the material properties are same as the values in Table 6.
**Original material properties are in Table 6.

For both low speed conditions (v=10 m/s) and critical speed conditions (v=78 m/s), the results show that ballast stiffness and damping and railpad stiffness and damping will not have an obvious influence on either rail surface displacement or critical speed. All the results for
displacement obtained from different conditions have a difference within 2 percent. The maximum forces on top of the ballast also remain the same values as when calculated for the original track components properties. In addition, since the values of ballast stiffness and railpad stiffness are much larger than the soil stiffness, the results change little within typical values of the track components properties.

In order to investigate how these parameters perform in the model, values which are beyond the typical values of these properties are chosen (Table 8).

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Ballast</th>
<th>Railpad</th>
<th>Results by Original Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changed Values</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rail Surface Disp.</td>
<td>14.0 mm</td>
<td>14.0 mm</td>
<td>20.0 mm</td>
</tr>
<tr>
<td>(low speed)</td>
<td></td>
<td></td>
<td>14.0 mm</td>
</tr>
<tr>
<td>Force on Ballast Top</td>
<td>10250 N/m</td>
<td>10250 N/m</td>
<td>10250 N/m</td>
</tr>
<tr>
<td>(low speed)</td>
<td></td>
<td></td>
<td>10250 N/m</td>
</tr>
<tr>
<td>Rail Surface Disp.</td>
<td>25.8 mm</td>
<td>25.8 mm</td>
<td>29.1 mm</td>
</tr>
<tr>
<td>(critical speed)</td>
<td></td>
<td></td>
<td>25.4 mm</td>
</tr>
<tr>
<td>Force on Ballast Top</td>
<td>13000 N/m</td>
<td>13000 N/m</td>
<td>13000 N/m</td>
</tr>
<tr>
<td>(critical speed)</td>
<td></td>
<td></td>
<td>13000 N/m</td>
</tr>
</tbody>
</table>

According to the results, it is shown that only railpad stiffness makes contributions to rail surface displacements. By decreasing the railpad stiffness, for both low speed condition and critical speed condition, rail surface displacement will increase. But the properties of other track components do not have an obvious influence on track performance.

5.3.3 Critical Speed Effect

In order to study the ground-borne vibrations generated by high speed rail, the velocity of a train needs to rise up to the critical speed. Critical speed, in this research, is the most important parameter. Therefore, it is necessary to investigate how the parameters in the model affect the values of critical speed. The values of material properties in Table 6 are still being used in this
analysis. Ballast stiffness and damping, railpad stiffness and damping have almost no influence on critical speed effect. The key parameters for the value of critical speed are soil properties. Therefore, the analysis is only performed on soil modulus and Poisson’s ratio. The effect of soil elastic modulus and Poisson’s ratio on critical speed is demonstrated in Table 9.

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Soil Modulus</th>
<th>Soil Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Critical Speed (m/s)</td>
<td>30 MPa</td>
<td>0.45</td>
</tr>
<tr>
<td>Changed Values</td>
<td>20 MPa</td>
<td>0.35</td>
</tr>
<tr>
<td>Initial Critical Speed (m/s)</td>
<td>70</td>
<td>85</td>
</tr>
<tr>
<td>Changed Values</td>
<td>40 MPa</td>
<td>0.25</td>
</tr>
<tr>
<td>Initial Critical Speed (m/s)</td>
<td>95</td>
<td>90</td>
</tr>
</tbody>
</table>

*Initial Critical Speed: the lowest speed when the critical speed effect occurs.

As it can be seen, by increasing soil elastic modulus, i.e. from 20 MPa to 40 MPa, the critical speed increases. On the other hand, the smaller the Poisson’s ratio, the higher the critical speed. From the literature review, the critical speed is when the speed of moving train reaches the speed of Rayleigh wave of subgrade soil. Rayleigh waves have a speed slightly less than shear waves depending on the elastic constants of the material. The typical speed is of the order of 2 to 5 km/s. Moreover, the velocity of a shear wave is controlled by the shear modulus of soil: $V_s = \sqrt{G/p}$, where $G$ is the shear modulus, $p$ is the solid’s density. Decreasing the Poisson’s ratio and increasing the soil modulus will increase the value of shear modulus. Therefore, the speed of shear wave increases, as well as the speed of Rayleigh wave. Finally, it results in a higher initial critical speed.

### 5.4 Train Speed Effect

In order to study ground-borne vibrations generated by high speed rail, the effect of the velocity of train is an important parameter that will affect vibration level, area of the influenced region and the mode of vibration. The railway track geometry and components properties in the
model remain the same as before. In order to explore how the velocity of train has an influence on vibration level, the displacements of the rail surface are calculated under different velocities of trains.

The results of rail surface vertical displacement changed with train velocities are plotted in Figure 29. When a train is running at lower speed conditions (5, 10, 20, 30, and 40 m/s), the displacements of both soil and rail surface change very little. For example, the values of rail surface displacements are 15.6, 15.6, 15.62, 15.63, 15.7 mm. However, increasing the speed of train gives rise to a significant increase in vibration level. The rail surface displacements for higher speed (50, 60, 70, 75, 78, and 80 m/s) are 15.9, 16.1, 16.8, 19.5, 22, 26 mm. The results fit the previous research very well (Figure 30) and also show again that trains will encounter a significant increase in vibration level when running at critical speed.

![Figure 29. Maximum soil and rail surface vertical displacement changed with velocities of train](image-url)
The soil surface vertical displacement contours are shown in Figure 24, which describes three speed conditions (subsonic, supersonic and transonic). The result of low speed condition with 5m/s is plotted in Figure 31(a). The critical speed effect as it starts to develop can be described as in Figure 31(b). The plot of Figure 31(c) is when the critical speed effect is fully developed in the subgrade soil.

Figure 30. Maximum soil surface displacement for different train speeds [Madshus, 2004]
Figure 31. Contour of soil surface vertical displacement under different train speeds: (a) \( v = 5 \) m/s; (b) \( v = 80 \) m/s; (c) \( v = 83 \) m/s
As shown in Figure 31, at low speed \((v=5 \text{ m/s})\), the displacement pattern is mainly symmetric. The loading type is similar to static loading and the displacement field moves with the train. The displacement pattern resembles the static response of track under static train loading. As the train speed increases to \(80\text{m/s}\), the displacement field starts to grow and propagates to the surrounding area. The displacement region still moves with the train; however, the shape is tending to be dynamic. When the train speed completely reaches critical speed which is \(83\text{m/s}\) in this condition, both the area of the influenced region and the mode of vibration become fully dynamic response. The area of influenced region is much larger than that of the low speed condition. The responses of soil no longer remain like static loading, but a cone-shaped displacement field.

### 5.5 Chapter Summary

According to the results, the best domain size, among those considered, for the model to study critical speed effect in this research would be \(160\text{m}\) in width and \(20\text{m}\) in depth. The element with \(32.5\text{ cm}\) length of side is adequate to be applied for the model. For the soil properties, by increasing soil elastic modulus and Poisson’s ratio, the rail surface displacement increases. In addition, changing the properties of track components within their typical values does not have an obvious influence on track performance. As an important parameter for the model, increasing the speed of trains could enhance the vibration level and enlarge the influenced field, particularly when train speed approaches critical speed.
Chapter 6
Verification

The purpose of verification is to test and improve the accuracy of the proposed 3D dynamic train-track interaction model (Sandwich model) by comparing the results from benchmark FEM software ABAQUS. It would require a considerable consumption of both time and money to verify the proposed model by field tests, although that would be desirable for future work. Benchmark software is standard, most widely used and reliable program packages. Hence, 3D FEM simulation will serve as a backup to the 3D dynamic train-track interaction model in order to significantly decrease the risk of this proposed research. 3D Finite Element Modeling for verification is performed with commercial FEM software (ABAQUS®). If the results of verification are comparable to the results of the proposed model, the risk of putting the proposed model into use could decrease significantly.

This current verification by ABAQUS is only the initial verification for this research. This is for a number of reasons. In order to simulate the whole system, the proposed model includes complicated train model and rail surface roughness, which are hard to simulate in ABAQUS. Also, due to the introduction of 2.5D FEM, the computational time by MATLAB is reduced significantly. However, in ABAQUS, the number of elements in 3D domain has to be thousands of times more than that in the proposed model. The computational time could be unacceptable. Therefore, the verification herein is only conducted for low speed conditions to reproduce the results obtained by the proposed model. The results from dynamic loading will not be simulated exactly. In addition, field tests should be conducted in the future to improve and further verify this model.
In the ABAQUS modeling, the domain size and element size should retain the same values when carrying out the sensitivity analysis. In addition, in order to avoid boundary condition problems, the method is to enlarge the domain of model. As long as \( L > cT/2 \), where \( c \) is the wave velocity of the infinite medium, and \( T \) is the calculation time, the artificial boundary can be ignored. Dating back to 1968, Alterman (1968) applied the method of enlarging domain size to simulate impact loads, and the results showed that the method is able to eliminate the boundary effects. For static loading, the domain with a width of 160m, which is the same as used in Chapter 5, is large enough to conduct the verification without boundary reflection.

6.1 Modeling Procedures in ABAQUS

The modeling procedures in ABAQUS, as for other finite element method software packages, can be divided into different modules. Each module serves its purpose separately. For instance, the function of PART module is to sketch the geometry of each part in the model; PROPERTY module enables users to define the material properties that will be used in the model; STEP and MESH modules allow users to choose analysis type and generate mesh, respectively. After finishing creation of the model, the JOB module will generate an input file and submit it to solvers. Finally, the post process will perform in VISUALIZATION module.

6.1.1 Elements

ABAQUS has an extensive element library to provide a powerful set of tools for solving many different problems. All elements used in ABAQUS are divided into different categories. The modeling space is divided into 3D space and 2D planar space, and so on. The available element types based on different research can be categorized as beam elements, shell elements and solid elements, etc. In this chapter, only solid elements and spring and dashpot elements are introduced.
Solid Elements

Solid elements in two and three dimensions are available in ABAQUS. The two-dimensional solid element allows modeling of plane and axisymmetric problems. In three dimensions, the isoparametric hexahedron element is the most common, but in some cases, complex geometry may require tetrahedral elements. Those elements are generally recommended to fill in awkward parts of the mesh. In order to obtain more accurate results, an 8-node hexahedron element (C3D8R) is used here (Figure 32). C3 means three dimensions; D8 means twenty nodes; R is short for reduced integration. Reduced integration can decrease time for calculation and enhance computational accuracy for contact analysis.

![Figure 32. Solid elements](image)

Spring and Dashpot Elements

Spring and dashpot elements are widely used in this model. For instance, they are utilized for the railpads between the rail and the sleeper, ballast between sleepers and subgrade soil. Three spring elements available in ABAQUS are SPRING1, SPRING2 and SPRINGA elements.
SPRING1 is applied between a node and ground, acting in a fixed direction. SPRING2 is applied between two nodes, acting in a fixed direction. The SPRINGA element is available in both ABAQUS/Standard and ABAQUS/Explicit. SPRINGA acts between two nodes, with its line of action being the line joining the two nodes. The spring behavior can be linear or nonlinear in any of the spring elements in ABAQUS. The three types of spring in ABAQUS are shown in Figure 33.

![spring elements](image)

Figure 33. Node ordering on element (manual)

### 6.1.2 Analysis Type

General static analysis is used for static loading, which here means the low speed condition. Dynamic explicit is the analysis type for 3D FEM modeling to test the train speed effect. Dynamic explicit is appropriate when the time interval is very small and easier to converge. The current analysis is clearly a short-time dynamic problem when the train is running at critical speed. So, the effect of critical speed can be accurately analyzed owing to the characteristics of this analysis type. A static analysis allows dividing load or boundary conditions into small increments and performs an incremental analysis. For a dynamic analysis, time-dependent load or displacement is defined and will be incremented in time.
General Static Analysis

General static analysis can be used to analyze linear or nonlinear response and is performed to analyze static behavior, such as deflection due to a static load. General analysis steps can be included in an ABAQUS/Standard or ABAQUS/Explicit analysis. A static step uses time increments until the calculation time ends. The default time period is 1.0 units of time, representing 100 percent of the applied load.

According to the previously-cited research and analysis, the low speed condition can be regarded as static loading. General static analysis is performed to calculate the responses generated by the static train load.

Dynamic Explicit

The explicit dynamics has been used to calculate critical speed effect. The explicit scheme appeared to be very computationally demanding and is more appropriate to use when the time interval is very small. Modeling the critical speed effect is a short-time dynamic problem (several second or less than 1 second). It is easier to achieve convergence based on an explicit scheme if the behavior of the structure is unknown. However, it is hard to simulate a comprehensive model for train model and moving load. The solution is to make a constant point load occur at different times and locations. If the time interval between the occurrences of two loads is small enough, a whole sequence of loads can be regarded as a moving load.
6.1.3 Model in ABAQUS

The model used in ABAQUS is shown in Figure 34. The rail is located on the top of railpad, which is simulated as surface-to-surface contact with spring stiffness and damping. Then the sleepers are on the ballast that is modeled as another surface contact with spring stiffness and damping. The typical geometry of tie is shown in Figure 35, and rail profile geometry is shown in Figure 36. The whole track model is placed on the subgrade soil as shown in Figure 37.

![Track model in ABAQUS](image)

Figure 34. Track model in ABAQUS
Figure 35. Typical geometry of tie

<table>
<thead>
<tr>
<th>Tie</th>
<th>Full scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tie length</td>
<td>102 inches</td>
</tr>
<tr>
<td>Tie width</td>
<td>9 inches</td>
</tr>
<tr>
<td>Tie Height</td>
<td>7 inches</td>
</tr>
<tr>
<td>Spacing</td>
<td>21 inches</td>
</tr>
<tr>
<td>Tie Gap</td>
<td>12 inches</td>
</tr>
</tbody>
</table>

Figure 36. Rail profile geometry (unit: m)
6.2 Modeling Results

6.2.1 Soil Properties

The consideration of soil property variations in ABAQUS is only performed under low speed conditions, which can be regarded as static loading condition. Figure 38 shows the soil displacement field shape in the ABAQUS results, which are the same shape as that in the results from the proposed train-track interaction model.

The displacement results obtained from ABAQUS for the low speed condition are listed in Table 10. No matter the variation tendency or the displacement, the values are comparable for the proposed 3D train-track interaction model and the ABAQUS results. In addition, according to the plots in Figure 38, the soil surface displacement contours obtained by both the proposed model and ABAQUS possess similar shapes. The largest surface displacements are under wheel load and the displacement fie does not propagate to a distant area. According to the similarity, the proposed model in MATLAB code can be reasonable.
### Table 10 Rail Surface Displacements by Changing Soil Properties (Static Loading)

<table>
<thead>
<tr>
<th>Soil Modulus (MPa)</th>
<th>Rail Surface Displacement (mm)</th>
<th>Rail Surface Displacement from Proposed Model (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>14.62</td>
<td>14.00</td>
</tr>
<tr>
<td>40</td>
<td>12.27</td>
<td>12.40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Poisson’s Ratio</th>
<th>Rail Surface Displacement (mm)</th>
<th>Rail Surface Displacement from Proposed Model (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>16.01</td>
<td>16.90</td>
</tr>
</tbody>
</table>

Figure 38. Soil surface displacement contour for low speed condition: (a) proposed model, (b) ABAQUS results
6.2.2 Track Components Properties

For static loading, the damping of railpad and ballast should not contribute to the system performance. Only the effect of stiffness of railpad and ballast are investigated in this section. For original properties, the rail surface displacement is 14.62mm. The results generated under changed values of stiffness of railpad and ballast are shown in Table 11.

Table 11 Rail Surface Displacements by Changing Stiffness of Ballast and Railpad

<table>
<thead>
<tr>
<th>Railpad Stiffness (MN/m)</th>
<th>Rail Surface Displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>14.7</td>
</tr>
<tr>
<td>Ballast Stiffness (MN/m)</td>
<td>Rail Surface Displacement (mm)</td>
</tr>
<tr>
<td>80</td>
<td>14.64</td>
</tr>
</tbody>
</table>

As the results indicate in Table 11, reducing railpad stiffness and ballast stiffness will finally cause an increase in rail surface vertical displacement. Railpad stiffness makes greater contributions to rail surface displacements. The results that fit the results from the proposed model well indicate that the proposed model is reasonable for the tested conditions.

6.2.3 Train Speed Effect

In order to investigate the critical speed effect, the solver, dynamic explicit, is used here. The moving load is simulated as a constant point load that occurs at different times and locations. If the time interval of occurrence of two adjacent loads is small enough, a whole sequence of loads can be regarded as a moving load. Here, this study is only used to conduct some qualitative analysis. The tasks in this section are just testing the variation trend of soil displacements under both low speed and critical speed conditions. Also, it needs to be confirmed that the higher soil modulus will generate higher critical speeds.
Due to the limitation of current available computing ability, the train speed effect cannot be exactly reproduced. In further research, the dynamic responses generated by high speed conditions should be obtained by higher performance workstation.

As can be seen from Figure 39 and Figure 40, a critical speed effect occurs when the train running at critical speed. Rail surface displacement is only 5 mm under the low speed condition (v=10 m/s). However, the displacement rises up to 10 mm when the velocity of the train is 100 m/s. In addition, the displacement field enlarges and propagates to the vicinity of the ground like a wave.

![Figure 39. Train speed, V=10 m/s (soil modulus = 20 MPa)](image-url)

Figure 39. Train speed, V=10 m/s (soil modulus = 20 MPa)
Figure 41 shows the displacement field when the train is running at 100 m/s. It can be observed that both the displacement area and the vibration level decrease significantly, i.e. the train-induced vibration is attenuated at a fast rate. However, as discussed previously, the resonating effect of vibration at the same speed (100 m/s) can be detected in Figure 40 when the soil Young’s modulus is 20 MPa, a significantly softer soil compared to the one presented in Figure 41. This comparison indicates that the stiffer the subgrade soil, a higher train speed is needed to create critical conditions.
6.3 Chapter Summary

In this chapter, results generated by ABAQUS model fit the results from the proposed model very well. The proposed model is initially verified for a limited set of conditions.
Chapter 7

Critical Layer Prediction

In this chapter, the location of the critical layer and the effect of depth of critical layer are investigated using the proposed model. The influence of the train on the critical layer and how ground waves propagate in the critical layer is presented.

All the other parameters in the model are kept the same as they were for the sensitivity analysis presented in Chapter 5. The subgrade soil layers are divided into four layers all with depths of 5m. Elastic modulus in each separated layer is independent and can be changed one by one. Three sets of soil properties are used in this analysis. By shifting soil layers of different properties up and down, the location of the critical layer can be predicted. Then, the depth of critical layer can also be confirmed by changing the thickness of critical layer.

7.1 Critical Effect from Different Groups of Soil

Three groups of soil with different elastic modulus, as used in this study, are listed in Table 12. All the layers have a modulus of 29 MPa in Group 1. Group 2 has a change in soil modulus of 8 MPa for the first layer. Group 3 has a stiffer soil for the lower three layers with a soil modulus of 60 MPa. Moreover, in order to further investigate which part dominates the critical speed effect, the first layers of Groups 2 and 3 (elastic modulus: 8MPa) are shifted from the first layer to the rest of layers.
Table 12 Three Groups of Soil Stiffnesses (Young’s Moduli) in this Study

<table>
<thead>
<tr>
<th>Layer</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1 Stiffness (MPa)</td>
<td>29</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Layer 2 Stiffness (MPa)</td>
<td>29</td>
<td>29</td>
<td>60</td>
</tr>
<tr>
<td>Layer 3 Stiffness (MPa)</td>
<td>29</td>
<td>29</td>
<td>60</td>
</tr>
<tr>
<td>Layer 4 Stiffness (MPa)</td>
<td>29</td>
<td>29</td>
<td>60</td>
</tr>
<tr>
<td>Corresponding Figure</td>
<td>42</td>
<td>43</td>
<td>44</td>
</tr>
</tbody>
</table>

The figures below (Figure 42, Figure 43 and Figure 44) show the surface vertical displacement plots and contours of soil surface vertical displacement generated under different soil properties. In these three figures, the two different speeds represent the moment when the critical phenomenon starts and just fully develops.

Figure 42. Rail surface vertical displacement plot and contour of soil surface vertical displacement of Group 1: (a) v=80 m/s; (b) v=82 m/s
Figure 43. Rail surface vertical displacement plot and contour of soil surface vertical displacement of Group 2 (original position of first layer): (a) $v=42$ m/s; (b) $v=44$ m/s
Figure 44. Rail surface vertical displacement plot and contour of soil surface vertical displacement of Group 3: (a) v=42 m/s; (b) v=44 m/s

7.2 Location of Critical Layer

The vibration response of the rail surface and soil surface, solved for the transition from sub-critical speeds to critical speeds, has been plotted in the previous section. As an example, Figure 42(a) describes the rail surface vibration response along the direction of train movement when the train is running at 80m/s. Two wheel loads under each bogie of the car are close to symmetric shape, and the responses of the rail surface beyond the location of the car approach to zero deformation, which is similar to static loading. In addition, the responses of vertical soil surface displacement (the red area) start to propagate to the surrounding area. However, the red areas, which are the largest displacements just under the two bogies, are still in their original shape and location. Nevertheless, the responses are thoroughly different when the velocity of
train increases to 82m/s. From Figure 42(b), the rail surface responses transform into a non-symmetric shape and are undulating, rather than zero responses beyond the location of the car. Moreover, the soil surface responses grow into a cone-shaped state and larger region of influence. When the track responses go through such a transition, the current speed of train can be regarded as the critical speed. For the sake of investigating the location of critical layer, comparison and analysis on the numerical values of critical speed of different soil properties are performed. The comparison is based on the principle of control value by changing one value each time. Through several sets of comparisons, the location of the critical layer can be confirmed.

From Figure 42 and Figure 43, the only variation of the soil properties is changing elastic modulus of the first layer from 29 MPa to 8 MPa. The critical speed significantly decreases from 82m/s to 44m/s. What can be thus surmised is that the soil layer with 8 MPa modulus contributes to this change. However, the contribution of the other three layers does not reflect in this comparison. In order to consider that, the elastic modulus of the other three layers has been changed to 60 MPa. The results are illustrated in Figure 44. The critical speed in this condition is still 44m/s. Although the rail and soil surface displacement reduce a very small amount, which is caused by the modulus change, the critical speed remains the same. Therefore, the condition at which the critical effect occurs is not changed at all. The analysis shows that the first layer with a depth of 5m is dominant for the critical speed effect.

However, it still cannot be concluded that the first layer is the unique cause for critical speed effect, because the softest layer, namely the layer with an elastic modulus of 8 MPa, is located on the top of the soil layers. When the soft stratum moves to the second stratum or lower, the results may also be influenced. For purposes of further investigating which part dominates the critical speed effect, the weak layer is shifted from the first layer to the other soil layers. The
details of the soil properties are listed in Table 13. The results generated from the 3D model are also shown in Figure 45 and Figure 46.

<table>
<thead>
<tr>
<th>Group</th>
<th>Layer</th>
<th>Group 4</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>Stiffness (MPa)</td>
<td>29</td>
<td>60</td>
</tr>
<tr>
<td>Layer 2</td>
<td>Stiffness (MPa)</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Layer 3</td>
<td>Stiffness (MPa)</td>
<td>29</td>
<td>60</td>
</tr>
<tr>
<td>Layer 4</td>
<td>Stiffness (MPa)</td>
<td>29</td>
<td>60</td>
</tr>
</tbody>
</table>

Figure 45. Rail surface vertical displacement plot and contour of soil surface vertical displacement of Group 4 (the soft layer is the second layer): \( v = 82 \text{ m/s} \)

Figure 46. Rail surface vertical displacement plot and contour of soil surface vertical displacement of Group 5 (the soft layer is the second layer): \( v = 117 \text{ m/s} \)

Figure 45 and Figure 46 only depict the conditions such that the critical phenomenon just fully develops in the model. From Figure 45, the critical speed is 82m/s, which is constant with the value in group 1. The rail surface displacement increases slightly from 3.2mm to 3.8mm.
Thus, critical speed will not be changed if the soft layer is at a lower position. The displacement will increase, which can be easily explained by the reduction in soil modulus. It is also noted that the results will also not be significantly changed by moving the soft layer to another position, such as the third layer and fourth layer. Combining the first two layers or any other combinations with 8 MPa and 29 MPa, the results will not change, and all of the conclusions above can be still applied. Figure 46 provides a further backup to confirm the conclusions. The critical speed effect is dominated by the first layer of the subgrade soil. The displacements of rail and soil are influenced slightly by the other three lower layers, but still are governed by the first layer. The critical speed of trains is only determined by the first layer of the soil.

7.3 Depth of Critical Layer

The conclusion that the surface layer is the dominant layer for critical speed effect is obtained with a five-meter surface layer in the previous section. However, the depth of the critical layer cannot be decided, because the conclusion from previous section only points out that the location of critical layer is within the surface 5 meters of subgrade. In order to explore the effect of thickness of the surface layer, three different depths of surface layers (2m, 3m, and 4m) are used in this study. If the critical speed is changed, it implies that the conditions of occurrence for critical speed are also changed. The first layer is no longer the unique layer that dominates the critical effect. The soil properties of group 3 remain as previously used. The first layer in group 3 is 5 meters with elastic modulus of 8 MPa. The results obtained from the first layer thicknesses of 2m, 3m and 4m are presented in Figure 47.
Figure 47. Rail surface vertical displacement plot and contour of soil surface vertical displacement:
(a) depth of first layer=2m; (b) depth of first layer=3m;
(c) depth of first layer=4m; (d) depth of first layer=5m
In Figure 47, the results are generated under the original soil property conditions. The critical speed is 44m/s. The other three conditions with different depths of first layer are also analyzed at the train speed of 44m/s.

It can be observed that the critical effects do not happen under all of the three soil layer conditions. Figure 47(a) has the least rail surface displacement. That can only be explained if the lower and stiffer soil layer has effect on the results. The rail surface displacement in Figure 47(b) increases, however, the displacement region of the soil surface is still in the static loading type. In Figure 47(c), some undulating responses in soil surface can be detected, but the critical speed effect is not still fully developed as in Figure 47(d). From the experimental results above, a train running on the 5m depth of first layer with a speed of 44m/s will induce critical speed effect. However, the other three depths less than 5m will not generate the same results. Combining the conclusion that the location of critical layer is within 5 m of the subgrade soil, it can be concluded that the depth of critical layer is around 5m depth from soil surface.

7.4 Chapter Summary

In this chapter, it is shown that critical speed decreases if the soil modulus reduces. In addition, the critical layer is the layer with five-meter depth from the soil ground surface. That means only the properties of the first five meters of subgrade soil determine the condition of occurrence for the critical speed effect.
Chapter 8
Conclusions and Future Study

8.1 Conclusions

Based on the results of a series of analyses using the proposed model, trains running at critical speed will induce a significant increase in vibration level on both rail track and subgrade soil. The vibration level depends on train speed and soil properties. The track components also have important effects on vibrations. Verification using benchmark FEM software ABAQUS indicates that the proposed model is reasonable under low speed conditions. However, it still needs to be further verified by field tests and computer modeling, particularly for high speed conditions.

With regards to the train speed effect, it was observed that the rail surface and soil surface vertical displacements increased with the train speed. Also, when a train is operating at a speed less than the critical speed, the vibration level increases slowly with increasing train speed. However, the vibration level increases significantly when the train speed approaches or exceed the critical speed.

With respect to the soil properties, by increasing the soil elastic modulus, the initial critical speed is increased. By reducing the Poisson’s ratio, the initial critical speed is also increased. Very soft clay could generate the critical speed effect easily, because the speed of Rayleigh wave in soft clay is much lower than that in harder clay.

Also, the location and depth of the critical layer was found to be within five meters from the soil surface. That is to say, only the properties of the first five meters of subgrade soil determine the conditions under which the critical speed effect occurs. Therefore, in order to
mitigate the influence of high speed train induced vibration, a main approach would be to treat the first five meters of the subgrade soil.

8.2 Limitations of the Current Work and Recommendations for Future Study

The current goal is to obtain overall track dynamic responses. Therefore, ballast is only modeled as spring/dashpot system in this research. The effect of ballast mass should be further studied in future work.

In the current sensitivity analysis, only one property is changed for each run. However, the combined effects of changing two or more properties should be investigated. How these parameters act together, and the relative effect in changing system responses for each parameter, should be further explored.

With regards to verification, only the low speed condition is verified in this thesis. Verification for the critical speed condition will be implemented in prospective future research. In addition, the train model is only simulated as point loading in this thesis. More forward steps are needed to simulate the train model in ABAQUS. Also, the results should be compared with field tests in order to obtain final verification for the proposed train-track interaction model.

In this study, the critical effect was found to occur with a surface layer with a depth of five meters, modeled as containing only one soil type. The effect needs to be further researched when the surface layer, with a depth of five meters, contains two or more different soil types.
References


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Appendix A

User Interface Design

In order to improve the efficiency of inputting parameters in this model, and also to facilitate others to use this model in the future, a user interface is introduced here (Figure 48).

The user interface allows users to quickly input all the parameters in this model and save the input as txt files. The files can be directly imported to MATLAB for further calculation. The user interface is coded by EXCEL VBA. It can fit to any screen with any resolution and is easy to use.

Figure 48. Sandwich model input interface
By clicking images visualized in the user interface, corresponding user forms for inputting parameters will pop out in the right of the screen. The user forms that pop out in the right of the screen are available for importing parameters for corresponding parts of the sandwich model. Then click "save" in the end of the user form. A text file would be generated automatically and saved to the same path as this executable program, no matter where the user places it.
Appendix B

ABAQUS Input File

An ABAQUS input file, as used for this thesis, is included here. The details of elements, nodes and each track component are removed.

*Heading
** Job name: Job-28 Model name: Model-1
** Generated by: Abaqus/CAE 6.11-1
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
** PARTS
**
*Part, name=rail
*Node

*Element, type=C3D8R

*Nset, nset=_PickedSet2, internal, generate
   1, 9982, 1
*Elset, elset=_PickedSet2, internal, generate
   1, 5280, 1
** Section: rail
*Solid Section, elset=_PickedSet2, material=rail

,*
*End Part
**
*Part, name=sleepers
*Node

*Element, type=C3D8R

*Nset, nset=_PickedSet2, internal, generate
   1, 198, 1
*Elset, elset=_PickedSet2, internal, generate
   1, 84, 1
** Section: sleepers
*Solid Section, elset=_PickedSet2, material=sleepers

,*
*End Part
**
*Part, name=soil
*Node

*Element, type=C3D8R

*Nset, nset=_PickedSet2, internal, generate
** Section: soil
*Solid Section, elset=_PickedSet2, material=soil

** ASSEMBLY

**
*Assembly, name=Assembly

**
*Instance, name=rail-1, part=rail
*End Instance

**
*Instance, name=sleepers-1, part=sleepers
*End Instance

**
*Instance, name=soil-1, part=soil
*End Instance

**
*Instance, name=sleepers-1-lin-1-2, part=sleepers
  0., 0., 0.53
*End Instance

**
*Nset, nset=_PickedSet1517, internal, instance=soil-1, generate
  1, 29369, 8
*Elset, elset=_PickedSet1517, internal, instance=soil-1, generate
  1, 24851, 7
*Nset, nset=_PickedSet1518, internal, instance=rail-1
13, 14, 15, 16, 17, 18, 19, 20, 45, 46, 47, 48, 49, 50, 51, 52
*Nset, nset=_PickedSet2277, internal, instance=rail-1

*Elset, elset=_PickedSet2277, internal, instance=rail-1

*Nset, nset=_PickedSet2278, internal, instance=sleepers-1
1, 2, 3, 4, 5, 6, 7, 8, 9, 190, 191, 192, 193, 194, 195, 196
197, 198

*Elset, elset=_PickedSet2278, internal, instance=sleepers-1
1, 2, 3, 4, 81, 82, 83, 84

*Surface, type=ELEMENT, name=CP-1-soil-1
_CP-1-soil-1_S4, S4
*Elset, elset=_CP-1-sleepers-1_S6, internal, instance=sleepers-1, generate
1, 83, 2
*Elset, elset=__PickedSurf1515_S5, internal, instance=rail

*Elset, elset=__PickedSurf1515_S3, internal, instance=rail

*Elset, elset=__PickedSurf1515_S4, internal, instance=rail-1, generate 3, 2236, 7

*Elset, elset=__PickedSurf1515_S6, internal, instance=rail-1, generate 4, 2230, 14

*Surface, type=ELEMENT, name=_PickedSurf1515, internal
__PickedSurf1515_S5, S5
__PickedSurf1515_S3, S3
__PickedSurf1515_S4, S4
__PickedSurf1515_S6, S6
*Elset, elset=__PickedSurf1516_S4, internal, instance=sleepers-1, generate 2, 84, 2

*Surface, type=ELEMENT, name=_PickedSurf1516, internal
__PickedSurf1516_S4, S4
*End Assembly
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** MATERIALS
**
*Material, name=rail
*Density 7800..
*Elastic 2e+11, 0.3
*Material, name=sleepers
*Density 1500..
*Elastic 2e+08, 0.3
*Material, name=soil
*Density 1500..
*Elastic 2.9e+06, 0.45
**
** INTERACTION PROPERTIES
**
*Surface Interaction, name=sleeperstorail 1..
*Surface Behavior, pressure-overclosure=LINEAR 9e+07,
*Surface Interaction, name=soiltosleepers 1..
*Surface Behavior, pressure-overclosure=LINEAR 1.6e+08,
**
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Displacement/Rotation
*Boundary
  _PickedSet2277, 1, 1
  _PickedSet2277, 3, 3
** Name: BC-3 Type: Displacement/Rotation
*Boundary
  _PickedSet2278, 1, 1
  _PickedSet2278, 3, 3
** Name: soil Type: Symmetry/Antisymmetry/Encastre
*Boundary
  _PickedSet1517, ENCASTRE
**
** INTERACTIONS
**
** Interaction: CP-1-soil-1-sleepers-1
*Contact Pair, interaction=soiltosleepers, type=SURFACE TO SURFACE
  CP-1-sleepers-1, CP-1-soil-1

** ---------------------------------------------------------------
**
** STEP: Step-1
**
*Step, name=Step-1
*Static
  1., 1., 1e-05, 1.
**
** LOADS
**
** Name: Load-1  Type: Concentrated force
*Cload
  _PickedSet1518, 2, -85000.
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step

*End