EXPERIMENTAL DETERMINATION OF THE DISTRIBUTED DYNAMIC COEFFICIENTS FOR A HYDRODYNAMIC FLUID FILM BEARING

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by

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Abstract

Most current rotor bearing analysis utilizes lumped parameter bearing coefficients to model the static and dynamic characteristics of fluid film bearings. By treating the stiffness and damping properties of the fluid film as acting upon the axial centerline of the rotor, these models are limited in their analysis to first order lateral rotor-bearing motion. The development of numerical methods that distribute the dynamic properties of the fluid film around the bearing circumference allow for higher order analysis of the motion between the bearing and rotor. Assessment of the accuracy of the numerical method used to calculate distributed dynamic fluid film bearing coefficients is performed by developing a novel hydrodynamic journal bearing test rig and experimental testing procedure capable of obtaining measured distributed dynamic coefficients over a range of bearing operating conditions.

The instrumented bearing test rig is used to measure the dynamic bearing displacement and fluid film pressure responses from application of an externally applied excitation force. Least squares solution to a system of perturbed pressure equations, populated by measured displacement and pressure responses, is used to determine the hydrodynamic stiffness and damping properties for a finite region of the bearing surface. Incremental rotation of pressure sensors embedded in the body of the test bearing allow for measurement of the fluid film circumferential pressure distribution which is used to calculate a set of experimentally determined dynamic bearing coefficients.

Distributed bearing coefficients derived from experimental measurements are compared to numerically calculated distributed coefficients as well as to lumped parameter coefficients generated from experimental and numerical methods found in the literature. Overall, the numerically calculated distributed coefficients successfully model both the circumferential distribution and the operating conditions of the experimental distributed bearing coefficient values and show reasonable correlation to results obtained through lumped parameter methods. Excitation frequency independence is validated through experimental testing over multiple frequencies, and damping cross term inequality of the numerically distributed bearing coefficients is validated by lumped coefficient analysis found in the literature. While uncertainty and variation of the test rig dimensional and operating parameters have some effect on the accuracy with which the numerical methods model the experimental results, the most significant source of dissimilarity in numerical and experimental results comes from test rig specific features not captured in the numerical methods, such as bearing surface wear and bearing-shaft misalignment.
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List of Symbols

- $B_{ij}$: Dimensionless damping coefficient, $i, j = x, y$
- $C$: Bearing radial clearance
- $D$: Journal diameter
- $F$: Force
- $F_b$: Body Forces
- $K_{ij}$: Dimensionless stiffness or spring coefficient, $i, j = x, y$
- $L$: Bearing length
- $P$: Pressure
- $R$: Journal radius, $R = D / 2$
- $Re$: Reynolds number
- $S$: Sommerfeld number
- $U$: Velocity
- $W$: Applied load
- $b_{ij}$: Damping coefficient of fluid film, $i, j = x, y$
- $e$: Journal eccentricity
- $h$: Film thickness
- $k_{ij}$: Stiffness or spring coefficient of fluid film, $i, j = x, y$
- $q$: Volumetric flow rate
- $r$: Radial bearing clearance
- $t$: Time
- $u, v, w$: Velocity components in the respective $x, y, z$ coordinates
- $x, y, z$: Vertical, horizontal, and axial coordinates
- $\Omega$: Angular velocity of journal
Journal eccentricity ratio, $\varepsilon = \varphi / C$

- $\varepsilon$: Journal eccentricity ratio, $\varepsilon = \varphi / C$
- $\varphi$: Attitude angle
- $\mu$: Lubricant absolute viscosity
- $\rho$: Lubricant density
- $\theta$: Angular coordinate
- $\tau$: Shear stress
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Chapter 1

Introduction

1.1 Motivation

Fluid film bearings have long been used in machine design to transfer loads from moving to stationary objects. The use of sliding bearings can be dated back to the eighteenth century during the early days of the Industrial Revolution with the first experimental work on hydrodynamic lubrication being performed in the mid-1800s. The low wear and vibration attenuating characteristics and the ability to support high rotation speeds make hydrodynamic bearings an attractive option for numerous applications. Current applications range from the connecting rod bearings in automobiles to three-foot diameter driveshaft bearings in high speed turbogenerators.

Industries continue to demand higher speeds and lower vibration thresholds from their products, and as they do, uses for fluid film bearings continue to arise. With the ever increasing demand for computer memory capacity, hard disk drive design continues to require greater recording density. A factor limiting increased hard drive density is the degradation in head reading accuracy caused by ball bearing vibrations. Currently being developed is a next-generation hard drive spindle using magnetic fluid bearings to improve rotational accuracy and reduce high frequency vibrations [1]. Another recent fluid bearing application is the development of rim-driven podded propulsion units for use on mid-to-large sized water crafts [2]. The propulsion units use a combination of large diameter journal (radial) and thrust (axial) bearings to mount the drive’s motor in a ring around the tips of the rotor blades. This design allows for
propulsion systems with smaller weight and size, and equal or improved efficiency when compared to current conventional hub driven propulsors.

As the number of applications utilizing fluid film bearings grows, so does the need to provide accurate methods to model their behavior. Accurate analysis of the dynamic motion and properties for larger diameter rotor-bearing applications, or those with thin walled housings and spindles, require modeling efforts to take into consideration the effects of non-uniform forcing around the circumference of the bearing. Classic bearing modeling techniques represent the bearing rotor as a single point mass located at the geometric center of the shaft. Figure 1.1 shows how the classic lumped parameter bearing coefficients are applied to a rotor bearing system. Forcing vectors acting upon the rotor are then treated as passing through the shaft and acting upon this theoretical point.

Figure 1-1. Classical application of dynamic stiffness and damping coefficients for a rotor-bearing system.
Treating the rotor as a single point mass limits dynamic analysis to purely lateral, or first order, shaft and bearing motion, thereby ignoring the potential effects of higher order modes. For numerical and analytical methods of calculating bearing stability, knowledge of only first order shaft vibrational modes is sufficient. However, if information beyond standard bearing stability analysis is required, as in modeling the vibration and radiated sound from a fluid bearing system, knowledge of the higher order circumferential modes may be required. In a related study on electric motors, Cameron, Lang, and Umans [3] show that the \(0^{th}\), \(2^{nd}\), and \(4^{th}\) order vibrational modes of the stator, illustrated in Figure 1.2, are responsible for most of the radiated acoustic noise. Current lumped parameter bearing coefficients would be unable to predict accurately the radiated noise levels from these electric motors, as they are only able to model lateral displacements of the shaft, incapable of predicting breathing \((n = 0)\) and higher order noise radiating shaft vibrations.
Figure 1-2. Circumferential vibration modes: (a) 0\textsuperscript{th} or breathing mode, (b) 1\textsuperscript{st} or lateral mode, (c) 2\textsuperscript{nd} order mode, and (d) 4\textsuperscript{th} order mode.

With a lack of methods available to represent accurately the dynamic coefficients of a fluid film bearing for use with higher order numerical analysis, current studies remain limited in their ability to predict force transfers through a fluid bearing. To address this problem, Campbell proposed and developed a distributed bearing coefficient method [4]. This numerical method couples the motion between the bearing and shaft using dynamic properties of the fluid film calculated over a series of circumferentially distributed bearing surface nodes rather than by net film properties acting on the shaft centerline. By using a finite difference approximation of the Reynolds equation, the bearing properties for each circumferentially distributed bearing node are calculated. The research contained herein attempts to experimentally validate this newly proposed method of numerically calculating distributed bearing coefficients, thereby confirming
its usage as a more accurate representation of the bearing dynamics when used in large scale system-level analytical or numerical models.

1.2 Background

By definition a bearing is a device that constrains the relative motion between two or more moving machine parts. In the case of a typical hydrodynamic plain journal bearing, a rotating shaft (rotor) is supported by a surrounding stationary sleeve (stator). At operating speeds, a thin film of lubrication allows this system to support a radial load with no direct contact between the rotor and stator, producing a low friction, low wear method of supporting angular motion. Eccentric operation of the bearing creates a convergent radial clearance between the shaft and bearing surfaces, as illustrated in Figure 1-3. Shaft rotation and viscous effects drag the lubrication fluid through this converging gap, forming a region of high pressure. The pressure generated by this action separates the rotor from the stator and provides the load carrying capacities of the bearing.
In order to design and build better rotor-bearing systems, a convention was needed to accurately describe the effects a fluid film bearing has on the kinematics of a system. The concept of using stiffness and damping coefficients to represent the dynamic response characteristics of a fluid film bearing originated in the mid-1920s with the work of Stodola and Hummel [5]. They introduced these dynamic coefficients as a means to quantify the effect oil film compliance had on their calculations of rotor critical speed. As computational and experimental tools improved, so did methods for determining bearing dynamic coefficients. Over the years, numerous methods and refinements have increased the accuracy with which both theoretical and experimental dynamic bearing coefficients are determined.
1.2.1 Theoretical Methods

In the early development of theoretical methods for calculating dynamic bearing coefficients, the only available solution of the Reynolds equation was the Sommerfeld long bearing solution. The Sommerfeld method assumes an infinitely long bearing, thereby allowing lubricant end leakage to be ignored. This simplified version of the Reynolds equation allows for a closed form solution for hydrodynamic calculations but at the expense of the accuracy with which the dynamic coefficients are calculated. Advances in computing power during the 1950s allowed for the utilization of more computationally intensive calculations, leading to the development of numerical solutions to the full Reynolds equation.

Lund and Sternlicht [6] performed a theoretical dynamic analysis of a two-bearing rotor system. Among the simplifications made to ease the complexity of calculations in this analysis were that the lubricant viscosity was assumed constant during steady state system operation and that the two-bearing system was treated as a one-dimensional system by concentrating the rotor mass to a single point located equidistant between the two bearings. The former simplification is a valid assumption under all but the most extreme operating conditions, while the latter assumes a perfectly aligned and rigid rotor as well as identical fluid bearings. The hydrodynamic fluid film force used to determine the dynamic coefficients is obtained by solving a finite difference approximation of the Reynolds equation, and numeric differentiation is used to find the coefficients from small bearing displacements.

Warner [7] presented a method for calculating the stiffness and damping coefficients by simplifying the Reynolds equation using the Ocvirk short bearing approximation. The short bearing approximation assumes circumferential lubrication flow through the bearing to be negligible, simplifying the Reynolds equation to an easily integrable form, allowing for an exact solution to be obtained. Partial differentiation of the resulting solution leads to determination of
the coefficients. This method provides efficient coefficient estimation over a wide range of operating conditions as well as the ability to assume constant lubricant viscosity. However, the assumptions made also limit the validity of this method to plain journal bearings of short length. White [8] later presented a one-dimensional finite element approach to include the effects of non-constant lubricant temperature and viscosity as well as the effect of turbulent fluid flow.

Later, Lund [9] improved upon his earlier method by including a pad assembly technique to allow for the analysis of tilting pad bearings. These bearings, as opposed to the plain interior face of a journal bearing, have multiple pivoting partial arc faces, each producing a converging fluid gap (Figure 1.4). This method superimposes the calculated fluid forces from each individual pad to determine journal eccentricity. Once the locations and angles of each pad relative to the journal are determined, the dynamic coefficients are calculated for each pad. Usually these coefficients are reduced to the standard eight coefficients (four stiffness and four damping) by assuming a pad vibration frequency. Parsell [10] further examined the frequency dependence of these synchronously reduced coefficients and showed that they generally provide adequate results for most rotor-dynamic stability analyses.
Later refining his previous methods of using numerical differentiation to obtain the coefficients, Lund [5] used a perturbation method to expand the fluid forces around an equilibrium position to determine the coefficients as true gradients. By linearizing the coefficients through this expansion of the bearing forces, small bearing displacement amplitudes must still be assumed.

For large diameter tilting pad bearings, the inertia and mass effects of the individual pad can become significant to the calculation of the dynamic bearing coefficients and the overall stability of the rotor bearing system. Rouch [11] describes an approach that incorporates these effects into Lund’s pad assembly technique by taking into account the flexibility of each pad at its pivot point. By treating the pivotal stiffness of each pad as a spring in series with the fluid film,
the net stiffness and damping of the bearing is reduced. Rouch confirms this through several numerical analyses for a typical five pad bearing.

Up to this point the dynamic bearing coefficients discussed have all been linearized, limiting the accurate usage of these coefficients to that of small bearing excitation forces and displacements. More recently, Choy, Braun, and Hu [12] and Rho and Kim [13] examined nonlinear bearing characteristics and their effects on rotor dynamics. Choy et al. compared the frequency response functions and orbit patterns of several bearing systems calculated from both linear and nonlinear equations. By simulating these bearings over various operating conditions, they concluded that dynamic responses become increasingly nonlinear at extreme operating conditions (low bearing load, high bearing load, and unstable operation) and that nonlinear analysis methods provide more accurate numerical modeling of experimental work performed at these conditions. Rho and Kim continued this work by expanding the scope of the analysis to include external disturbances and excitations applied to the bearing system. They focused on accurately determining the locations of areas of cavitation in the fluid film that have an effect on the oil film forces and dynamic coefficients. From their study Rho and Kim concluded the gap between linear and nonlinear analysis widens as excitation amplitude and frequency increases.

1.2.2 Experimental Methods

In order to substantiate the results of numerical methods and bearing performance calculations before introduction into final machinery, a method of measuring bearing dynamic coefficients and system stability is needed. Because collecting physical data from existing fluid bearing machinery is not always economical, or even practical, experimental testing facilities have been created in an effort to provide a controlled, yet flexible, means to measure dynamic bearing coefficients over a wide range of operating conditions.
The simplest method to measure the stiffness coefficients is incremental loading. By applying small changes to the applied static load during bearing operation and measuring the x and y displacements of equilibrium journal position, the bearing stiffness coefficients can be calculated by application of Hooke’s Law. Parkins [14] presents results from this method, showing coefficient values comparable with those obtained using a more complicated two-shaker dynamic excitation method.

In order to determine damping coefficients, an external force must be used to excite the rotor bearing system and induce relative motion between the rotor and bearing. The main methods of excitation are the use of unbalanced rotational forces, transient excitation, and electromagnetic vibrators. The last two are the most commonly used and are discussed in this section.

Transient excitation methods involve measuring the response of rotor bearing systems to either a force impulse or step function. Work performed by Nordmann and Schollhorn [15] used a rigid rotor supported by two fixed journal bearings. An impact hammer of known mass is used to strike the rotating shaft. The resulting decaying displacement response is then recorded in the horizontal and vertical directions at each bearing. Because the impulsive force can be thought of as the summation of multiple single frequency signals, the displacement of the shaft can be considered a response to these multiple signals. The individual bearing coefficients for each frequency are then found by Fourier analysis.

Jiang et al. [16] attempted to measure the dynamic coefficients of a large diameter journal bearing using the methods presented by Nordmann. An existing 480 mm steady state testing rig was modified with electromagnetic exciters and displacement sensors to add dynamic measurement capabilities. Impulse forces were exerted on the shaft in two separate directions and the displacement responses recorded. After vibrations caused by the unbalanced response of the shaft were removed from the data, the stiffness and damping coefficients were calculated
using a least squares algorithm. Comparison of the results to theoretical values showed direct
stiffness coefficients on the same order of magnitude, but cross stiffness and all damping terms
were smaller than predicted theoretical values by several orders of magnitude. The main sources
of error were identified as the following: sensitivity in the least square method to the unbalanced
response and the inability to completely remove this response from the data signal, inaccuracy in
measurement of the force amplitude, and lack of stiffness of the test rig.

Another method using a similar approach is described by Goodwin [17]. Instead of using
an impact hammer to excite an impulse force, a calibrated link is used to induce a step function
input. The link is attached to the rotating shaft and a steadily increasing load is applied. The link
is calibrated to “break” at a known load, instantaneously releasing the applied load and inducing
free damped vibrations in the shaft.

The most common, and arguably the most flexible, method of excitation is by means of
an electromagnetic shaker. The typical testing apparatus consists of a floating test bearing
centered between a pair of fixed support bearings. A static load and dynamic excitation force is
introduced to the test bearing, and displacement measurements are recorded. Excitation is
required in both the horizontal and vertical directions. This excitation can be performed using
either two separate single shaker measurements or simultaneous excitation in both directions with
two shakers. The single shaker method requires more involved mathematics to calculate the
coefficients, while the dual shaker method requires a more complicated experimental procedure
of adjusting the phasing between the two shakers.

Morton [18] measured the dynamic coefficients of a large diameter test bearing and
compared experimental results to those calculated theoretically. Testing was conducted at
rotation speeds low enough to ensure laminar lubrication flow, allowing oil inertia and turbulence
effects to be ignored. Sinusoidal dynamic bearing excitation was provided via a single
electrohydraulic vibrator that could be positioned to supply either vertical or horizontal forcing.
The use of an external excitation force allowed for the investigation of dynamic bearing characteristics at frequencies non-synchronous to that of the shaft rotation speed. Experimental results, when compared to his theoretical results, showed an underestimation of the bearing stiffness and damping at high eccentricities and an overestimation at lower eccentricities. Morton hypothesized that air being drawn into the oil film could have caused this discrepancy.

Further work on large diameter bearings was performed by Ha and Yang [19], whose research involved an experimental investigation of the effects of excitation frequency on the dynamic coefficients. Ha and Yang noted that the effects of such parameters as lubrication turbulence, viscosity variation, and temperature on the dynamic coefficients could be quite substantial in large fluid film bearings and therefore lead to difficulties in analytical predictions. In an effort to produce data for the calibration of a numerical prediction model, they constructed an experimental setup consisting of a 300.91 mm diameter five-pad test bearing to be tested under a variety of operating conditions. Static and dynamic forces were applied to the floating bearing housing via hydraulic shakers. The excitation frequency, static load, and shaft rotation speed were varied throughout the experiment, and dynamic coefficients were calculated from recorded bearing displacements. They observed that as the excitation frequency ratio increased, stiffness coefficients decreased and damping coefficients increased. Also noted was the decrease in both the stiffness and damping coefficients in the direction of bearing load with an increase in shaft rotational speed (or decrease in bearing load) while the transverse coefficients remained more or less unchanged.

1.3 Scope of Research

The majority of current rotor bearing analysis uses lumped parameter methods when determining dynamic bearing coefficients. These lumped parameter methods calculate dynamic
bearing coefficients as acting upon on the shaft’s axial centerline. It has been shown that these methods can provide accurate results for systems with lower frequency excitation and smaller shaft diameters. For large diameter systems, however, the vibrational modes around the circumference of the shaft and housing play a larger role in the dynamic force exchange, resulting in inaccuracies if not properly accounted for. Campbell’s work provides a numerical method for calculating dynamic bearing coefficients that allows for accurate analysis of the dynamic coupling of higher order rotor and stator modes by distributing the coefficients around the circumference of the bearing surface.

The objective of the research set forth in this dissertation is to assess experimentally the accuracy of Campbell’s distributed dynamic bearing coefficient method by comparing results obtained from this numerical method to ones measured from a large diameter test bearing. Further measurements will also be taken to calculate the classical lumped parameter coefficients, and these results compared to predicted values.

The first step in this research is to design and construct a large diameter journal bearing test bed capable of measuring both lumped and distributed dynamic bearing coefficients. The test bearing consists of a 101.6 mm diameter, 50.8 mm long, plain journal bearing with a 0.249 mm nominal radial clearance. Static loading to the bearing is provided by hanging mass. The rotor, supported by two precision angular ball bearings, is spun by a 1.5 kW AC drive motor. Complete specifications and drawings for the test bearing and bed are found in the appendix.

Secondly, lumped and distributed dynamic bearing coefficients are obtained from measurements collected from the test rig. Lumped dynamic coefficients are determined by using two separate experimental methods found in the literature. One method calculates stiffness coefficients from steady state displacement in response to changes in the applied static bearing load. The other method calculates stiffness and damping coefficients from small elliptical bearing displacement orbits about a point of static equilibrium produced from known harmonic
excitation forces. Distributed coefficients are obtained by integrating directional pressure coefficients, generated from perturbated film pressure responses to bearing motion, over finite sections of the bearing surface area. For both the lumped and distributed methods, testing is performed for multiple excitation frequencies and bearing static loads to determine dynamic coefficients over a range of operating conditions.

Finally, numerical distributed coefficient results are calculated for each experimental bearing operating condition. Comparison between experimentally obtained distributed coefficients and those calculated numerically is used to confirm the capability of the distributed coefficient numerical method to accurately model the hydrodynamic forces acting around the fluid film bearing circumference. Experimental and numerical result discrepancy between distributed methods is evaluated against experimental and numerical lumped method results to identify potential sources of measurement and methodology error.

Experimental validation of the numerical method used to produce the distributed dynamic bearing coefficients allows the numerical method to be used with confidence in future rotor-bearing analysis, adding the capability of modeling higher order shaft and bearing vibrational modes. For this purpose, a novel fluid film bearing test rig and experimental method capable of measuring the distribution of hydrodynamic fluid film forces around the test bearing circumference is developed.
Chapter 2

Numerical Bearing Coefficients

Numerical bearing coefficients are typically obtained by solving for the induced fluid film pressure field created in the converging clearance gap that occurs between the journal and bearing surfaces. Integration of this pressure field over the entire film circumference generates the classical one dimensional, or lumped parameter, dynamic bearing coefficients. By stipulating incremental integration limits around the fluid film surface area, these lumped bearing coefficients can be distributed around the bearing circumference, thereby allowing for the characterization of fluid film stiffness and damping properties as a function of angular position. This section presents the derivation of the distributed dynamic bearing coefficients from the governing equations of fluid mechanics.

2.1 Introduction

Numerical calculation of hydrodynamic bearing impedance properties requires knowledge of the distribution of the fluid film high pressure region generated by the relative velocities of the bearing and shaft surfaces. The equations used to describe the dynamics of the fluid within the bearing are derived from simplification to the general fluid mechanic equations governing the conservation of mass, momentum, and energy. For hydrodynamic lubrication the Reynolds equation, based on the work of Osborne Reynolds, is the governing equation for thin film lubrication. Solution of the Reynolds equation provides a fluid film pressure distribution from which all other bearing characteristic parameters can be calculated [20].
Knowledge of the film pressure distribution over the bearing surface can be used to calculate the reactionary bearing forces from small displacement bearing motion. Relation of the reactionary forces to the changes in film pressure and bearing displacement are used to develop the dynamic stiffness and damping properties of the bearing fluid film.

2.2 General Fluid Mechanics and the Reynolds Equation

Complete line by line derivations of the Reynolds equation from first principles can be found in many fluid mechanics texts [20, 21]. This section provides a review of the general derivation architecture, highlighting the major principles and assumptions.

The principles governing the fundamentals of fluid mechanics and viscous fluid flow are the conservation laws of mass (continuity equation), momentum (Navier-Stokes equations), and energy (energy equation). Application of these equations, with the proper assumptions, to general fluid film geometries is used to derive equations for bearing fluid film flow rates and pressure distributions.

The Navier-Stokes equations, shown below in Equations 2.1 through 2.3 for Cartesian coordinates, are the general equations of motion governing the conservation of momentum for a Newtonian fluid.

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \left( \frac{\partial}{\partial x} \tau_{xx} + \frac{\partial}{\partial y} \tau_{yx} + \frac{\partial}{\partial z} \tau_{zx} \right) - \frac{\partial p}{\partial x} + F_{Bx} \tag{2.1}
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \left( \frac{\partial}{\partial x} \tau_{xy} + \frac{\partial}{\partial y} \tau_{yx} + \frac{\partial}{\partial z} \tau_{zy} \right) - \frac{\partial p}{\partial y} + F_{By} \tag{2.2}
\]

\[
\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \left( \frac{\partial}{\partial x} \tau_{xz} + \frac{\partial}{\partial y} \tau_{yz} + \frac{\partial}{\partial z} \tau_{zz} \right) - \frac{\partial p}{\partial z} + F_{Bz} \tag{2.3}
\]
The general equation for conservation of mass for a compressible fluid in Cartesian coordinates is:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0.
\] (2.4)

Applying Equations 2.1 through 2.3 to the sliding bearing geometry of Figure 2-1 and employing the following assumptions:

- Negligible fluid inertia and body forces compared to viscous effects
- Negligible pressure variation across the fluid film
- Laminar fluid flow
- Negligible curvature effects
- Constant fluid density and viscosity

the equations for the volumetric flow rates can be derived:

\[
\frac{\partial P}{\partial x} = \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right),
\] (2.5)

\[
\frac{\partial P}{\partial y} = \frac{\partial}{\partial z} \left( \mu \frac{\partial v}{\partial z} \right), \text{ and}
\] (2.6)

\[
\frac{\partial P}{\partial z} = 0^*.
\] (2.7)

* While the Reynolds equation is valid in all three directions, the assumption of negligible pressure variation across the fluid film thickness reduces pressure to a function of only the \(x\) and \(y\) directions.
Integrating the partial differential Equations 2.5 and 2.6 twice with respect to the film thickness and evaluating the constants of integration for non-zero bearing and shaft sliding motion in the radial and circumferential directions produces expressions for the $x$ and $y$ direction fluid velocity distributions:

$$u = \frac{1}{2\mu} \frac{\partial P}{\partial x} (z^2 - zh) + \left(1 - \frac{z}{h}\right) U_a + \frac{z}{h} U_b \quad \text{and}$$

$$v = \frac{1}{2\mu} \frac{\partial P}{\partial y} (z^2 - zh).$$

The bearing volumetric flow rates of Equations 2.10 and 2.11 are obtained by integration of the fluid velocity distributions across the film thickness:

$$q_x = -\frac{h^3}{12\mu} \frac{\partial P}{\partial x} + \frac{U_a + U_b}{2} h \quad \text{and}$$

$$q_y = -\frac{h^3}{12\mu} \frac{\partial P}{\partial y}.$$

Figure 2-1. Shaft and bearing sliding surfaces separated by arbitrary fluid film gap $h$. 
Integrating the continuity equation (Equation 2.4) over the film thickness and combining with Equations 2.10 and 2.11 produces a general expression of the Reynolds equation:

\[
\frac{\partial}{\partial x} \left( \frac{\rho h^3}{12\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h^3}{12\mu} \frac{\partial P}{\partial y} \right) = \text{Poiseulle terms: net flow rate due to pressure gradients.}
\]

\[
\frac{1}{2} \frac{\partial}{\partial x} \left[ \rho (U_a + U_b) \right]h = \text{Couette term: net flow rate due to shear.}
\]

\[
\frac{1}{2} \rho h \frac{\partial}{\partial x} (U_a + U_b) + \frac{1}{2} \rho (U_a + U_b) \frac{\partial h}{\partial x} + \frac{1}{2} (U_a + U_b) h \frac{\partial P}{\partial x} = \text{Physical stretch.}
\]

\[
\frac{1}{2} \rho h^3 \frac{\partial}{\partial x} \left( \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{h^3}{12\mu} \frac{\partial P}{\partial y} \right) = \frac{1}{2} U \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t}. \tag{2.12}
\]

Physical interpretation for each of these terms is provided by Khonsari [20], but for most practical applications the stretch, density wedge, and local expansion terms can be neglected, reducing the general Reynolds expression to the more commonly seen form:

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{12\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{h^3}{12\mu} \frac{\partial P}{\partial y} \right) = \frac{1}{2} U \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t}. \tag{2.13}
\]

In polar coordinates the equation takes the form:

\[
\frac{1}{R} \frac{\partial}{\partial \theta} \left( \frac{h^3}{12\mu} \frac{1}{R} \frac{\partial P}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12\mu} \frac{\partial P}{\partial z} \right) = \frac{1}{2} U \frac{\partial h}{\partial \theta} + \frac{\partial h}{\partial t}. \tag{2.14}
\]

The above Reynolds equation derivation makes the assumption of laminar fluid flow and ideal no-slip boundary conditions, which for the majority of standard rotor-bearing applications is valid. Further refinement of bearing prediction capabilities has been performed to extend the
Reynolds equation to account for the effects of turbulent fluid flows [22], bearing and shaft surface roughness [23], and thermal variation [24].

2.3 Solution of the Reynolds Equation

Solving the Reynolds equation allows determination of the hydrodynamic pressure distribution within the fluid film bearing. Closed form analytic solutions can be found for the limiting cases of infinitely long and infinitely short bearings. For bearings with lengths significantly smaller than their diameter, fluid flow in the direction of shaft rotation is negligible and therefore has little influence on the film pressure. The infinitely short approximation, proposed by Ocvirk [25], applicable to bearings with lengths significantly smaller than their diameter, neglects the $x$ terms of Equation 2.14, leaving a one dimensional differential equation easily integrable by analytical methods. In similar fashion, the infinitely long approximation, or Sommerfeld solution, for bearings with lengths significantly larger than their diameter, neglects the $y$ terms of Equation 2.14 as fluid flow in the direction of rotation dominates that of flow in the axial direction.

For finite length bearings, defined as bearings with length-to-width ratios between 0.5 and 2 [20], where the infinitely long and infinitely short bearing approximations are unsatisfactory, numerical methods are required to find a full solution of the Reynolds equation. In addition, numerical solutions can be used to calculate solutions to partial arc bearings, found individually in many industrial applications or in series as part of a tilting pad bearing.

Accurate solution of the Reynolds equation requires boundary condition selections that appropriately characterize the behavior of the fluid film. Conditions relating to bearing end boundaries and the presence of oil supply grooves are well established. More difficult, and still under debate, are the selection of boundary conditions that apply to film rupture (cavitation)
within the divergent portion of the fluid film. For numerical applications the Swift-Steber (or Reynolds) boundary condition is currently used widely. It specifies a condition of zero pressure gradient at the location of film rupture:

\[ \frac{\partial p}{\partial \theta} = 0, \]  

(2.16)

defining a smooth pressure distribution at the cavitation boundary and producing solutions to the Reynolds equations with fairly reasonable agreement to experimentally measured data [26].

2.4 Dynamic Coefficients

The hydrodynamic forces produced by the induced fluid film dictate the bearing dynamic operating parameters. For the coordinate system shown in Figure 2-2 the horizontal and vertical reactionary forces may be expressed by integrating the fluid pressure over the bearing surface area:

\[
\begin{align*}
F_x & = - \frac{L}{2} \int_{\theta_1}^{\theta_2} p \left\{ \cos \theta \right\} Rd \theta dz, \\
F_y & = - \frac{L}{2} \int_{\theta_1}^{\theta_2} p \left\{ \sin \theta \right\} Rd \theta dz,
\end{align*}
\]  

(2.17)

where \( p \) is the fluid film pressure distribution, \( R \) is the shaft radius, and the limits of integration are the axial (\( L \)) and circumferential (\( \theta \)) bearing surface area boundaries over which the force is to be calculated [4, 5].
Figure 2-2. Plain journal bearing coordinate system.

The analysis is simplified by assuming a rigid shaft, effectively ignoring effects of radial shaft translation on the hydrodynamic bearing properties, making these reactionary forces functions of bearing center position and displacement only. Lund presents the development of lumped parameter spring and damping coefficients for journal bearings [27]. Analytical representations of the reactionary bearing forces are simplified by examining small amplitude bearing center motion about a point of static equilibrium. A first order Taylor series expansion is
used to define the horizontal and vertical forces as linear functions of instantaneous bearing center displacements \((x, y)\) and velocities \((\dot{x}, \dot{y})\):

\[
F_x = F_{x_0} + \left( \frac{\partial F_x}{\partial x} \right)_0 \Delta x + \left( \frac{\partial F_x}{\partial y} \right)_0 \Delta y + \left( \frac{\partial F_x}{\partial \dot{x}} \right)_0 \dot{x} + \left( \frac{\partial F_x}{\partial \dot{y}} \right)_0 \dot{y}, \quad \text{and}
\]

\[
F_y = F_{y_0} + \left( \frac{\partial F_y}{\partial x} \right)_0 \Delta x + \left( \frac{\partial F_y}{\partial y} \right)_0 \Delta y + \left( \frac{\partial F_y}{\partial \dot{x}} \right)_0 \dot{x} + \left( \frac{\partial F_y}{\partial \dot{y}} \right)_0 \dot{y}.
\]

The dynamic stiffness and damping coefficients are defined by the partial derivatives evaluated at static equilibrium:

\[
k_{xx} = \left( \frac{\partial F_x}{\partial x} \right)_0, \quad b_{xx} = \left( \frac{\partial F_x}{\partial \dot{x}} \right)_0 \]

\[
k_{xy} = \left( \frac{\partial F_x}{\partial y} \right)_0, \quad b_{xy} = \left( \frac{\partial F_x}{\partial \dot{y}} \right)_0
\]

\[
k_{yx} = \left( \frac{\partial F_y}{\partial x} \right)_0, \quad b_{yx} = \left( \frac{\partial F_y}{\partial \dot{x}} \right)_0
\]

\[
k_{yy} = \left( \frac{\partial F_y}{\partial y} \right)_0, \quad b_{yy} = \left( \frac{\partial F_y}{\partial \dot{y}} \right)_0
\]

Perturbation analysis is employed to solve for these coefficients. By perturbing the fluid film, the film thickness variable \(h\) defined in the derivation of the Reynolds equation, may be rewritten as:

\[
h = h_0 + \Delta h,
\]

where the individual terms of the perturbated film thickness are defined in relation to the bearing geometry and coordinate system shown in Figure 2-2:

\[
h_0 = C + x_0 \cos \theta + y_0 \sin \theta,
\]

\[
\Delta h = \Delta x \cos \theta + \Delta y \sin \theta, \quad \text{and}
\]

\[
\frac{\partial h}{\partial t} = \Delta \dot{x} \cos \theta + \Delta \dot{y} \sin \theta.
\]
Perturbation of the film thickness leads to a perturbed film pressure expression

\[ p = p_0 + \Delta p, \]  

(2.25)

where the static pressure is defined by \( p_0 \) and the fluctuating pressure by

\[ \Delta p = p_x \Delta x + p_y \Delta y + p'_x \Delta \dot{x} + p'_y \Delta \dot{y}. \]  

(2.26)

Substitution of Equations 2.21 through 2.26 into the Reynolds equation (Equation 2.15) and retaining only first order terms produces:

\[ R\{ p_0 \} = \frac{1}{2} \Omega \frac{\partial h_0}{\partial \theta} = \frac{1}{2} \Omega \frac{\partial h_0}{\partial \theta} (-x_0 \sin \theta + y_0 \cos \theta), \]  

(2.27)

\[ R\{ p_x \} = -3 \frac{\cos \theta}{h_0} \frac{1}{2} \Omega \frac{\partial h_0}{\partial \theta} - 3 \left( \frac{h_0^3}{12 \mu} \frac{\partial p_0}{\partial \vartheta} \right) \frac{\cos \theta}{h_0} - \frac{\Omega}{2} \sin \theta, \]  

(2.28)

\[ R\{ p_y \} = -3 \frac{\sin \theta}{h_0} \frac{1}{2} \Omega \frac{\partial h_0}{\partial \theta} - 3 \left( \frac{h_0^3}{12 \mu} \frac{\partial p_0}{\partial \vartheta} \right) \frac{\sin \theta}{h_0} - \frac{\Omega}{2} \cos \theta, \]  

(2.29)

\[ R\{ p'_x \} = \cos \theta, \]  

(2.30)

\[ R\{ p'_y \} = \sin \theta, \]  

(2.31)

where the operator on the left hand side of each equation is defined as:

\[ R\{ \} = \frac{\partial \left( \frac{h_0^3}{12 \mu R \vartheta \vartheta} \right)}{\partial \vartheta} + \frac{\partial \left( \frac{h_0^3}{12 \mu \vartheta \vartheta} \right)}{\partial \vartheta}. \]  

(2.32)

Expressions for the eight stiffness and damping coefficients as functions of the directional pressure coefficients (\( p_x, p_y, p'_x, \) and \( p'_y \)) are found by combining Equations 2.17, 2.18, 2.19, 2.20, and 2.26:

\[ k_{xx} = -\frac{1}{2} \int_{\theta_0}^{\theta_1} p_x \left\{ \frac{\cos \theta}{\sin \theta} \right\} Rd\vartheta dz, \]  

(2.33)

\[ k_{yy} = -\frac{1}{2} \int_{\theta_0}^{\theta_1} p_y \left\{ \frac{\cos \theta}{\sin \theta} \right\} Rd\vartheta dz, \]  

(2.34)
By setting the limits of integration for Equations 2.33 through 2.36 to encompass the entire fluid film surface area, the traditional lumped parameter bearing coefficients, illustrated in Figure 1-1, are obtained. Campbell [4] extends this method for incorporation with finite element analysis by setting the limits of integration to correspond to the locations of individual finite element model nodes, as illustrated in Figure 2-3, thereby creating a distribution of the stiffness and damping properties positioned around the bearing circumference, enabling higher order structural analysis of the shaft/bearing interface. Campbell goes on to apply this distribution method to tilting pad bearings and provides a case study comparing accelerance transfer functions computed from both distributed coefficient derived models and conventional synchronously reduced coefficient derived models. Presented results show significant differences between computed bearing transfer accelerances, resonance frequencies, and damping levels calculated from each model type, demonstrating the importance of appropriately distributing the fluid film stiffness and damping properties for higher order rotor-dynamic models.
Figure 2-3. Circumferential shaft and bearing nodes between which distributed dynamic bearing coefficients are calculated.
Chapter 3

Determination of Experimentally Derived Bearing Coefficients

This chapter explains the experimental methods used to obtain the distributed dynamic bearing coefficients presented in Chapter 4. The general design approach of the experimental test apparatus is discussed and the instrumentation and testing methodology is described. Finally, the test validation and troubleshooting procedures performed on the final test rig design are reviewed. Detailed engineering specifications for the constructed test rig can be found in the appendix.

3.1 Test Apparatus Overview

A comprehensive comparison of numerically derived and experimentally derived distributed dynamic bearing coefficients requires an experimental test rig capable of producing and measuring bearing and fluid film dynamic responses over a wide range of bearing operating conditions. The fluid film bearing test rig developed for this research incorporates the typical design considerations required of any floating bearing test setup while implementing a novel measurement method capability that allows for the experimental determination of circumferentially distributed dynamic bearing coefficients.

3.1.1 Test Rig

Experimental bearing test rigs used to measure bearing coefficients typically fall into one of two basic arrangements: fixed bearing-free shaft designs and free bearing-fixed shaft designs. Fixed bearing-free shaft designs represent the configurations found in most conventional rotating
machinery. Here, hard mounted bearings are used to support a shaft that is free to rotate as well as translate radially. Free bearing-fixed shaft designs represent the inverse condition, utilizing a rotating radially fixed shaft and a free, or floating, bearing that is restricted in rotation but free to radially translate. Examples of each design configuration are shown in Figures 3-1 and 3-2.

It can be shown analytically that dynamic coefficients derived from each of these configuration methods are equivalent [28]. However, for laboratory testing, free bearing-fixed shaft designs provide a less complex design approach as only a single non-rotating test bearing is required. Furthermore, by separating the rotating and translating components of the test rig, the application of static and dynamic loading is greatly simplified. For these reasons, the free bearing-fixed shaft configuration is used for this research.

Figure 3-1. Schematic diagram of a fixed bearing-free shaft fluid film bearing test rig design.
Steps taken to ensure minimal transverse shaft motion include the use of large mass support bearing housings (Figure 3-3), radially stiff support bearings (Figure 3-4), and precision shaft machining and balancing operations. The fundamental frequencies of the shaft and shaft supports are designed to occur several orders of magnitude above that of the shaft rotation and dynamic loading operating frequencies. Precision duplex angular contact ball bearings are selected for the support housings due to their angular rigidity and ability to restrict shaft deflection from static and dynamic loading applied to the test bearing housing.
Figure 3-3. Test shaft supported at either end by large mass support bearing housings.
Figure 3-4. Fixed-end support bearing housing with angular contact duplex ball bearings exposed.

The floating fluid film test bearing and accompanying housing is positioned directly between the support bearings (Figure 3-5). The bearing and housing components are designed to minimize external forces and constraints applied to the bearing, thereby isolating external loading on the bearing housing to the calibrated input forces of the static and dynamic loading systems. Rigid connections to the test bearing are avoided. When connections to the bearing are required, as in the case of instrumentation wiring and lubrication hosing (Figure 3-6), these connections are made as light and flexible as possible to avoid adding unwanted stiffness and damping constraints to the system.

The floating fluid bearing is the preferred setup for the measurement of dynamic fluid film bearing properties. The nature of this design, however, introduces the potential issue of lateral bearing instability. While the induced hydrodynamic fluid film of plain journal bearings is quite stiff in the radial direction, it is extremely compliant in the axial direction, leaving an
unconstrained bearing susceptible to lateral pitch instability at low eccentricity operating conditions. This issue is discussed in further detail in Section 3.4.2.

Figure 3-5. Floating test journal bearing mounted between ball bearing support housings.
Figure 3-6. Bearing test rig utilizing minimally constraining measurement instrumentation and lubrication supply connections.
3.1.2 Measurements and Instrumentation

The objective of the method developed in this research is to experimentally obtain distributed dynamic bearing coefficients by measuring the fluid film pressure fluctuations induced by a known bearing displacement. Therefore, the test bearing and housing must be instrumented in such a way to measure both the dynamic film pressure around the bearing circumference and the relative motion between the bearing and shaft. Knowledge of dynamic loading frequency and magnitude, shaft angular frequency, lubrication temperature, and lubrication inlet pressure are also important for quantifying the operating condition of the bearing during data acquisition, allowing for a direct comparison to numerically calculated bearing coefficients. In addition, whenever possible, measures are taken to minimize and quantify the effects that the required instrumentation could have on the fluid film dynamics and general rig operation.

Inductive proximity sensors mounted to the exterior of the bearing housing are used to measure the relative motion of the test bearing (Figure 3-7). These sensors are mounted in orthogonal pairs on each side of the test bearing housing (Figure 3-8) to measure the vertical and horizontal relative displacement between the bearing and shaft. Mounting orthogonal pairs of sensors on either side of the test bearing housing allows for measurement of axial bearing misalignment and conical bearing motion during operation. The non-contact attribute of inductive sensors allows for the measurement of relative bearing motion without the additional constraint of a physical shaft-bearing connection. By mounting the sensors outside the bearing housing, measurements can be collected without disrupting the fluid flow within the induced hydrodynamic film.

Strain gauge pressure transducers mounted within the body of the test bearing measure the pressure distribution of the hydrodynamic fluid film via small diameter pressure taps. The
sensors are aligned axially along the face of the bearing at a fixed angular position, as shown in Figure 3-9. Incremental rotation of the test bearing within the housing allows measurement of the axial pressure distributed at multiple angular positions to create a complete map of the fluid film pressure over the surface of the bearing face (Figure 3-10). The small diameter pressure taps are used to measure the fluctuating film pressure with both minimal disruption to the load bearing region of the fluid film and minimal effect on the sensor dynamic measurement performance. Pressure losses and phase lags in the measurements, introduced by the small diameter tap and sensor cavity (Figure 3-11), are modeled as a second order spring-mass system. Based on the geometry of the cavity, a natural frequency of 15.9 kHz is calculated. As the natural frequency of the cavity occurs several orders of magnitude above that of the test rig maximum shaft rotation and shaker excitation frequencies, signal attenuation and phase lag of the measured dynamic pressure can be assumed negligible.

Further discussion on test rig and instrumentation specifications used to collect the operating conditions during experimental data collection are found in the appendix. With the exception of the lubrication inlet pressure and outlet temperature, which are monitored and recorded at regular intervals throughout the testing, all measurements are recorded continuously during rig operation at a sample rate of 20 kHz by a National Instruments PXI-1000B data acquisition chassis.
Figure 3-7. Non-contact inductive displacement sensor positioned to measure relative displacement between bearing housing and shaft surface.
Figure 3-8. Orthogonally positioned displacement sensors mounted to either side of test bearing housing.
Figure 3-9. Axially aligned pressure sensors mounted in test bearing.
Figure 3-10. Schematic diagram of lubrication inlet and pressure sensor angular orientations with incremental rotation of test bearing housing.
3.2 Experimental Method

This section describes the experimental methods, assumptions, and data processing techniques used to extract distributed bearing stiffness and damping coefficients from the
measured pressure and displacement data collected by the bearing test rig as described in Section 3.1.

3.2.1 Overview

The previous chapter set forth a method for numerically calculating dynamic bearing coefficients by integration of directional fluid film pressures obtained from a finite difference approximation of the Reynolds equation:

\[
\begin{align*}
    k_{xx} &= -\int_{-\theta}^{\theta} p_x \left\{ \cos \theta \right\} Rd\theta dz, \\
    k_{yy} &= -\int_{-\theta}^{\theta} p_y \left\{ \sin \theta \right\} Rd\theta dz, \\
    k_{xy} &= -\int_{-\theta}^{\theta} p_y \left\{ \cos \theta \right\} Rd\theta dz, \\
    k_{yx} &= -\int_{-\theta}^{\theta} p_x \left\{ \sin \theta \right\} Rd\theta dz, \\
    b_{xx} &= -\int_{-\theta}^{\theta} p'_{x} \left\{ \cos \theta \right\} Rd\theta dz, \text{ and} \\
    b_{yy} &= -\int_{-\theta}^{\theta} p'_{y} \left\{ \sin \theta \right\} Rd\theta dz, \\
    b_{xy} &= -\int_{-\theta}^{\theta} p'_{y} \left\{ \cos \theta \right\} Rd\theta dz, \\
    b_{yx} &= -\int_{-\theta}^{\theta} p'_{x} \left\{ \sin \theta \right\} Rd\theta dz,
\end{align*}
\]

where \( p_x \), \( p_y \), \( p'_x \), and \( p'_y \) are the directional pressure coefficients defined by a first order Taylor series expansion of the fluid film pressure response to a perturbation in film thickness:

\[
p = p_0 + \Delta p = p_0 + p_x \Delta x + p_y \Delta y + p'_x \Delta \dot{x} + p'_y \Delta \dot{y}.
\]
the fluctuating bearing displacement and fluid film pressure responses from an externally applied excitation force.

3.2.2 Dynamic Loading and Responses

Fluctuating displacement and pressure responses are obtained by applying dynamic loading to the hydrodynamic bearing via two orthogonally positioned electromagnetic shakers attached directly to the test bearing housing. Sinusoidal input forces \( F_a \) and \( F_b \), of equal frequency but independently variable amplitude and phase angle, are applied to the bearing housing. Using

\[
F_x = F_a \cos \left( \frac{\pi}{4} \right) + F_b \cos \left( \frac{\pi}{4} \right) \quad \text{and} \quad (3-6)
\]

\[
F_y = F_a \sin \left( \frac{\pi}{4} \right) - F_b \sin \left( \frac{\pi}{4} \right), \quad (3-7)
\]

the orthogonal input forces \( F_a \) and \( F_b \) are transformed to the established Cartesian coordinate system producing vertical and horizontal forces \( F_x \) and \( F_y \), as illustrated in Figure 3-12. The net applied vertical and horizontal dynamic forces induce an elliptical bearing displacement orbit centered about the point of steady state operating eccentricity, as schematically represented in Figure 3-13. Adjustments of the relative amplitudes and phase angles for \( F_a \) and \( F_b \) produce elliptical bearing excitation orbits of varying axis widths and angles.
Figure 3-12. Transformation of input shaker excitation forces into global Cartesian coordinate system.
Figure 3-13. Schematic diagram of elliptical bearing displacement response from externally applied harmonic excitation.

For small bearing displacements about a point of static equilibrium, the nonlinear and non-isotopic fluid film bearing properties can be approximated as linear [29]. Modeling the dynamics of the fluid film as a linear system allows for the assumption that the net applied dynamic forces $F_x$ and $F_y$ will produce harmonic bearing displacement and film pressure responses of the same frequency. Written in complex notation, the time harmonic input forces and bearing responses are as follows:

$$ F_i(t) = \text{Re}\left\{F_0 e^{i(\omega t - \varphi)}\right\}, $$

(3-8)
\[ F_y(t) = \text{Re}\left\{ F_0 e^{i(\alpha - \psi_y)} \right\}, \]  
(3-9)

\[ x(t) = \text{Re}\left\{ X_0 e^{i(\alpha - \phi_x)} \right\}, \]  
(3-10)

\[ y(t) = \text{Re}\left\{ Y_0 e^{i(\alpha - \phi_y)} \right\}, \]  
(3-11)

\[ \dot{x}(t) = \text{Re}\left\{ -\frac{X_0}{\omega} e^{i\left[\alpha - \phi_x - \frac{\pi}{2}\right]} \right\}, \]  
(3-12)

\[ \dot{y}(t) = \text{Re}\left\{ -\frac{Y_0}{\omega} e^{i\left[\alpha - \phi_y - \frac{\pi}{2}\right]} \right\}, \]  
(3-13)

\[ p(t) = \text{Re}\left\{ P_0 e^{i(\alpha - \psi_p)} \right\}, \]  
(3-14)

where \( \psi_x \) and \( \psi_y \) are the phase lag angles relative to a chosen datum associated with the external forces \( F_x \) and \( F_y \), and \( \phi_x \), \( \phi_y \), and \( \phi_p \) are the phase lag angles associated with the bearing displacement and pressure responses respectively.

### 3.2.3 Measurement of Bearing Responses

Accurate experimental calculation of the directional pressure coefficients of Equation 3-5 requires precise measurement of displacement and pressure responses from an externally applied harmonic force. While extensive design efforts are made to minimize unwanted sources of noise in the bearing test rig, complete elimination of such sources is not possible. Figure 3-14 shows typical displacement and pressure time histories collected by the on-bearing instrumentation. These data contain not only the displacement and pressure responses from the forced harmonic excitation, but also signal content from many other sources. Viewing of the data in the frequency domain, as shown in Figure 3-15, allows for identification of the major noise sources. Shaft imbalance and support bearing noise appear as harmonics of the fundamental shaft rotational
frequency. Pitch instability of the bearing, as a result of the unconstrained floating bearing design, appears as broadband low frequency noise and is discussed in more detail in Section 3.4.2. In many cases the rotation frequency of the shaft drive motor and lubrication pump motor are present in the data and appear at their respective fundamental rotational frequencies. Present only in the displacement measurements is background noise caused from the out-of-roundness of the shaft. The surface roughness and out-of-roundness of the shaft, also called shaft run-out, is recorded in the displacement measurements as the displacement sensors, mounted directly to the exterior of the bearing housing, measure the instantaneous distance between the bearing and shaft surface. In addition to measuring relative motion of the bearing, the sensors also record any irregularities in the surface finish of the rotating shaft. Measurement of shaft run-out appears as harmonics of the fundamental shaft rotation frequency. In most of the collected data sets 60 Hz electrical noise is also present.
Figure 3-14. Time series of recorded bearing displacement and fluid film pressure measurements.
Design and construction methods, such as isolation of the test rig from shaft and pump motors, precision shaft machining and balancing, and use of super precision ABEC-7 support bearings, significantly limit external sources of bearing vibration. Yet, a certain level of unwanted noise still remains. To avoid spectral overlap with remaining noise sources, frequency selection of the external harmonic excitation is chosen from a portion of the spectrum with low
signal to noise ratio. Since most of the previously listed noise sources are directly related to the operating conditions of the bearing rig, selection of forcing frequencies can be easily determined in advance to avoid spectral overlap of such noise sources. Selection of the forcing frequencies from the low noise portions of the spectrum helps maximize the signal-to-noise ratios of the displacement and pressure signals corresponding to the forced excitation. Furthermore, these selections allow for the assumption that the amplitude and phase angles collected from displacement and pressure measurements at the frequency of excitation are solely attributed to the dynamic forcing provided by the electromagnetic shakers.

3.2.4 Bearing Displacement Interpolation

In an ideal bearing, rotor and shaft centerlines are parallel at all times during excitation (Figure 3-16a), resulting in radial bearing displacement independent of axial position. However, actual fluid bearings experience imbalances, instabilities, and uneven wear (discussed further in Section 3.4.2) resulting in conical bearing motion when excited (Figure 3-16b), thereby creating an axial component to the bearing displacement.
Figure 3-16. Two dimensional schematic diagram of (a) purely radial bearing motion and (b) conical bearing motion.

Section 3.1.2 illustrates how bearing displacement measurements are collected on each side of the test bearing outside of the fluid film. To obtain bearing displacement values for positions within the fluid film, specifically at the axial locations of the bearing embedded pressure sensors from measurement of the elliptical orbit paths at each end of the test bearing, requires a method of interpolation.

An interpolation algorithm to obtain bearing displacement orbits at each of the pressure sensor axial locations is developed by combining a geometry based line-plane intersection solution [30] with the test rig dimensions and measured dynamic elliptical orbits. By calculating the intersection of the line formed by the instantaneously measured displacement orbits and the
radial planes of reference for each axial pressure measurement position, an instantaneous displacement value is obtained, as shown in Figure 3-17. Stepping through the recorded displacement time histories produces a trace of the elliptical displacement orbits associated with each axial pressure measurement.

Figure 3-17. Geometry of line-plane intersection to determine instantaneous bearing displacement values at locations of axial pressure measurements.
3.2.5 Solving for Directional Pressure Coefficients

The directional pressure coefficients, $p_x$, $p_y$, $p'_x$, and $p'_y$, are calculated for each individual node of the fluid film mesh by solution of Equation 3.5. Measured fluctuating fluid pressure and bearing displacement responses from a given excitation orbit are used to populate the known variables of the equation, leaving only the four pressure coefficients as unknowns. By adjusting the amplitude and relative phases of the two electromagnetic shakers, a change in the net applied harmonic excitation force is achieved. This results in a second excitation orbit, as shown in Figure 3-18. The adjustment process is repeated with the resulting recorded pressure and displacement responses from each discrete excitation orbit plugged into Equation 3.5, creating a system of linear simultaneous equations as shown in matrix form in Equation 3.15. A least squares solution [31, 32] of this over-determined system of equations is then calculated to obtain the four directional pressure coefficients for the given pressure measurement location.

\[
\begin{bmatrix}
    p_1(t) - p_0 \\
    p_2(t) - p_0 \\
    p_3(t) - p_0 \\
    \vdots \\
    p_n(t) - p_0
\end{bmatrix} =
\begin{bmatrix}
    \Delta x_1(t) & \Delta y_1(t) & \Delta \dot{x}_1(t) & \Delta \dot{y}_1(t) \\
    \Delta x_2(t) & \Delta y_2(t) & \Delta \dot{x}_2(t) & \Delta \dot{y}_2(t) \\
    \Delta x_3(t) & \Delta y_3(t) & \Delta \dot{x}_3(t) & \Delta \dot{y}_3(t) \\
    \vdots & \vdots & \vdots & \vdots \\
    \Delta x_n(t) & \Delta y_n(t) & \Delta \dot{x}_n(t) & \Delta \dot{y}_n(t)
\end{bmatrix}
\begin{bmatrix}
    p_x \\
    p_y \\
    p'_x \\
    p'_y
\end{bmatrix}
\]

\[
3.15
\]
3.2.6 Construction of Pressure Coefficient Distribution

Equations 3.1 through 3.4 show determination of the dynamic bearing coefficients by integration of the directional pressure coefficients over the fluid film bearing surface area. Section 3.2.5 presents the process used to obtain directional pressure coefficients from recorded displacement and pressure responses about a single location of pressure measurement. Ideally, replication of this process over the entire surface area of the bearing can be used to produce a distribution of the directional pressure coefficients over the entire bearing circumference, to be integrated accordingly. Unfortunately, constraints in the design of the bearing housing, specifically overlap of the fixed lubrication inlet connection and shaker attachment locations (Figure 3-19), restrict the range of bearing housing angular rotation, limiting circumferential pressure distribution measurements to a 54° partial arc of the full fluid film distribution (Figure 3-10). As Equations 3-1 through 3-5 use an angular distribution of fluctuating pressures values to

Figure 3-18. Measured harmonic excitation forcing vectors and resulting bearing displacement responses.
calculate experimental distributed bearing coefficients, the coefficient results presented in Chapter 4 are limited to the 54° arc for which pressure measurements are collected.

Figure 3-9 shows the orientation of the pressure sensors within the fluid film bearing. This configuration allows simultaneous dynamic pressure measurements across the bearing width, thereby providing an axial fluid film pressure distribution at a given circumferential angular position. Incremental rotation of the test bearing housing repositions the embedded pressure sensors to multiple angular positions. Repositioning of the bearing mounted displacement sensors and electromagnetic shakers for each angular bearing position allows for a constant orientation between the dynamic displacement measurements and excitation forces and the applied static load to be maintained (Figure 3-19). Measurement of the axial fluid film pressure distributions at multiple locations around the bearing circumference allows for the construction of a fluid film pressure distribution over a portion of the bearing surface area.
While the pressure sensors are able to rotate independently of the displacement sensors and applied static and dynamic load locations, the test bearing design requires the angular position of the lubrication inlet port to rotate with the pressure sensors. To minimize the effect of the oil inlet position on the hydrodynamic properties of the fluid film bearing, rotation angles are limited only to angles that orient the inlet port within the low-pressure/non-load bearing portion of the fluid film (Figure 3-20). A more in depth discussion of oil inlet effects is presented in Section 3.4.3.
3.2.7 Calculation of Distributed Bearing Coefficients

Repetition of the directional pressure coefficient solution method described in Section 3.2.5 for each angular pressure measurement location described in Section 3.2.6 is used to produce directional pressure coefficients \((p_x, p_y, p'_x, \text{ and } p'_y)\) for each measurement node over the surface area of the bearing.

Integration of these terms, as shown in Equations 3.1 through 3.4, across the bearing axial width and incremental theta angles (graphically depicted in Figure 2-3) produces a set of
experimentally derived distributed stiffness and damping coefficients. These results are presented in Chapter 4.

3.3 Test Setup and Operating Conditions

The coefficient extraction algorithm of Section 3.2 is used to investigate the effect of various bearing operating parameters on the distributed stiffness and damping coefficients. By comparing these experimentally derived distributed bearing coefficients to those obtained numerically, bounds of validity for the numerical method can be determined.

The results presented in this dissertation show the effects of bearing loading, dynamic excitation, and lubrication inlet pressure. Static bearing loading is varied between 316 N and 532 N. This is limited on the low end by bearing instability effects of a lightly loaded bearing [33] and on the high end by the maximum safe working load limits of the static load hardware of the test rig. To maximize signal-to-noise ratio and ensure measurable bearing displacement responses, dynamic excitation frequencies are chosen to correspond with the peak force output of the electromagnetic shakers. From this frequency range, the specific frequencies of 42, 52, 62, and 70 Hz are selected to avoid overlap with shaft rate harmonics and other significant noise sources. Bearing lubrication inlet pressures are kept constant for the main portion of the testing. Further, inlet pressures are set at the lowest possible flow rates in order to minimize unwanted external hydrostatic forces being applied to the bearing, while still providing a fully developed fluid film around the bearing circumference. Further discussion of this optimization process is discussed in Section 3.4.4.
3.3.1 Bearing Geometry

A plain journal bearing design is used for this research. The bearing, shown in Figures 3-21 and 3-22, is fabricated from stock bearing bronze. Due to the relatively narrow bearing profile (L/D = 0.5) and large lubrication flow volumes caused by a wide clearance gap, the use of axial or circumferential lubrication grooves is not required. Rather, a single circular supply hole, positioned in the low pressure region of the bearing, is able to be used for bearing lubrication. This method of lubrication supply allows for not only greater simplicity in the bearing design and fabrication, but for improved correlation between the test rig design specifications and numerical model input parameters. A summary of the test bearing specifications is shown in Table 3-1.
Figure 3-21. Plain journal test bearing removed from bearing housing.
Table 3-1. Test bearing specifications.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Design Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing Type</td>
<td>Plain journal bearing</td>
</tr>
<tr>
<td>Bearing Material</td>
<td>Bronze</td>
</tr>
<tr>
<td>Lubrication Supply</td>
<td>Single 6.35 mm (0.25 in) Dia inlet port</td>
</tr>
<tr>
<td>Length</td>
<td>50.8 mm (2.000 in)</td>
</tr>
<tr>
<td>Diameter</td>
<td>101.6 mm (4.000 in)</td>
</tr>
<tr>
<td>Mean radial clearance</td>
<td>0.249 mm (0.0098 in)</td>
</tr>
<tr>
<td>Preload</td>
<td>0</td>
</tr>
</tbody>
</table>

3.3.2 Lubrication Specifications

The test bearing is lubricated with Mobil Velocite Oil No. 6, a light-weight, petroleum-based, detergent-free spindle oil. Lubrication temperature is regulated using an air cooled heat
exchanger (Figure 3-23) integrated into the lubrication pump loop. Lubrication is measured at the bearing housing exit spouts during data collection to monitor temperature fluctuation during test rig operation. The duration of test rig operation is controlled to keep lubrication temperatures within acceptance tolerances, thereby ensuring reasonably constant viscous properties of the fluid film.

The absolute viscosity of the oil is measured using a Brookfield LVDV-II+ Pro Viscometer. The viscosity is tested at three temperatures, corresponding to measured oil operating temperatures during data collection over several rotating speeds, the results of which are shown in Table 3-2.

Figure 3-23. Air cooled heat exchanger used to regulate temperature of lubrication supply.
Table 3-2. Lubrication viscosity testing results.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Speed (rpm)</th>
<th>Viscosity (Pa·s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.70</td>
<td>6</td>
<td>0.0170</td>
</tr>
<tr>
<td>24.70</td>
<td>12</td>
<td>0.0167</td>
</tr>
<tr>
<td>24.70</td>
<td>30</td>
<td>0.0170</td>
</tr>
<tr>
<td>24.70</td>
<td>30</td>
<td>0.0167</td>
</tr>
<tr>
<td>24.70</td>
<td>12</td>
<td>0.0160</td>
</tr>
<tr>
<td>24.70</td>
<td>6</td>
<td>0.0170</td>
</tr>
<tr>
<td>25.90</td>
<td>6</td>
<td>0.0160</td>
</tr>
<tr>
<td>25.90</td>
<td>12</td>
<td>0.0159</td>
</tr>
<tr>
<td>25.90</td>
<td>30</td>
<td>0.0150</td>
</tr>
<tr>
<td>25.90</td>
<td>30</td>
<td>0.0159</td>
</tr>
<tr>
<td>25.90</td>
<td>12</td>
<td>0.0159</td>
</tr>
<tr>
<td>27.00</td>
<td>6</td>
<td>0.0151</td>
</tr>
<tr>
<td>27.00</td>
<td>12</td>
<td>0.0151</td>
</tr>
<tr>
<td>27.00</td>
<td>30</td>
<td>0.0151</td>
</tr>
<tr>
<td>27.00</td>
<td>30</td>
<td>0.0151</td>
</tr>
<tr>
<td>27.00</td>
<td>12</td>
<td>0.0152</td>
</tr>
<tr>
<td>27.00</td>
<td>6</td>
<td>0.0154</td>
</tr>
</tbody>
</table>

3.3.3 Test Plan

A test plan is developed to systematically and efficiently collect the required displacement and pressure measurements over a range of operating conditions and bearing angles. To determine the appropriate test ranges for each experimental factor, pre-test experimentation is conducted. Upon completion of the pretesting, the experimental factors are ranked according to the complexity and time required to change the factor from one setting to another. Simple operations such as adjustment of the frequencies and magnitudes of the applied dynamic loading can be performed quickly from the LabVIEW Control panel (see Figure 3-24) and are adjusted most frequently. More complex procedures, such as the angular rotation of the pressure sensors,
which require repositioning of the static and dynamic load mechanisms and reorientation of the displacement sensors, are positioned in the test plan to be adjusted as infrequently as possible.

Over the course of the testing, periodic sensor calibrations and continuous monitoring of lubricant temperature and pressure are performed to ensure consistency in measurements throughout the duration of the data collection. Table 3-3 highlights the key points of the 700 run test plan used to collect the data presented in this dissertation. The combination of shaft speeds and bearing loads produce bearing operating conditions at Sommerfeld numbers of 0.137, 0.171, and 0.230.

Figure 3-24. Screen shot of LabVIEW control panel used to adjust dynamic excitation parameters and display instantaneous bearing displacement measurements.
Table 3-3. Journal bearing test rig operating conditions for which distributed dynamic bearing coefficients are calculated.

<table>
<thead>
<tr>
<th>Operating Condition Variables</th>
<th>Operating Condition Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular Pressure Sensor Locations (deg)</td>
<td>171, 180, 198, 207, 216, 225</td>
</tr>
<tr>
<td>Dynamic Excitation Frequencies (Hz)</td>
<td>42, 52, 62, 70</td>
</tr>
<tr>
<td>Bearing Excitation Orbits</td>
<td>9 Distinct Orbits</td>
</tr>
<tr>
<td>Static Bearing Loads (N)</td>
<td>316, 423, 532</td>
</tr>
</tbody>
</table>

3.3.4 Determination of Bearing Operating Center

Figure 3-7 illustrates the fastening method in which the non-contact displacement sensors are mounted to the external surface of the bearing housing side shields. This sensor configuration measures the relative vertical and horizontal motion between the bearing and shaft. A bearing alignment procedure is performed to calibrate these relative displacement measurements into a bearing displacement referenced from the bearing centerline. From this absolute displacement measurement, bearing eccentricity and attitude angle are calculated. In addition, the bearing alignment procedure allows for quantification of temperature related effects on the bearing clearance.

Prior to the alignment procedure, the lubrication pump loop is turned on to circulate heated oil to the bearing, bearing housing, and shaft to warm the test rig componentry. Once nominal operating temperature is established, the oil pump is shut off and a vertical load is applied to the bearing housing in order to achieve metal-to-metal contact between the bearing and shaft, as shown in Figure 3-25a. Offset measurements from the vertical and horizontal displacement sensors are then recorded from this position. Removal of the applied static load allows the weight of the bearing and housing to produce a second point of shaft/bearing contact on the other side of the bearing, as shown in Figure 3-25b. Again offset measurements are recorded. Assuming circular cross section and parallel axial centerline geometries of the shaft
and bearing, the midpoint between the two points of metal-to-metal contact is used to determine the location of the bearing hot operating center.

This measurement process is repeated before and after each angular repositioning of the pressure sensors, as pressure sensor repositioning requires reorientation and recalibration of the displacement sensors at each new angular position. This measurement process also allows for quantification of the changes in the bearing clearance gap over the duration of the data collection due to fluctuations in the operating temperature causing expansion and contraction of the shaft and bearing diameters.

Figure 3-25. Schematic diagram of the (a) maximum and (b) minimum vertical bearing displacements used to calculate the test bearing hot operating center.
3.4 Rig Verification and Trouble Shooting

Validation and verification of the test rig performance is required to ensure the accuracy of collected measurements. The following sections, while not all inclusive, highlight several design efforts, experimental pretests, and in situ troubleshooting performed during the rig development and data collection process to improve the accuracy and understanding of the measured dynamic bearing coefficients.

3.4.1 Noise Source Identification

As explained earlier in this chapter, derivation of experimentally obtained dynamic coefficients relies on precision measurement of the bearing displacement and film pressure fluctuations at the frequency of external excitation. To ensure these measurements of bearing dynamics are correctly quantified, comprehensive understanding of the bearing rig internal sources of noise is required. Characterization and understanding of the sources of noise produced during the steady state operation of the test rig allow for the adjustment of operating and experimental parameters, ensuring that externally applied excitation frequencies, used to generate displacement and pressure fluctuations, do not coincide with frequencies of high background noise.

Characterization of the test rig internal noise sources begins with instrumentation of the major rig components with accelerometers. These components include the bearing housing, each support bearing pedestal, both the shaft and pump motors, the suspended static load platform, and the rig test plate. The rig is turned on and acceleration data is collected at various operating conditions by making incremental changes to the shaft rotational speed, pump motor speed, and
static load. Results from this testing are then used to identify sources of noise observed in the bearing displacement and film pressure measurements.

Figures 3-14 and 3-15 show typical raw time series data and spectral content from the measured bearing displacement and fluid film pressure respectively. The DC offset for each signal, represented in the 0 Hz amplitude of the spectral plots, corresponds to the static, or steady state, film pressure of the pressure measurement and the displacement sensor standoff for the displacement measurement. The second largest peak, occurring at 42 Hz, is the measured response to the applied external dynamic loading. Excitation frequencies of 42, 52, 62, and 70 Hz are selected to avoid overlap with sources of unwanted noise induced by normal rig operation.

The major sources of unwanted noise in the signal are due to shaft rotational imbalance and bearing instability. Shaft imbalance effects appear in the spectrum as harmonics of the fundamental shaft rotation frequency. In addition, for the displacement measurements, shaft out-of-roundness also appears at rotation frequency harmonics. This is because the displacement sensor, fixed to the floating bearing housing, measures not only the relative displacement between the bearing and shaft but also the periodic measurement of the circumferential shaft surface profile. Bearing instability, discussed further in Section 3.4.3, appears as a low frequency swath centered around 5 Hz. Reduction in bearing eccentricity ratio, either through reduced static loading or increased shaft rotation rates, increases the level of bearing instability.

Other minor sources of noise include lubrication pump rotational frequencies, drive motor frequencies, and drive belt tooth frequencies. However, these sources occur at significantly lower amplitudes and typically outside the frequency ranges of interest.
3.4.2 Bearing Instability and Wear

By nature, the hydrodynamic fluid films for plain journal bearings are quite stiff in the radial direction but relatively susceptible to pitch instability due to the parabolic axial pressure distribution produced. The parabolic pressure distribution creates a point of instability which has the potential to produce a lateral rocking motion or bearing misalignment when the bearing is tested in a floating-bearing rig setup. For practical applications, lateral instability is not a significant design concern, as typical fluid film bearing designs employ the bearings in series, in connection with a thrust bearing, or in a fixed-bearing configuration [29].

To address the instability of the bearing, various rig design methods have been employed to constrain the motion of the floating test bearing to move only in the radial direction. The most commonly used methods are adding axially oriented tensioned wires [34], pitch stabilizers [35], or smooth rollers [36] to connect the bearing housing to the stationary rig side fixtures (Figure 3-26). These methods provide additional lateral stiffness, restricting the side-to-side motion of the bearing, while exerting minimal external restrictions to the radial bearing motion.
Figure 3-26. Axial constraint wires as a method to provide lateral stiffness to a floating test bearing assembly.

Measurements of the hydrodynamic fluid film pressures for the test bearing show an asymmetric axial pressure distribution, as illustrated in Figure 4-4. Radial bearing displacements confirm that the unconstrained floating bearing is operating at a slight lateral tilt. Displacement measurements also show laterally uneven eccentricity changes in response to the variation in the applied vertical loading. This lateral tilt and out-of-plane motion is attributed to a change in the steady state operating alignment of the test bearing in reaction to uneven bearing surface wear (Figure 3-27). CMM measurements of the test bearing inner surface show a wear pattern consistent with the measured results (Figure 3-28).
Lateral tilt and out-of-plane motion of the test bearing prevent utilization of the aforementioned axial bearing constraint methods, as steady state and dynamic bearing displacement responses no longer act purely in the vertical plane. Without the ability to address bearing instability through lateral restraints, the test bearing is prone to lateral rocking motion during operation. Partial suppression of bearing lateral instability is achieved by avoiding low eccentricity bearing operating conditions. Differences in dynamic properties due to bearing instability related eccentricity variation are assumed to average out during conversion of the measured data to the frequency domain.
3.4.3 Lubrication Supply Pressure Effects

Maintaining full-film hydrodynamic lubrication requires lubrication be supplied to the bearing at a rate equal to or greater than the rate at which it escapes from the axial bearing ends. The method in which lubrication is delivered to the bearing can significantly impact the fluid film dynamic properties. Positioning lubrication supply ports in the low pressure region of the bearing, opposite the area of maximum load, will minimize disruption of the induced hydrodynamic film pressure distribution. The hydrostatic pressures generated from the lubrication supply flow rates can add unwanted bearing forces that affect the dynamics of the bearing, illustrated in Figure 3-29.
Figure 3-29. Schematic diagram of the hydrodynamic film pressure and lubrication supply line pressure acting on the rotor-bearing system.

To minimize the hydrostatic force effects generated by inlet supply pressures, lubrication supply line pressures are set as low as possible while maintaining flow volumes large enough to distribute sufficient oil to support a fully established hydrodynamic fluid film. Other than possible bearing cooling effects, supply flow rates beyond those required to achieve a fully flooded operating condition will not improve bearing performance [20].

Figure 3-30 shows measurements of the fluid film axial pressure distribution as the supply pressure is varied. At a 0 Pa lubrication line pressure, no lubrication is supplied to the bearing, resulting in direct metal-on-metal contact between the bearing and rotating shaft. As the line pressure is increased, the measured axial pressure distribution shows the development of the hydrodynamic fluid film. Plotting the bearing centerline pressure versus supply inlet pressure...
shows the fluid film transition from oil-starved to optimally-flooded to hyper-flooded conditions. The optimal supply line pressure, which minimizes hydrostatic supply line forces while supplying adequate lubrication flow rates, is identified as the lowest possible line pressure before the increased film pressure effects of bearing starvation are noticeable.

![Graph showing fluid film pressure vs lubrication supply line pressure](image)

Figure 3-30. The effect of lubrication supply line pressure variation on the measured hydrodynamic fluid film pressure.

### 3.4.4 Angular Bearing Positioning Effects

Measurement of the fluid film pressure circumferential distribution using the designed fluid film bearing and measurement system requires rotation of the test bearing and housing to
position the embedded pressure sensors at multiple angular locations around the bearing circumference. Calculation of the experimental bearing coefficients from the measured fluid film pressure fluctuations assumes that the dynamic properties of the fluid film and the operating conditions of the test bearing at each rotated bearing position are the same and that asymmetries in the test bearing, both designed and otherwise, have negligible effects.

An ideal plain journal bearing, free from lubrication supply features and surface irregularities, produces dynamic responses invariant to angular bearing rotation. Bearing surface features, such as the position of the lubrication inlet port, pressure sensor annuli, and bearing bore out-of-roundness have the potential to noticeably influence the dynamic properties of the induced hydrodynamic fluid film. As the bearing housing is rotated to each angular measurement position, the asymmetric features of the bearing rotate as well, influencing different regions of the fluid film at each location. The two asymmetric bearing features with the greatest potential to affect the static and dynamic bearing properties are the previously discussed lubrication inlet port and bearing surface wear.

Bearing design and operating procedure can minimize the influence of the lubrication inlet port on the fluid film properties. As previously discussed, positioning the lubrication supply inlet in the low pressure regions of the bearing and minimizing supply line pressures are employed in the test rig design. However, the ability of the lubrication inlet to produce a fully developed hydrodynamic fluid film must also be considered. The use of a single hole design limits the effectiveness of the port to distribute lubrication over the full axial length of the bearing. Low recirculation rate, due to large radial clearance and narrow bearing length, requires care be taken when rotating the axial position of the inlet port to a location far enough up stream of the converging oil wedge to provide full axial lubricant distribution and allow full development of the fluid film pressure region. Nearly parabolic axial distributions of experimental pressure
measurements, as shown in Figure 3-31, at all seven angular bearing rotation positions confirm a fully developed fluid film for all operating conditions.

![Graph](image)

**Figure 3-31.** Near-parabolic axial distribution of the experimental pressure measurements suggesting the formation of a fully developed fluid film.

Effects of bearing surface wear are seen in both the axial distribution of the static pressure (Figure 4-4) and in the steady state axial alignment of the bearing. As the electromagnetic shakers that provide the dynamic excitation force called for in the experimental method are hard mounted to the bearing housing, misalignment of the bearing produces a lateral dynamic forcing component. As the bearing is rotated through its angular measurement positions, the location of the bearing surface wear moves relative to the fluid film pressure
distribution, potentially producing varying levels of bearing misalignment and lateral dynamic forcing.

To identify potential effects of these bearing asymmetries on the bearing dynamic responses, displacement responses to a common excitation force were measured for each of the seven angular measurement positions. Figure 3-32 presents the measured vertical and horizontal dynamic bearing displacement responses at each of the seven angular bearing housing positions to an applied 51.4 N, 52 Hz excitation force. The consistency of the displacement response amplitudes across the bearing housing rotation angles suggests that the test bearing exhibits similar dynamic properties at each of the bearing rotation positions. These results do not confirm that the asymmetric features of the bearing have no effect on the bearing dynamic properties, only that they exhibit a consistent effect on the dynamic bearing responses, independent of bearing rotation position.
3.4.5 Summary of Rig Verification

Accurate experimental determination of fluid film properties for comparison with numerical results requires understanding the major external and internal forces acting on the test rig. Through test rig verification and validation, the major sources of experimental error are identified and, for the most part, either corrected or avoided in the collection of the data presented in Chapter 4. Isolation of applied harmonic excitation responses from internal test rig noise sources and lubrication supply effects allows for precise determination of test bearing hydrodynamic film properties. Discrepancies between experimental and numerical results attributed to non-uniform bearing surface wear are discussed in Chapter 5.
Chapter 4

Results and Discussion

4.1 Steady State Operating Conditions

Experimental determination of the distributed dynamic bearing coefficients is obtained through measurement of the dynamic displacement and pressure responses from dynamic loading about an operating point of static equilibrium. As fluid film bearing dynamics depend on the operating conditions of the rotating system, measurement of the static bearing displacements and pressure distributions are compared to numerical static displacement and pressure predictions to confirm that the test bearing is operating as expected and to ensure that dynamic stiffness and damping coefficients taken at these points of static equilibrium can be compared directly to equivalent numerically derived coefficients.

Validation of numerical methods is performed by comparing bearing property values derived from both numerical and experimental methods over a range of operating conditions. The results of this comparison, presented in this chapter, are plotted as a function of Sommerfeld number, or bearing characteristic number, $S$. The Sommerfeld number is a dimensionless number used in lubrication analysis to capture the fundamental design and static operating parameters of a hydrodynamic bearing and is defined in Equation 4.1 as follows:

$$S = \frac{\mu_{NDL} R^2}{W C}$$  \hspace{1cm} (4.1)

where

$\mu = \text{absolute viscosity (Pa\cdot s)}$, 

$\mu_{NDL} = \text{dynamic viscosity (Pa\cdot s)}$, 

$R = \text{bearing radius}$, 

$W = \text{bearing width}$, 

$C = \text{bearing clearance}$.
\[ N = \text{shaft speed (rev/s)}, \]
\[ D = \text{shaft diameter (m)}, \]
\[ L = \text{shaft length (m)}, \]
\[ W = \text{bearing load (N)}, \]
\[ R = \text{shaft radius (m)}, \text{ and} \]
\[ C = \text{radial bearing clearance (m)}. \]

### 4.1.1 Static Bearing Displacements

Tables 4-1 and 4-2 quantify the differences between experimental and numerical static displacement results in terms of bearing eccentricity and attitude angle. Bearing eccentricity is defined as the radial distance from the bearing center to the shaft center, and attitude angle is defined as the angle between the lines of centers and the direction of the applied static load. Eccentricity ratio is the ratio of the eccentric bearing center displacement to the radial bearing clearance. These tables show good agreement between experimental and numerical results with percent differences ranging from 2.29\% to 3.60\% for bearing eccentricity and -4.16\% to 3.35\% for attitude angle.
Table 4-1. Experimental vs. numerical bearing eccentricity values.

<table>
<thead>
<tr>
<th>Run Num</th>
<th>Sommerfeld Number</th>
<th>Experimental Eccentricity</th>
<th>Numerical Eccentricity</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.213</td>
<td>0.711</td>
<td>0.694</td>
<td>2.29</td>
</tr>
<tr>
<td>2</td>
<td>0.200</td>
<td>0.722</td>
<td>0.704</td>
<td>2.46</td>
</tr>
<tr>
<td>3</td>
<td>0.189</td>
<td>0.732</td>
<td>0.714</td>
<td>2.51</td>
</tr>
<tr>
<td>4</td>
<td>0.181</td>
<td>0.742</td>
<td>0.721</td>
<td>2.80</td>
</tr>
<tr>
<td>5</td>
<td>0.172</td>
<td>0.749</td>
<td>0.729</td>
<td>2.66</td>
</tr>
<tr>
<td>6</td>
<td>0.164</td>
<td>0.758</td>
<td>0.737</td>
<td>2.81</td>
</tr>
<tr>
<td>7</td>
<td>0.158</td>
<td>0.765</td>
<td>0.743</td>
<td>2.92</td>
</tr>
<tr>
<td>8</td>
<td>0.151</td>
<td>0.771</td>
<td>0.749</td>
<td>2.87</td>
</tr>
<tr>
<td>9</td>
<td>0.145</td>
<td>0.780</td>
<td>0.756</td>
<td>3.08</td>
</tr>
<tr>
<td>10</td>
<td>0.140</td>
<td>0.786</td>
<td>0.760</td>
<td>3.26</td>
</tr>
<tr>
<td>11</td>
<td>0.135</td>
<td>0.793</td>
<td>0.766</td>
<td>3.38</td>
</tr>
<tr>
<td>12</td>
<td>0.130</td>
<td>0.799</td>
<td>0.771</td>
<td>3.49</td>
</tr>
<tr>
<td>13</td>
<td>0.126</td>
<td>0.804</td>
<td>0.776</td>
<td>3.53</td>
</tr>
<tr>
<td>14</td>
<td>0.122</td>
<td>0.809</td>
<td>0.780</td>
<td>3.57</td>
</tr>
<tr>
<td>15</td>
<td>0.118</td>
<td>0.814</td>
<td>0.785</td>
<td>3.60</td>
</tr>
</tbody>
</table>

Table 4-2. Experimental vs. numerical bearing attitude angles.

<table>
<thead>
<tr>
<th>Run Num</th>
<th>Sommerfeld Number</th>
<th>Exp Attitude Angle (deg)</th>
<th>Num Attitude Angle (deg)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.213</td>
<td>46.6</td>
<td>45.1</td>
<td>3.35</td>
</tr>
<tr>
<td>2</td>
<td>0.200</td>
<td>45.5</td>
<td>44.3</td>
<td>2.69</td>
</tr>
<tr>
<td>3</td>
<td>0.189</td>
<td>44.7</td>
<td>43.6</td>
<td>2.47</td>
</tr>
<tr>
<td>4</td>
<td>0.181</td>
<td>43.6</td>
<td>43.1</td>
<td>1.29</td>
</tr>
<tr>
<td>5</td>
<td>0.172</td>
<td>43.2</td>
<td>42.5</td>
<td>1.61</td>
</tr>
<tr>
<td>6</td>
<td>0.164</td>
<td>41.9</td>
<td>41.9</td>
<td>-0.03</td>
</tr>
<tr>
<td>7</td>
<td>0.158</td>
<td>41.4</td>
<td>41.4</td>
<td>-0.05</td>
</tr>
<tr>
<td>8</td>
<td>0.151</td>
<td>40.9</td>
<td>40.9</td>
<td>-0.02</td>
</tr>
<tr>
<td>9</td>
<td>0.145</td>
<td>40.5</td>
<td>40.4</td>
<td>0.24</td>
</tr>
<tr>
<td>10</td>
<td>0.140</td>
<td>39.8</td>
<td>40.0</td>
<td>-0.47</td>
</tr>
<tr>
<td>11</td>
<td>0.135</td>
<td>39.0</td>
<td>39.6</td>
<td>-1.54</td>
</tr>
<tr>
<td>12</td>
<td>0.130</td>
<td>38.2</td>
<td>39.1</td>
<td>-2.37</td>
</tr>
<tr>
<td>13</td>
<td>0.126</td>
<td>37.7</td>
<td>38.8</td>
<td>-2.91</td>
</tr>
<tr>
<td>14</td>
<td>0.122</td>
<td>37.1</td>
<td>38.4</td>
<td>-3.55</td>
</tr>
<tr>
<td>15</td>
<td>0.118</td>
<td>36.5</td>
<td>38.0</td>
<td>-4.17</td>
</tr>
</tbody>
</table>

Figure 4-1 shows the relative displacement of the bearing center for an incrementally increasing applied static load as measured from the test rig shaft center and plotted on a rectangular coordinate system. Figure 4-2 plots these same measurements as a function of eccentricity ratio and Sommerfeld number. The general behavior of these data is as expected,
with bearing eccentricity increasing with applied static load. Even with the somewhat limited static loading range, Figure 4-1 reveals that the curvature of the experimental results behaves as expected, with eccentricity increasing into the positive-x / positive-y quadrant as the static load on the bearing is increased and attitude angle decreasing with increased bearing eccentricity.

Figure 4-1. Experimental and numerical bearing center loci for multiple operating conditions.
Both Figures 4-1 and 4-2 do, however, show a small but distinct offset between the experimental and numerical results. This discrepancy is believed to stem from inaccuracy in experimental determination of bearing operating center using the method presented in Section 3.3.4. Bearing operating center is calculated by averaging measured bearing displacement at positions of maximum and minimum vertical displacement as shown in Figure 3-25. Assumed in this measurement method is perfect shaft and bearing circularity and zero horizontal offset from center at each position. However, uneven surface wear present in the test bearing (Figure 3-28) invalidates these assumptions, potentially causing unaccounted for offset in experimental bearing center loci measurements.
4.1.2 Static Fluid Film Pressure Profiles

Fluid film pressure distributions for each of the axial measurement planes (bearing centerline, \( Z = 12.7 \) mm, and \( Z = -12.7 \) mm) are presented in Figure 4-3. Figures 4-4 and 4-5 show plots of the axial pressure distributions for two angular bearing positions around the estimated location of minimum film thickness. Experimental fluid film pressures at the axial bearing ends (\( Z = 25.4 \) mm and \( Z = -25.4 \) mm) are assumed atmospheric. Experimental measurements and corresponding numerical predictions are shown for Sommerfeld values of 0.2298, 0.1713, and 0.1365.

In general, the behavior of the static pressure results is as expected. Circumferential pressure distributions are at a maximum just prior to the angular location of minimum film thickness and decrease with angular separation from this point. Axial pressure exhibits a parabolic distribution with peak pressure values around the bearing centerline that reduce to atmospheric pressure toward the axial bearing ends. Increased static loading elevates overall fluid film pressure distribution values due to changes in bearing eccentricity.

Experimentally measured and numerically calculated results show reasonable similarities in overall film pressure levels. The major difference between the two results is that while the numerical model predicts an axial symmetric pressure distribution (Figure 4-6), the experimental measurements show a distinct axial asymmetry (Figure 4-7) associated with uneven bearing surface wear as discussed in Section 3.4.2. Direct comparisons between experimental and numerical results illustrating the axial bias toward one side of the bearing are found in Figures 4-4 and 4-5. Figure 4-3 shows that in addition to an axial bias in the pressure distribution, there is also an angular offset of circumferential distribution that varies with axial position. The experimental circumferential pressure distribution for the \( Z = 12.7 \) mm measurement plane shows the location of peak film pressure to occur at approximately the same angular bearing position as
shown in the numerically calculated results. As the measurement plane moves across the bearing axis in the negative-$z$ direction, the angular offset between experimental and numerical circumferential distribution peaks increases. Table 4-3 presents estimates of the circumferential distribution offset for each of the three applied static loads. These results show that the degree to which the circumferential pressure distribution is offset varies nearly linearly with axial position. For all operating conditions and axial positions, the numerical method underpredicts the angular location of fluid film rupture.
Figure 4-3. Experimentally measured and numerically calculated circumferential fluid film pressure distributions for axial measurement planes $Z = 12.7$ mm, 0.0 mm, and -12.7 mm.
Figure 4-4. Experimentally measured and numerically calculated axial film pressure distributions for angular measurement position $\theta = 198^\circ$. 

Measurement Position, $\theta = 198^\circ$
Figure 4-5. Experimentally measured and numerically calculated axial film pressure distributions for angular measurement position $\theta = 207^\circ$. 
Figure 4-6. Numerical film pressure distribution for Sommerfeld number of 0.1713 over experimentally measured bearing surface.
Figure 4-7. Experimental film pressure distribution for Sommerfeld number of 0.1713 over measured bearing surface.

Table 4-3. Angular bearing position of experimental and numerical peak fluid film pressures.

<table>
<thead>
<tr>
<th>Axial Position (mm)</th>
<th>Sommerfeld Number</th>
<th>Experimental Angle (deg)</th>
<th>Numerical Angle (deg)</th>
<th>Angular Difference (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z = 12.7</td>
<td>0.2298</td>
<td>198.3</td>
<td>198.3</td>
<td>0.0</td>
</tr>
<tr>
<td>Z = 12.7</td>
<td>0.1713</td>
<td>196.1</td>
<td>197.6</td>
<td>-1.5</td>
</tr>
<tr>
<td>Z = 12.7</td>
<td>0.1365</td>
<td>195.3</td>
<td>196.4</td>
<td>-1.1</td>
</tr>
<tr>
<td>Z = 0.0</td>
<td>0.2298</td>
<td>204.1</td>
<td>197.6</td>
<td>6.5</td>
</tr>
<tr>
<td>Z = 0.0</td>
<td>0.1713</td>
<td>201.5</td>
<td>196.8</td>
<td>4.7</td>
</tr>
<tr>
<td>Z = 0.0</td>
<td>0.1365</td>
<td>199.4</td>
<td>196.4</td>
<td>3.0</td>
</tr>
<tr>
<td>Z = -12.7</td>
<td>0.2298</td>
<td>209.9</td>
<td>198.3</td>
<td>11.6</td>
</tr>
<tr>
<td>Z = -12.7</td>
<td>0.1713</td>
<td>209.5</td>
<td>197.6</td>
<td>11.9</td>
</tr>
<tr>
<td>Z = -12.7</td>
<td>0.1365</td>
<td>205.1</td>
<td>196.4</td>
<td>8.7</td>
</tr>
</tbody>
</table>
4.2 Dynamic Distributed Bearing Coefficients

The distributed dynamic bearing coefficients for the test bearing specified in Table 3-1 are calculated both numerically and experimentally using the methods described in Chapters 2 and 3. These numerical and experimental methods produce a set of four stiffness coefficients \((K_{xx}, K_{xy}, K_{yx}, \text{ and } K_{yy})\) and four damping coefficients \((B_{xx}, B_{xy}, B_{yx}, \text{ and } B_{yy})\) for each circumferential calculation node or pressure measurement location respectively. As the dynamic bearing properties of the test bearing at a given operating condition are fixed, the distribution of the stiffness and damping properties around the bearing circumference produce bearing coefficients dependant on the angular resolution of these nodal and pressure measurement locations. Numerical coefficient angular resolution is set to 0.75 degrees to provide accurate calculation for the finite difference algorithms used. This finer resolution produces a smoother distribution curve for presenting the numerical dynamic coefficient results. The experimental coefficient resolution is set to 9 degrees due to design constraints of the test rig. This coarser resolution produces choppy distribution of the experimental dynamic coefficients. To allow for direct comparison of experimental and numerical results, the distributed dynamic bearing coefficients presented in this chapter are normalized by their respective angular measurement resolutions.*

* In addition to the industry standard non-dimensionalization of the dynamic bearing coefficients by radial clearance, static load, and (for damping terms) shaft angular velocity:

\[
K_{ij} = \frac{r}{W}k_{ij}
\]

\[
B_{ij} = \frac{\alpha}{W}b_{ij}
\]
4.2.1 Numerical Method Sensitivity Analysis

The numerically derived bearing coefficients presented in this chapter are calculated using experimental testing specification inputs to provide a direct comparison to the experimentally derived bearing coefficients. The nominal values of the experimental testing specifications for bearing dimensions, fluid characteristics, and operation conditions can be found in Tables 3-1 and 3-3. While efforts are made, both in the test rig design and operation, to minimize the variability and uncertainty of these parameters, a certain level of coefficient value variation is unavoidable. Table 4.4 shows the minimum, average, and maximum values the testing input parameters used to calculate the numerical bearing coefficients. The effects of these input value variations in the final numerical stiffness and damping coefficient values are shown in the following figures. Not included in this sensitivity analysis are the effects of test rig specific parameters unaccounted for in the numerical model, such as lubricant supply location and bearing housing tilt which are discussed in subsequent sections.

Table 4-4. Measured experimental design and operating specifications used for numerical model input parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum Value</th>
<th>Average Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid Viscosity (Pa·S)</td>
<td>0.0152</td>
<td>0.0160</td>
<td>0.0169</td>
</tr>
<tr>
<td>Fluid Density (kg/m3)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Shaft Rotation Speed (Hz)</td>
<td>17.99</td>
<td>18.15</td>
<td>18.32</td>
</tr>
<tr>
<td>Bearing Length (mm)</td>
<td>50.77</td>
<td>50.80</td>
<td>50.83</td>
</tr>
<tr>
<td>Bearing Diameter (mm)</td>
<td>101.57</td>
<td>101.60</td>
<td>101.63</td>
</tr>
<tr>
<td>Bearing Radial Clearance (mm)</td>
<td>0.231</td>
<td>0.249</td>
<td>0.267</td>
</tr>
</tbody>
</table>

Figures 4-8 and 4-9 present the potential range of numerically derived stiffness and damping coefficients based on the measurement and operational variability of the input parameters. The numerical coefficient distributions of these figures are calculated using the combination of measured specification values from Table 4-4 that produce maximum and minimum peak distribution values. These bounds provide a range of numerical values
representing the variation of the experimental design and operating specifications for each operating condition. The input parameters with significant effect on the final numerical coefficient values are radial bearing clearance and fluid viscosity as shown in Figures 4-10 through 4-13. As the numerical method assumes a perfectly round and axially constant bearing diameter, bearing surface discontinuities are not able to be accounted for directly in the calculations. The large range of bearing clearance inputs is used to take into account asymmetry of the bearing surface caused by shaft contact wear as discussed in Section 3.4.2. Variations in viscosity relate directly to changes in fluid temperature during experimental testing. The heat generated by viscous shear of the fluid is unable to be fully dissipated through the lubrication system heat exchanger, resulting in a gradual rise in fluid temperature during rig operation. The variation of bearing length, bearing width, and shaft rotation speed has minimal effect on the final numerical coefficient curves (Figures 4-14 through 4-15), as range of values for these input parameters can be kept quite small by tight machining tolerances, accurate static measurements, and a correctly sized drive motor.
Figure 4-8. Differences in numerical stiffness coefficients due to total experimental input parameter variation for a Sommerfeld number of 0.1713.
Figure 4-9. Differences in numerical damping coefficients due to total experimental input parameter variation for a Sommerfeld number of 0.1713.
Figure 4-10. Differences in numerical stiffness coefficients due to experimental radial bearing clearance variation for a Sommerfeld number of 0.1713.
Figure 4-11. Differences in numerical damping coefficients due to experimental radial bearing clearance variation for a Sommerfeld number of 0.1713.
Figure 4-12. Differences in numerical stiffness coefficients due to experimental lubricant viscosity variation for a Sommerfeld number of 0.1713.
Figure 4-13. Differences in numerical damping coefficients due to experimental lubricant viscosity variation for a Sommerfeld number of 0.1713.
Figure 4-14. Differences in numerical stiffness coefficients due to experimental bearing length, bearing width, and shaft rotation speed variation for a Sommerfeld number of 0.1713.
Figure 4-15. Differences in numerical damping coefficients due to experimental bearing length, bearing width, and shaft rotation speed variation for a Sommerfeld number of 0.1713.

4.2.2 Numerical and Experimental Distributed Coefficient Comparisons

Comparisons between the numerical and experimental distributed dynamic bearing coefficients for the 101.6 mm plain journal bearing are made at three distinct operating conditions: Sommerfeld numbers of 0.2298, 0.1713, and 0.1365 corresponding to applied static...
loads of 316 N, 423 N, and 532 N respectively. Coefficient results obtained through the numerical and experimental methods are presented in Figures 4-16 through 4-21 as functions of angular bearing location. Numerical coefficients are calculated for both average value and maximum/minimum bounds based on experimental input parameter variation, as discussed in Section 4.2.1, and plotted over the high pressure region of the fluid film. Experimental coefficients are calculated for each of the four dynamic excitation frequencies used during testing and plotted over the angular region where fluid film pressure measurements were made.
Figure 4-16. Distributed dynamic bearing stiffness coefficients for a Sommerfeld number of 0.2298.
Figure 4-17. Distributed dynamic bearing damping coefficients for a Sommerfeld number of 0.2298.
Figure 4-18. Distributed dynamic bearing stiffness coefficients for a Sommerfeld number of 0.1713.
Figure 4-19. Distributed dynamic bearing damping coefficients for a Sommerfeld number of 0.1713.
Figure 4-20. Distributed dynamic bearing stiffness coefficients for a Sommerfeld number of 0.1365.
Figure 4-21. Distributed dynamic bearing damping coefficients for a Sommerfeld number of 0.1365.

Calculation of the experimental distributed dynamic bearing at excitation frequencies of 42, 52, 62, and 70 Hz shows that variation in externally applied forces has only a small effect on the dynamic properties of the fluid film. The experimental results show that excitation at 62 Hz produces higher overall direct ($K_{xx}$, $K_{yy}$) and cross-coupled ($K_{xy}$, $K_{yx}$) stiffness coefficients for all three operating conditions. Excitation at 70 Hz is shown to produce lower overall direct and
cross-coupled coefficients with 42 and 52 Hz falling in between. This excitation frequency
dependence can be seen to a lesser extent for the $B_{xx}$ coefficient plots shown in Figure 4-17.

In general, the coefficient results obtained experimentally show good correlation to those
obtained numerically. The major similarities and differences between the numerical and
experimental coefficients remain constant over each of the tested operating conditions. As
expected, both methods show increases in overall level as the Sommerfeld number decreases.
However, experimental coefficients increase at a greater relative rate. As a whole, agreement
between numerical and experimental stiffness coefficients is better than agreement between the
damping coefficients.

Polynomial curve fitting of the experimental data is used to provide quantitative
comparison of the numerical and experimental distributed coefficient results, examples of which
are shown in Figures 4-22 and 4-23. Table 4-5 provides percent differences of the peak
coefficient level and angular offset for each of the distributed dynamic bearing coefficients at
each of the operating conditions.

Of note, results from the numerical model do not produce equivalent damping coefficient
cross terms, $B_{xy}$ and $B_{yx}$, a condition often used as an accuracy check for lumped parameter
dynamic coefficient calculation methods [37]. Numerical $B_{yx}$ distribution peak values are
consistently greater than $B_{xy}$ peak values for each of the three bearing operating conditions.
Experimental methods produce nearly equivalent damping cross terms for the 423 and 532 N
loading conditions and larger peak $B_{xy}$ values for the 316 N condition.

* Further discussion of damping coefficient cross term inequality, within the context of distributed bearing
coefficients, is found in Section 4.3.2.
Figure 4-22. Fourth order polynomial curve fit of experimental distributed dynamic stiffness coefficient results for a Sommerfeld number of 0.2298.
Figure 4-23. Fourth order polynomial curve fit of experimental distributed dynamic damping coefficient results for a Sommerfeld number of 0.2298.
Table 4-5. Comparison of peak value and angular offset for numerical and experimental distributed dynamic bearing coefficient results.

<table>
<thead>
<tr>
<th>Dyn Coeff</th>
<th>Sommerfeld Number</th>
<th>Exp Peak Value</th>
<th>Num Peak Value</th>
<th>Peak Diff.</th>
<th>Exp Peak Angle (deg)</th>
<th>Num Peak Angle (deg)</th>
<th>Angular Diff. (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{xx}$</td>
<td>0.2298</td>
<td>4.24</td>
<td>5.47</td>
<td>-1.23</td>
<td>199.20</td>
<td>196.77</td>
<td>3.52</td>
</tr>
<tr>
<td>$K_{yy}$</td>
<td>0.1713</td>
<td>6.17</td>
<td>7.10</td>
<td>-0.93</td>
<td>199.10</td>
<td>196.77</td>
<td>2.64</td>
</tr>
<tr>
<td>$K_{xy}$</td>
<td>0.1365</td>
<td>7.73</td>
<td>8.73</td>
<td>-1.01</td>
<td>198.30</td>
<td>196.77</td>
<td>1.53</td>
</tr>
<tr>
<td>$K_{yx}$</td>
<td>0.2298</td>
<td>5.72</td>
<td>5.35</td>
<td>0.37</td>
<td>208.40</td>
<td>203.52</td>
<td>2.22</td>
</tr>
<tr>
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<td>6.98</td>
<td>6.17</td>
<td>0.82</td>
<td>207.10</td>
<td>203.52</td>
<td>2.39</td>
</tr>
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<td>6.92</td>
<td>0.89</td>
<td>206.40</td>
<td>203.52</td>
<td>2.88</td>
</tr>
<tr>
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<td>5.30</td>
<td>6.12</td>
<td>-0.81</td>
<td>211.30</td>
<td>201.27</td>
<td>8.12</td>
</tr>
<tr>
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<td>3.27</td>
<td>1.06</td>
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</tr>
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<td>215.60</td>
<td>208.77</td>
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<td>206.52</td>
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<td>11.48</td>
</tr>
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<td>222.90</td>
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</tr>
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<td>3.55</td>
<td>1.21</td>
<td>219.20</td>
<td>208.02</td>
<td>11.18</td>
</tr>
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<td>1.69</td>
<td>3.65</td>
<td>235.20</td>
<td>210.27</td>
<td>20.02</td>
</tr>
<tr>
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<td>4.86</td>
<td>1.58</td>
<td>3.28</td>
<td>230.80</td>
<td>210.27</td>
<td>18.59</td>
</tr>
<tr>
<td>$B_{yx}$</td>
<td>0.1365</td>
<td>4.50</td>
<td>1.51</td>
<td>2.99</td>
<td>226.90</td>
<td>210.27</td>
<td>16.63</td>
</tr>
</tbody>
</table>

4.3 Discussion of Results

In general, the numerical method used to calculate the distributed dynamic bearing coefficients shows good agreement with the results obtained through experimental measurement. Section 4.2.2 identifies the major discrepancies between the two methods, in particular, the angular offset of both the stiffness and damping coefficient distributions and the overall underprediction of the damping coefficients seen in results produced by using the distributed...
numerical method. To be discussed in this section are the additional investigation and experimentation performed to identify, characterize, and explain variation between the numerical and experimental bearing coefficient results.

4.3.1 Excitation Frequency Dependence

The numerical method used to calculate the dynamic bearing properties assumes the distributed stiffness and damping coefficient results to be independent of external vibration frequency [4]. To test these assumptions, experimental distributed bearing coefficients are calculated using multiple dynamic excitation frequencies. Results from this testing, presented earlier in Figures 4-16 through 4-21, show some dependency between coefficient value and dynamic excitation frequency, with 62 Hz excitation frequencies producing on average the highest stiffness and damping results and 70 Hz excitation the lowest.

Investigating this phenomenon further, experimental measurements are collected over a wider range of excitation frequencies. As discussed in Section 3.2.3, maximal shaker output and avoidance of frequency bands of high background noise are the main factors in the selection of the 42, 52, 62, and 70 Hz excitation frequencies. Force response at these values just above the shaker resonance frequency produce sufficiently larger bearing displacements capable of being accurately measured over the noise floor. Measured shaker force and bearing displacement amplitudes for the extended excitation frequency range are plotted in Figures 4-24 and 4-25. These plots demonstrate that as excitation frequency increases, force output of the dynamic shaker and, in turn, bearing displacement amplitude decrease. Distributed stiffness and damping coefficients are calculated from the extended excitation frequency range data set for a single nodal location and presented in Figures 4-26 and 4-27. The frequency independent numerical distributed coefficients for the corresponding nodal location are plotted for comparison.
Calculation of the distributed bearing coefficients over a wider range of excitation frequencies shows a mix of excitation frequency dependence among the coefficients. Aside from general scatter, the $K_{xx}$, $K_{yx}$, $B_{xx}$, $B_{yx}$, and $B_{yy}$ terms appear frequency independent. The $K_{xy}$, $K_{yy}$, and to a lesser extent, $B_{yy}$ terms appear to increase with increasing excitation. The fact that only $y$-direction terms show frequency dependence suggests a systemic error in coefficient calculation. From Figure 4-25 it can be seen that $x$-direction displacement decreases accordingly with reduced force output while $y$-direction displacement begins to level above 80 Hz. This relation is further illustrated in Figure 4-28, which plots the ratio of input force to output displacement for both the $x$ and $y$ directions. While the force to $x$-displacement ratio exhibits a consistent relation across the frequency range, the force to $y$-displacement exhibits erratic results.

From these results it can be concluded that the frequency variation seen in the distributed coefficients of Figures 4-16 through 4-21 are most likely attributed to a combination of random error and lack of sensitivity in the $y$-direction displacement measurements, rather than to a frequency dependence of the dynamic coefficients themselves. Variation of coefficient value with bearing excitation amplitude, caused by the frequency dependence of the electromagnetic shaker force output, could be a function of inaccuracies in the linearizing methods used to calculate the bearing coefficients and could warrant investigation in future research.
Figure 4-24. Measured harmonic force amplitude as a function of shaker excitation frequency.
Figure 4-25. Measured bearing displacement amplitude as a function of shaker excitation frequency.
Figure 4-26. Effect of dynamic excitation frequency variation on distributed stiffness coefficients calculated at a Sommerfeld number of 0.1713 and angular bearing position of 193.5°.
Figure 4-27. Effect of dynamic excitation frequency variation on distributed damping coefficients calculated at a Sommerfeld number of 0.1713 and angular bearing position of 193.5°.
4.3.2 Damping Cross Term Relation

Equivalency of damping cross terms, $B_{xy} = B_{yx}$, is often used as an accuracy check for lumped parameter dynamic coefficient calculation methods. While summation of the distributed numerical damping cross terms over the entire bearing circumference produce equivalent lumped parameter damping cross terms, the results of Section 4.2 show this equivalency relation does not hold true for the individual distributed coefficient elements. Figure 4-29 overlays circumferential distributions of representative numerically calculated $B_{xy}$ and $B_{yx}$ coefficients, illustrating the amplitude variation between the two terms.
Kato [37] provides conditions for which the \( B_{xy} = B_{yx} \) relation holds true. Kato states that for any type of finite width journal bearing, damping cross terms should be equivalent when calculated using the following conditions: (i) “The governing Reynolds equation is linear in pressure or regarded as linear in numerical calculations”; (ii) “Film thickness is given by \( h = c(1 + \kappa \cos \theta) \)”; and (iii) “Boundary condition is homogeneous such as \( p = 0 \) or \( \frac{dp}{dn} = 0 \), where \( n \) denotes a normal to the boundary”.

The numerically calculated distributed bearing coefficients of this research use (i) a perturbation method to linearly solve the Reynolds equation, (ii) a film thickness value defined by Equations 2-21 through 2-24, and (iii) a \( p = 0 \) boundary condition for the fluid film boundaries.
(film inception, film rupture, and axial bearing ends), thereby satisfying the three conditions stated by Kato. Kato further references work by Someya [38] in which the total fluid film pressure is calculated from the sum of two pressures: the pressure derived from journal squeeze action and the pressure derived from journal wedge action. Someya’s work shows $B_{sy}$ and $B_{sx}$ coefficient inequality when it is assumed that the pressure generated due to squeeze action and the pressure generated due to wedge action come from different regions of the fluid film.

The terms of the expanded Reynolds equation (Equations 2.12 and 2.13) correspond to the individual pressure generating devices for hydrodynamic bearings: specifically the local expansion, density wedge, and stretch actions, as well as the wedge and squeeze actions discussed by Kato. These actions relate changes in fluid flow rates and journal surface motion to the generation of fluid film pressure.

The works of Kato and Someya discuss these actions in the context of lumped parameter bearing coefficients and explain damping coefficient cross term inequality as a result of film pressure generation from individual actions being calculated from different regions of the fluid film. For the results presented in this dissertation, each of the numerically derived distributed bearing coefficient terms is calculated over the same fluid film pressure region. However, each individual coefficient value, relating to a specific circumferential node as illustrated in Figure 2-3, is calculated over a discrete angular region of the fluid film. Extending the assumption of Someya, that the contributions of each pressure generating action are dependent on angular bearing position, to the numerical method used to derive distributed bearing coefficients from discrete angular regions of the film pressure may explain the damping cross term inequality shown in Figure 4-29. If application of Someya’s assumption is valid, damping cross term inequality of the numerically derived distributed coefficients would be considered a property of the circumferential distribution of fluid film pressure and not a sign of numerical inaccuracy as associated with numerically derived lumped parameter coefficients.
Analytical substantiation suggesting that damping cross term inequality is a function of the circumferential separation of pressure generating mechanisms and not inaccuracy in the numerical method is demonstrated by summing each set of inequivalent distributed damping cross terms over the entire fluid film circumference to produce equivalent lumped parameter coefficient values.

Design of the bearing test rig does not allow for the experimental collection of measurements isolating the individual pressure generating actions, and therefore, experimental validation of this assessment remains an area of future work.

4.3.3 Lumped Dynamic Bearing Coefficients

Lumped parameter dynamic bearing coefficients are calculated from several different methods using experimental measurements and bearing parameters of the test rig to provide further insight into the differences found between the numerical and experimental distributed coefficient results. The numerically derived lumped parameter bearing coefficients are calculated using the methods presented in Chapter 2, but instead of integrating Equations 3.1 through 3.4 over incremental bearing angles, as was done to produce the distributed coefficients, the equations are integrated over the entire film surface. Experimentally derived lumped parameter coefficients are calculated using two different methods found in the literature. Kostrzewsky [28] presents an experimental method to obtain all eight bearing coefficients through dynamic excitation of the test bearing. In this method, measurements of small elliptical bearing displacements from a known harmonic excitation force are used to solve for the dynamic stiffness and damping coefficients. The second lumped parameter experimental method used is described by Goodwin [17]. This method obtains the bearing coefficients by measuring the steady state bearing displacement response from changes in the applied static bearing load. As only static
loading and bearing displacement measurements are used in this method, results are limited to only the four stiffness bearing coefficients.

The three lumped parameter methods described above are applied using the bearing parameter and operating conditions of the bearing test rig described in Chapter 3. Bearing coefficients for the two experimental methods are calculated over the same bearing operating conditions used for the distributed bearing coefficients ($S = 0.1365, 0.1713, \text{ and } 0.2298$). The numerical method is calculated over a wider range of bearing operating conditions ($0.05 < S < 5$) for illustrative purposes. Comparison of the results from these three methods are shown in Figures 4-30 and 4-31 with numerical coefficient results shown in solid green, experimental results using Kostrzewsky’s dynamic method in blue Xs, and experimental results using Goodwin’s steady state static loading method in red squares. A fourth set of coefficients, shown in solid pink, are taken directly from Someya’s Journal-Bearing Databook [31] and are plotted to put the calculated lumped parameter results of this section in the context of prior published coefficient values. While the bearing modeled by Someya is of slightly different specification, *these numerical results show strikingly similar stiffness and damping values to those experimentally and numerically derived in this research, thereby providing an independent source of validation to the measurements and methods presented in this dissertation.

* Someya models a similarly dimensioned ($L/D = 0.5$) cylindrical bearing, but this bearing utilizes two axial grooves located at 90 and 270 degrees for lubrication supply instead of a single inlet port that is used for the test rig of this research.
Figure 4-30. Comparison of stiffness coefficients for several lumped parameter determination methods: experimentally obtained using dynamic excitation (Kostrzewsky method), experimentally obtained using steady state static loading (Goodwin method), numerically obtained through circumferential summation of distributed coefficients (Chapter 2 method), and numerical results from literature (Someya data).
Figure 4-31. Comparison of damping coefficients for several lumped parameter determination methods: experimentally obtained using dynamic excitation (Kostrzewsky method), numerically obtained through circumferential summation of distributed coefficients (Chapter 2 method), and numerical results from literature (Someya data).

4.3.4 Comparison of Dynamic Bearing Coefficient Methods

Dynamic bearing coefficients are calculated using both lumped model methods found in the literature and distributed methods presented in Chapters 2 and 3. With each experimental
method, data is obtained from the same bearing test rig using different measurements and collection methods. By comparing results obtained from each method, insight into the source of numerical-experimental discrepancy may be identified, whether due to measurement, methodology, or the test rig itself.

Coefficient results from each method provide different information on the dynamic properties of the fluid film bearing, with lumped methods providing the dynamic properties of the system as a whole and distributed methods providing dynamic properties for finite angular portions of the bearing. From these differences a direct comparison of the results produced by these methods is not possible. Furthermore, as the test rig is only able to obtain pressure measurements for a limited range of bearing angles, the experimental distributed coefficients are unable to be reduced to lumped parameter coefficients. Despite these limitations, general comparisons of distributed results to results obtained using established lumped parameter methods can be used to help evaluate the accuracy of the distributed coefficient methods because a common experimental test rig and data acquisition system is used for all data collections.

Table 4-6 presents a summary of the coefficient value differences between numerical and experimental results for each of the experimental test methods. The values shown in this table for each coefficient and method are the bearing coefficient value differences between the numerical and experimental results, averaged over each of the operating conditions (and bearing angles for distributed coefficients) and calculated for each bearing coefficient and method. While Table 4-6 does not capture individual discrepancies due to loading or angular offset, it does provide a concise way to easily identify trending among the coefficients and methods.
Table 4-6. Difference between numerical and experimental coefficient values for several coefficient calculation methods.

<table>
<thead>
<tr>
<th>Dynamic Coefficients</th>
<th>Distributed Method</th>
<th>Lump Method (Dynamic)</th>
<th>Lump Method (Static)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K&lt;sub&gt;xx&lt;/sub&gt;</td>
<td>-2.05</td>
<td>-0.68</td>
<td>-0.42</td>
</tr>
<tr>
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<td>-0.15</td>
<td>2.31</td>
</tr>
<tr>
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<td>-7.54</td>
<td>-0.06</td>
</tr>
<tr>
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<td>-0.80</td>
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</tr>
<tr>
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<td>1.24</td>
<td>N/A</td>
</tr>
<tr>
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<td>0.88</td>
<td>1.90</td>
<td>N/A</td>
</tr>
<tr>
<td>B&lt;sub&gt;yx&lt;/sub&gt;</td>
<td>0.26</td>
<td>2.01</td>
<td>N/A</td>
</tr>
<tr>
<td>B&lt;sub&gt;yy&lt;/sub&gt;</td>
<td>0.70</td>
<td>4.41</td>
<td>N/A</td>
</tr>
</tbody>
</table>

By comparing average numerical-experimental coefficient differences using the values from Table 4-6, common trends in results obtained from the various methods can be identified:

- When compared to their equivalent numerical method, experimental y-direction displacement and velocity coefficients (K<sub>xy</sub>, K<sub>yy</sub>, B<sub>xy</sub>, and B<sub>yy</sub>) show comparatively larger coefficient values than x-direction motion terms (K<sub>xx</sub>, K<sub>yx</sub>, B<sub>xx</sub>, and B<sub>yx</sub>).

- Numerical method underpredicts B<sub>xy</sub>, B<sub>yx</sub>, and B<sub>yy</sub> damping values for each of the experimental method types.

Trending of coefficient values across common directionalities for all three experimental methods, each using a different method of collection, make coefficient calculation methods and data processing methods unlikely sources of error, leaving numerically unaccounted for test rig dynamics or sensor sensitivity as potential sources of error.

Underprediction of the numerical damping terms, occurring across method type and coefficient directionality and without correlation to directionally equivalent stiffness terms, suggests the bearing test rig may be exhibiting additional damping characteristics not being captured by the numerical methods. The use of the same dynamic displacement measurement data for both the distributed and lumped parameter dynamic experimental methods preclude data
collection and processing methods from being ruled out as sources of error. However, the fact that experimental distributed damping coefficient discrepancy varies with angular bearing housing position rather than occurring as a constant offset suggests the hydrodynamic properties of the test bearing to be the source of the unaccounted damping.

4.3.5 Calculation of Experimental Distributed Coefficients

Section 3.2 describes in detail the procedure used to obtain the experimental distributed dynamic bearing coefficients presented in Figures 4-16 through 4-21. This section will present and discuss the experimental results calculated at each intermediate step of the experimental method, starting with the measured displacement and pressure fluctuations, then using this data to populate a least square solution of the perturbed film pressure equation to obtain the directional pressure coefficients, and concluding with the integration of these pressure coefficients to obtain the final distributed dynamic bearing coefficients. Presented are the measured and calculated results for the test bearing operating at a Sommerfeld number of $S = 0.1713$ and 52 Hz external excitation frequency. The trends and characteristics discussed for this data set are representative of the results obtained for the other testing and operating conditions.

Horizontal and vertical displacement fluctuation amplitudes induced by applied harmonic forcing are shown in Figure 4-32 for three representative excitation orbits. The consistency of the horizontal and vertical fluctuation amplitudes across the angular bearing positions suggests the rotation of the test bearing (and the rotation of associated bearing non-uniformities) has minimal effect on the overall dynamic properties of the system.

Corresponding experimentally measured fluid film pressure fluctuations for the displacements of Figure 4-32 are shown in Figure 4-33. For comparison, numerical pressure differences are computed based upon the experimental bearing displacements. Similar to the
static pressure profiles in Section 4.1.2, but to a greater extent, the experimental pressure fluctuations show elevated amplitudes at the higher bearing angles. This suggests that the experimental film pressure distribution angularly extends well past the point where the numerical model predicts film rupture.

Figure 4-32. Fluctuating horizontal and vertical bearing displacement amplitudes from three unique harmonic excitation orbits measured for each incremental bearing rotation position.
Figure 4-33. Fluctuating fluid film pressure amplitudes from three unique harmonic excitation orbits measured at each incremental pressure sensor angular rotation position.

By using the displacement and pressure measurements from the six unique bearing excitation orbits, the directional pressure coefficients ($p_x$, $p_y$, $p'_x$, and $p'_y$) of Equation 3.5 are solved by least squares method. Individual pressure coefficient estimates are obtained for the twenty-one fluid film pressure measurement locations (three axial measurements collected at each
of the seven bearing rotation positions) and plotted in Figures 4-34 and 4-35, along with corresponding numerical results. The experimental pressure coefficient results exhibit many of the same characteristics seen in the experimental static and fluctuating pressure results, notably the axial distribution asymmetry and elevated values at higher bearing angles. Final experimental distributed bearing coefficients are obtained by integrating the directional pressure coefficients over discrete regions of the bearing surface. While integration over the entire bearing length averages out the axial asymmetric values of the pressure coefficients for each individual distributed bearing coefficient, it is expected that the underlying factors causing the asymmetry have an effect on the dynamic properties of the system as a whole.

Investigation of the intermediate values and calculations used to obtain the experimental distributed dynamic bearing coefficients allows for direct association of the elevated damping values presented in Section 4.2.2 to specific experimentally measured fluid film pressure distribution characteristics. Similarly elevated damping values are obtained from the experimental lumped parameter methods of Section 4.3.3, which do not utilize fluid film pressure measurements in their calculations. As such, it can be concluded that the higher than numerically predicted experimental damping is a product of numerically unaccounted for hydrodynamic properties of the test bearing and not due to measurement or method error.
Figure 4-34. Experimentally and numerically obtained displacement-related pressure coefficients ($p_x$ and $p_y$) for a Sommerfeld number of 0.1713.
Figure 4-35. Experimentally and numerically obtained velocity-related pressure coefficients ($p'_x$ and $p'_y$) for a Sommerfeld number of 0.1713.

4.3.6 Summary of Results

In this chapter, experimental and numerical distributed dynamic bearing stiffness and damping coefficients for a plain journal bearing of $L/D = 0.5$ are presented. Overall, the numerical method used to calculate the distributed coefficients shows good agreement with the
results obtained through experimental measurement for both coefficient amplitude and angular distribution. Sensitivity analysis shows that variation in numerical method radial bearing clearance and fluid viscosity input parameters have the greatest effect on numerical result uncertainty. However, the greatest source of discrepancy between numerical and experimental results is assessed to stem from bearing features and operating conditions not captured in the numerical input parameters, such as non-uniform surface wear and axial misalignment. Confirmation of numerical distributed coefficient frequency independence and damping cross term inequality is obtained through experimental and analytical analysis. In addition, distributed coefficient results are compared to results calculated from lumped parameter methods found in the literature to establish trends in coefficient results and to suggest sources of experimental and numerical result discrepancy.
Chapter 5
Summary, Conclusions, and Next Steps

A journal bearing test rig is developed to experimentally validate a numerical method used to calculate circumferential distributed stiffness and damping properties of a hydrodynamic fluid film bearing. Measurements from this test rig are used to obtain experimentally derived distributed bearing coefficients over a range of bearing operating conditions and dynamic excitation frequencies. Good agreement between coefficients obtained experimentally and those calculated numerically confirm the capability of the numerical method to accurately model the hydrodynamic forces acting around the fluid film bearing circumference. Discrepancies found between experimental and numerical results are attributed to the hydrodynamic effects of experimental test bearing features unique to this test rig.

5.1 Summary

Most current rotor bearing analysis utilizes lumped parameter bearing coefficients to model the static and dynamic characteristics of fluid film bearings. By treating the stiffness and damping properties of the fluid film as acting upon the axial centerline of the rotor, these models are limited in their analysis to first order lateral rotor-bearing motion. The development of a numerical method that distributes the dynamic properties of the fluid film around the bearing circumference allows for the higher order analysis of the motion between the bearing and rotor. Assessment of the accuracy of a numerical method used to calculate distributed dynamic fluid film bearing coefficients is performed by development of a hydrodynamic journal bearing test rig.
and experimental testing procedure capable of obtaining measured distributed dynamic coefficients over a range of bearing operating conditions.

Numerically calculated distributed dynamic bearing coefficients are obtained through finite difference approximation of the Reynolds equation, solving for the directional pressure coefficients of a first order Taylor’s series expansion of the perturbated film pressures. Integration of these pressure coefficients over finite regions of the fluid film produces a set of stiffness and damping coefficients distributed around the bearing circumference. Experimental distributed dynamic bearing coefficients are obtained in a similar fashion, but instead of numerically solving for the directional pressure coefficients, these values are obtained through direct measurement of film pressure and bearing displacement responses to an external harmonic excitation force.

A hydrodynamic bearing test rig and experimental test procedure are developed to collect the required bearing displacement, film pressure, and operating condition measurements to calculate the intermediate directional pressure and final distributed bearing coefficient values. The rig consists of a floating test bearing and housing centered between a pair of fixed support bearings. Dynamic excitation is applied to the bearing housing via a pair of orthogonally positioned electromagnetic shakers. Relative phase angle and amplitude of the shakers are varied to generate multiple bearing excitation orbits for which relative bearing displacement and fluid film pressure distribution measurements are collected. Data from these orbits are used to populate a least squares solution to the system of perturbated pressure equations to obtain the directional pressure coefficients for each angular bearing location. Integration of the pressure coefficients over angular pressure measurement spacing produces the experimental distributed bearing coefficient values.

Experimental distributed bearing coefficients are calculated using the test rig and collection methods for multiple bearing operating and testing conditions. These results are
compared to numerically calculated distributed coefficients as well as to lumped parameter coefficients generated by experimental and numerical methods found in the literature. Overall, the numerically calculated distributed coefficients successfully model both the circumferential distribution and the operating conditions of the experimental distributed bearing coefficient values and show reasonable correlation to results obtained through lumped parameter methods. Excitation frequency independence and damping cross term inequality of the numerically distributed bearing coefficients are validated through experimental testing over multiple frequencies and extrapolation of lumped coefficient analysis found in the literature. While uncertainty and variation of the test rig dimensional and operating parameters have some effect on the accuracy with which the numerical methods model the experimental results, the most significant source of numerical and experimental result dissimilarity comes from test rig specific features not captured in the numerical methods, such as the non-uniform bearing surface wear and bearing-shaft misalignment.

5.2 Conclusions

Experimental assessment of the circumferentially distributed dynamic fluid film bearing coefficients shows the numerical method used in this research to be a promising tool for higher order rotor-bearing analysis. The numerical method is able to accurately predict both the amplitude and angular distribution of the dynamic stiffness coefficients and to a lesser extent the dynamic damping coefficients. The primary shortcomings in the ability of the numerical method to model the stiffness and damping properties of the constructed hydrodynamic bearing test rig lie not with the theory or calculations used to distribute the fluid film properties but with the limitations in the capability of the method to capture the exact dimensional features and operating conditions of the experimental test bearing, specifically the non-uniform bearing surface wear.
which is assessed to affect the bearing alignment and fluid film pressure profile and in turn the circumferential distribution of the calculated experimental dynamic bearing coefficients.

The calculations used in the numerical distributed bearing coefficient methods are generalized to allow for the input of user defined bearing and operating parameters. For plain journal bearings the input parameters are: lubricant density, lubricant viscosity, bearing length, bearing diameter, bearing clearance, shaft speed, and static bearing load. To characterize the dynamics of a fluid film bearing with these seven parameters, multiple simplifying assumptions are applied. As discussed in Chapter 2, several assumptions about the lubricant fluid properties are made to allow for the use of the Reynolds equation to solve for the bearing film pressure. Calculated selection of lubricant type, bearing dimensions, and operating conditions can be used to ensure these assumptions are met. Characterization of the fluid bearing geometry from only bearing length, diameter, and clearance dimensions assumes the bearing and shaft have parallel axial centerlines and perfectly round cross sections. For a well machined and aligned bearing these ideal geometry assumptions would be applicable.

To accurately isolate and evaluate the process of circumferentially distributing the dynamic fluid film properties, an experimental test rig was designed to incorporate as many of the numerical assumptions as possible and to minimize the influence of design features not captured in the numerical model. Initially the experimental test rig met both of these design criteria as preliminary shakedown and pretest results showed good correlation to numerical calculations. However, by the time a full set of data was collected for the calculation of the experimental distributed coefficients, the surface of the test bearing had developed a significant wear spot. Effects from the bearing surface wear are seen both in the steady state and dynamic operation of the test rig.

Steady state effects from the bearing surface wear include misalignment of the bearing pitch and yaw and asymmetric film pressure distributions. The floating bearing design of the test
rig, used to minimize constraints on dynamic bearing motion, allows the test bearing to self align its steady state operating position. Axial misalignment of the test bearing is a consequence of the fluid film pressure redistribution caused by the asymmetric bearing surface wear. Dynamic effects due to bearing surface wear stem from the application of the dynamic loads on the misaligned bearing. Hard mounted shakers attached to the misaligned bearing housing produce lateral forces that result in an out-of-plane rocking motion rather than the assumed purely horizontal and vertical dynamic motion. These effects are captured in the experimental measurements and influence the final values of the calculated experimental distributed bearing coefficients.

As the numerical method is designed to model the dynamic properties of an ideal fluid film bearing, the unintentional bearing wear and alignment effects exhibited by the experimental test rig are unable to be accounted for, leading to discrepancies between the calculated numerical and experimental coefficient results. These discrepancies, in and of themselves, do not invalidate or decrease the utility of the numerical model but rather serve to provide insight into the dimensional tolerance and operational limits of the model. The extreme nature of the wear and misalignment seen in the test rig is not representative of typical real world bearing operating conditions. Therefore, the extent to which wear and alignment effects should be incorporated into the numerical model should be considered on an application-by-application basis.

While experimental evaluation of the distributed dynamic bearing coefficient methods is not able to quantify the accuracy of numerically produced results to a level where operating bounds of validity can be established, several promising conclusions can be drawn from the results, justifying further work on the topic:

1) Agreement between numerical calculation and experimental measurements demonstrate that the numerical method is able to accurately model the level and angular distribution of dynamic fluid film bearing stiffness coefficients.
2) The assumption of distributed coefficient excitation frequency independence is shown to be valid over a limited range of experimental frequencies.

3) Cross term inequality of the distributed dynamic damping coefficients exhibited in numerical results is not necessarily a sign of model inaccuracy as often associated with numerical lumped parameter bearing analysis methods. For distributed coefficients this inequality is likely a result of isolating the contributions of individual hydrodynamic pressure generating mechanisms when circumferentially distributing the dynamic bearing properties.

4) The inability of numerical model inputs to accurately capture the dimensional and operational experimental characteristics of the test rig, specifically bearing surface wear and associated steady state operating misalignment, is the primary contributor to numerical-experimental result discrepancy. Unaccounted for bearing surface wear and resulting effects on bearing dynamics are assessed to be the main source of discrepancy between experimental and numerical static operating center, coefficient amplitude, and coefficient angular distribution results.

5.3 Next Steps

The experimental evaluation performed in this research provides the first steps to validate the distributed dynamic bearing coefficient numerical method. However, design restrictions of the test rig limit the accuracy and range of operating conditions for which the experimental distributed bearing coefficients could be produced. Possible future work would include extending the capabilities of the test rig to further test the assumptions of the numerical model and to determine bounds of validity over a wider range of operating conditions. Specifically:

1) Adding an adjustable axial constraint system to restrict non-lateral bearing motion. The improved amplitude control over dynamic bearing displacements provided by lateral
constraint would allow for investigation into the nonlinearity of the fluid film properties through calculation of distributed bearing coefficients from various bearing displacement orbit magnitudes. Adjustability of the constraint system would allow for the quantification of alignment effects on distributed bearing properties through incremental application of angular misalignment.

2) Collecting fluid film pressure measurements around the entire bearing circumference. Measurement of the full experimental fluid film profile would provide information on the angular location of fluid film rupture and the extent of cavitation as well as data to calculate a complete set of distributed bearing coefficients. Experimental pressure data over the regions of fluid film rupture would allow for investigation into the applicability of the Reynolds and Gumbel boundary conditions for circumferentially distributed bearing coefficients used in higher order rotor-bearing analysis.

3) Improving test rig maximum static load, dynamic excitation, and shaft rotation speed capabilities. The ability to validate the distributed coefficients over a wider range of dynamic displacement responses and Sommerfeld values would allow for investigation into linear/nonlinear bearing motion and laminar/turbulent fluid flow assumptions.

4) Measuring the angular dependence of the major pressure generating mechanisms associated with the individual terms of the expanded Reynolds equation. Development of an experimental test procedure and apparatus capable of isolating the individual hydrodynamic pressure generating mechanisms could provide experimental validation of the assessment that cross term inequality of the numerically derived distributed damping coefficients is not related to numerical method inaccuracies.
References


Appendix

Test Rig Design

This appendix provides a detailed description of the hydrodynamic test rig and accompanying instrumentation used to obtain the data for this research. An overview of the test rig design is followed by a summary of the test bed's operating specifications and a component level review of the major test rig subsystems highlighting key design features.

A.1 Overview and Test Specifications

The hydrodynamic journal bearing test rig and data acquisition equipment used to collect the displacement and pressure measurements required to experimentally derive the distributed dynamic bearing coefficients presented in this dissertation is shown in Figure A-1. The test rig is capable of recording the relative displacements between the test bearing and shaft and the resulting pressure fluctuations of the fluid film around the circumference of the bearing over multiple bearing operating conditions specified by the testing procedures presented in Chapter 3. The test bed is capable of recording these measurements for a range of shaft rotational frequencies, lubrication inlet pressures, and applied static and dynamic loads as summarized in Table A-1.
Figure A-1. Hydrodynamic bearing test rig and data acquisition system.

Table A-1. Test bed operating specifications.

<table>
<thead>
<tr>
<th>Operating Condition</th>
<th>Operating Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied Static Load</td>
<td>0 - 158.8 kg (0 - 350 lbs)</td>
</tr>
<tr>
<td>Applied Dynamic Load</td>
<td>0 - 44.5 N (0 - 10 lbs)</td>
</tr>
<tr>
<td>Applied Dynamic Frequency</td>
<td>10 - 7,500 Hz</td>
</tr>
<tr>
<td>Shaft Rotational Frequency</td>
<td>0 - 2,125 rpm</td>
</tr>
</tbody>
</table>

The bearing test bed utilizes a fixed shaft-free bearing design. The rotating shaft is constrained at either end by radially stiff support bearings. These support bearings are rigidly housed in large mass support blocks that are mounted directly to the test bed. Angular rotation of
the shaft is provided via a variable speed DC motor by way of a toothed pulley connection. The
test bearing itself is located midway between the two support bearings. A two-piece bearing
housing encases the test bearing and provides attachment points for the loading mechanisms,
lubrication system, and on-bearing instrumentation. Static loads are applied to the bearing
housing via hanging mass. Dynamic loads are applied directly to the housing by two
orthogonally mounted electromagnetic shakers. Lubrication is circulated through the test bearing
by a closed circuit pump loop powered by a second variable speed electric motor.

A.2 Shaft and Shaft Support System

The function of the shaft and shaft support system is to axially locate the test bearing and
to provide the rotational motion to induce a hydrodynamic fluid film within the rotor-bearing
system. The shaft and supporting elements are designed to provide a radially stiff structure for
the test bearing while operating as noise and vibration free as possible.

The shaft assembly with ball bearing packs is presented in Figure A-2. Due to its ease of
machining and corrosion resistance, 4140 stainless steel is used to fabricate the shaft. The
nominal diameter of the shaft at the fluid bearing testing surface measures 101.6 mm (4.000 in)
and steps down to 50.00 mm (1.969 in) toward either end to accommodate the 210 series support
bearings. The first critical speed of the shaft is calculated from shaft dimensions to occur at
approximately 65,000 rpm, significantly above the test bed maximum operating speed of 2,125
rpm, ensuring the shaft will behave as a rigid body for all operating speeds. After machining, the
shaft is dynamically balanced to less than 1.8 gm-mm in all planes to reduce imbalance related
vibrations. Two sets of MRC 210 RDS ABEC 7 precision angular contact duplex bearings are
used to support the shaft rigidly at either end. The bearing packs are pressed onto the shaft in a
back-to-back arrangement. A preload of 300 N is applied by a lock nut to remove all internal bearing clearances.

Figure A-2. Section view of shaft assembly.

A36 plate steel pedestals house the bearing packs and function to support the shaft assembly. Figure A-3, a cut-away view of the support pedestals, illustrates the fixed-end/free-end bearing housing design used to axially locate the shaft. The drive-side pedestal clamps the outside edges of the bearing outer race, allowing the bearing to support thrust loads, providing axial positioning of the shaft. The free-end pedestal uses a looser housing fit (h5 housing bore tolerance vs. drive side js5 housing bore tolerance) and an axially unrestrained bearing outer race to allow the “free” end of the shaft to float in the axial direction, thereby preventing the buildup of thermal stresses induced by shaft elongation during rig operation.

Due to the relatively low operating speeds of the test rig, and therefore minimal heat dissipation requirements of the ball bearing packs, a lithium grease is used for the ball bearing lubricant, avoiding the complexities of a circulating oil lubrication system and allowing for a simplified self-contained housing bore design. Single lip radial shaft seals located at either end of the pedestal bearing bore are used to contain the lubrication within the bearing housing and help prevent debris from contaminating the bearings.
In an effort to further increase shaft stiffness and reduce shaft bending motion, the support pedestals are designed to fit as close to each other as possible, minimizing shaft length, allowing just enough axial space between housing ends to clear the width of the test bearing housing and external instrumentation. In constructing the support pedestals, the pedestal blocks are pinned and bolted to the base plate and then line-bored in the same machining operation to ensure precise collinearity of their centerlines. The entire testing rig fixture is then rigidly clamped to the laboratory test bed.
A.3 Test Bearing and Bearing Housing

The test bearing assembly consists of two main parts: (1) the hydrodynamic test bearing and (2) the bearing housing (Figure A-4). The test bearing is fabricated from bronze bearing stock to have an initial radial shaft clearance of 0.241 mm (.0095 in). The test bearing is encased by an aluminum housing which serves to contain the circulated oil and provide attachment points for the static and dynamic loading, lubrication inlet, and external instrumentation.
Figure A-4. Section view of test bearing assembly.

The test bearing contains three transducers mounted within the body of the bearing that measure the axial pressure distribution of the fluid film via small diameter tap holes. Rotation of the bearing housing about its axis rotates the embedded pressure transducers, allowing measurement of the film pressure distribution around the bearing circumference. Removable side
shields allow the external displacement instrumentation to remain stationary relative to the rotating test bearing.

Lubrication is provided to the fluid bearing by an oil inlet hole located in the low pressure region of the hydrodynamic film. After circulating through the bearing, the oil empties into the bearing housing cavity. There the oil is contained by non-contact labyrinth seals located in the housing side shields and exits through drain holes located at the bottom of the housing to be returned to the oil reservoir for filtering and recirculation.

A.4 Lubrication System

The lubrication system for the test rig is designed to continuously supply ISO 10 weight, detergent-free spindle oil to the hydrodynamic test bearing. Figure A-5 illustrates the circulation path of the oil with a schematic of the lubrication system. Oil is drawn from the supply reservoir and delivered to the test bearing by a gear pump driven by a 373 W (0.5 hp) variable speed direct current motor. The supply pressure is set by a combined adjustment of the pressure bleeder valve and gear pump motor speed. This setup allows relatively low oil flow volumes and pressures to be supplied to the test bearing while providing adequate cooling and flow volumes for ideal gear pump operation. An in-line 10 micron hydraulic filter, positioned between the oil reservoir and the gear pump, prevents unwanted particles from entering, and possibly damaging, the gear pump and test bearing.
The design of the lubrication attachment points of the bearing housing is critical to ensure the floating test bearing is isolated from external forces and vibrations. Flexible hosing is used to supply oil to the bearing housing from the pump. The lubrication inlet hosing is routed so that it does not exert any tensioning forces that could alter the steady state operating position of the floating test bearing (Figure A-6). Circulated oil exits the bearing housing through drain holes located on either side of the test bearing. A drain pan collects the exiting oil and routes the oil back to the supply reservoir for recirculation (Figure A-7).
Figure A-6. Minimally constraining lubrication supply line and bearing housing attachment.
A.5 Drive System

The test rig shaft assembly is spun by a 1.5 kW (2 hp) variable speed direct current drive motor by way of an internally toothed pulley belt. A 24:17 pulley ratio increases the torque supplied to the shaft allowing the drive motor to overcome the start-up friction in the test rig. This pulley belt configuration (Figure A-8) also helps to isolate the shaft assembly and test bearing from drive motor vibration.
A.6 Static and Dynamic Loading System

Static and dynamic loading is applied to the floating test bearing via external attachments to the test bearing housing. Vertical static loading is applied via a pulley/hanging mass system (Figure A-9). The vertically applied static load is adjusted by adding and removing calibrated weights from the hanging mass loading plate. Dynamic loading is applied directly to the bearing housing via two orthogonally mounted Wilcoxon F4 electromagnetic shakers. Each shaker is capable of exciting the bearing with dynamic loads up to 44.5 N (10 lbs) at frequencies ranging
from 10 Hz to 7,500 Hz. The amplitude and direction of the net applied dynamic force from the two shakers can be adjusted by varying the relative phase and amplitude of the two shakers. ICP quartz force ring sensors, mounted between the shaker and bearing housing, measure the vibrational force supplied by each shaker. Figure A-10 presents a schematic of the shaker orientation and force transducer arrangement. Multiple attachment points around the outside circumference of the bearing housing allow the static and dynamic loading points to remain directionally stationary for each angular rotation position of the test bearing.

Figure A-9. Static load support structure and loading plate.
A.7 Data Acquisition and Reduction

The on-rig instrumentation of the test bearing measures the relative bearing displacement and fluid film pressure. Two sets of orthogonally positioned Bently Nevada 3300 XL 8mm eddy current displacement sensors mounted to the bearing housing measure the motion of the bearing relative to the shaft (Figure A-11). The time-dependent angular position and rotational speed of the shaft are measured by a fifth displacement sensor recording the once-per-revolution signal produced by a machined notch in the shaft. Three Entran EPX-V03 absolute pressure sensors mounted in the body of the test bearing (Figure A-12) measure the pressure distribution of the hydrodynamic fluid film.
Figure A-11. Orthogonally positioned non-contact displacement sensors mounted to test bearing assembly.
Figure A-12. Test bearing with embedded pressure sensors.

Instrumentation inputs and outputs are monitored and managed by a LabVIEW Virtual Instrument (VI). Shaker frequency, relative phase angle, and left and right signal amplitude values are set by inputs on the VI control panel (Figure A-13). Sinusoidal outputs for the left and right shaker channels, based on the input settings, are sent from the VI to their respective electromagnetic shakers via a power amplifier. Displacement sensors measure the relative vertical and horizontal bearing displacements induced by the shaker excitation. Measured output is fed back to the VI where the spectral content of the vertical and horizontal displacement signals is calculated by fast Fourier transform. The phase and magnitude of each output signal, at the frequency of excitation, are plotted and compared to produce a plot of the shaker-induced bearing
orbit which is displayed on the VI main screen. Based on the ellipticity and angle of the excitation orbit, the shaker inputs (phase, left and right amplitudes) can be adjusted to produce unique bearing displacement orbits. The bearing rig instrumentation output signals are recorded by a National Instruments PXI-1000B data acquisition chassis. Data is recorded for a ten second record length at a sampling rate of 20 kHz. A screen shot of the data acquisition interface is presented in Figure A-14.

Figure A-13. Screenshot of LabVIEW VI used to control shaker excitation signals.
Figure A-14. Screenshot of National Instruments data acquisition interface.
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