SELF-POWERED THERMOACOUSTIC SENSOR FOR IN-PILE NUCLEAR REACTOR MONITORING

A Thesis in
Acoustics
by
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Abstract

Inspired by the unfortunate events of the Fukushima Daiichi nuclear disaster in Japan, 2011, the Pennsylvania State University began a collaboration with Idaho National Laboratories (INL) to embark upon the development of a self-powered sensor to monitor the temperature inside a nuclear reactor. A resonator manufactured by INL in accordance with dimensions and materials of a nuclear fuel rod was adapted to accommodate a thermoacoustic stack-based (standing wave) engine.

This thermoacoustic temperature sensor has resulted in a simple solution that is synergetic with the harsh temperatures of the nuclear reactor. Through electromagnetic radiation, these high temperatures provide the power needed to generate the thermoacoustic oscillations in the device. These acoustic pressure oscillations will propagate throughout a nuclear reactor by sound radiation and its frequency can be measured remotely, which is related to the temperature of the gas in the nuclear fuel rod. This temperature inferred from the frequency of the acoustic oscillation is an effective temperature of the nuclear fuel rod that the acoustic wave spatially averages across the length of the resonator. The interpretation of the thermoacoustic fuel rod resonance frequency as a representation of temperature will be also presented as a T-matrix approximation that properly weights the local sound speed in a resonator with a significant longitudinal temperature gradient.

The heat transfer within the device is also discussed, as acoustic streaming which is inherent to the thermoacoustically generated sound wave keeps the device in continuous operation. The streaming results in forced gas convection which removes heat from the ambient end of a stack and passes into the fluid surrounding the fuel rod. In conjunction with the electromagnetic radiation on the hot side of the stack, this eliminates the requirement for any hot or cold heat exchangers. This thermoacoustic sensor also has no physical moving parts and does not require any cables to or from the fuel rod for successful operation.
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“There must be no barriers to freedom of inquiry... There is no place for dogma in science. The scientist is free, and must be free to ask any question, to doubt any assertion, to seek for any evidence, to correct any errors. Our political life is also predicated on openness. We know that the only way to avoid error is to detect it and that the only way to detect it is to be free to inquire. And we know that as long as men are free to ask what they must, free to say what they think, free to think what they will, freedom can never be lost, and science can never regress.”

- J.R. Oppenheimer
Introduction

1.1 Motivation: The Fukushima Daiichi Nuclear Disaster

On Friday, March 11, 2011, at 2:46pm (Japan Standard Time), the Tōhoku region, on the east coast of northern Japan (see figure 1.1) experienced what would be known as the largest earthquake to be recorded in Japan, at magnitude 9.0 on the Richter scale [1]. The earthquake occurred in a region where the Pacific Plate is beneath the Eurasian and North American Plates, and an area of 400 kilometers long and 50 kilometers wide was ruptured [1]. This triggered an inevitable tsunami in excess of 15 metres tall [2] which soon followed the earthquake. As of November 28, 2012 there has been 15,870 deaths, 6,114 injured, 2,814 missing persons, 129,225 collapsed buildings, 254,204 ‘half-collapsed’ buildings and 728,724 partially damaged buildings as reported by a Japanese National Police Agency [3]. There was also significant damage and disruption to roads, bridges, gas and water supplies, telecommunications and electricity. It has been estimated by the World Bank that the damages could reach up to US$ 235 B [4].

If things were not bad enough, the nuclear power plants, in particular the Fukushima Daiichi nuclear power plant, suffered extensive and irreversible damage (these regions are also shown in figure 1.1). At the Fukushima Daiichi site, there were six operating units, each with a boiling water reactor. When the earthquake struck, three of the six reactors were operating and the others were in a periodic
inspection outage phase. In one of the reactors under inspection, all the fuel had been relocated to a spent fuel pool in the reactor building [2]. The seismic acceleration of the earthquake brought the three operating units to an automatic shutdown and since there was damage to the power transmission lines, the emergency diesel generators (EDG) were automatically started to ensure the cooling of the reactors and spent fuel ponds [2].

The situation was under control until the tsunami hit at about 45 minutes later with a maximum wave height of approximately 15m, which was greater than the sea wall height of 5m [2]. The influx of water submerged the EDGs, the electrical
switchgear, and DC batteries, resulting in the total loss of power to five of the six reactors [2].

Figure 1.2. Restricted Areas in Japan as of July 2012. [6].

This, in-turn, resulted in the loss of instrumentation that would otherwise be used to monitor and control the emergency. The ugly aftermath included high radiation exposure to the operators at the nuclear power plants, early contamination of food supplies, water and designation of several restricted areas in Japan[7] where the radiation levels have been rendered unsafe for living conditions as shown
in figure 1.2.

These series of unfortunate events are what has inspired this research. It has highlighted the need for a novel sensor/instrumentation system that can withstand similar or worse conditions as well as a back up power strategy to enable pumps to keep running and keep the reactor cores cool. While such systems and strategies will not necessarily avert a similar disaster, it will definitely become additional tools for the operators of the reactor when they experience a loss of the existing instrumentation within the nuclear reactor.

In particular, temperature sensors would be quite useful in similar disaster situations that can potentially occur. With temperature information, the operators can determine if the nuclear fuel is close to its melting point and if the mechanical components within the reactor are within their thermal expansion limits. Knowing the temperature of the surrounding cooling fluid would also be an indication of proper operation of the cooling systems. Furthermore, in a disaster situation a temperature measurement is indicative of the rate at which the temperature is increasing within the nuclear reactor. This can buy useful time to implement emergency cooling systems or evacuation from the site of a disaster.

Current temperature monitoring within a nuclear reactor is done using resistance temperature detectors (RTDs) and thermocouples [8]. These sensors are typically used in the inlet and outlet flows of the cooling fluid as well as inside the nuclear core (“in-pile”). Although these sensors have been used successfully in the nuclear industry for years, there are still drawbacks in using them in the harsh conditions in a nuclear reactor. The RTDs can be subject to cracks in the platinum element, low insulation resistance and cabling issues [8]. For in-pile thermocouples, there are frequent failures with high temperature measurements for long-term irradiations, including substantial signal drifts [9].

The limitations of these sensors also strengthen the need for developing new instrumentation for monitoring temperature that can co-exist with the existing strategies. One question which this thesis seeks to answer is: Is it possible to implement a self-powered sensor that could transmit data independently of electronic networks while taking advantage of the harsh operating environment of the nuclear reactor? Our attempt to answer this question provides the impetus for the following work described in this thesis.
1.2 A Thermoacoustic Solution

The concept of thermoacoustics, which exploits the interaction between heat and sound waves, offers an attractive solution toward the development of a novel, self-powered sensor system. A device known as a thermoacoustic engine produces a sound wave from heat flowing from a high temperature thermal reservoir to a colder one. Such a device can utilize the high heat energy from a nuclear reactor and convert this into an acoustic oscillation, whose frequency can be correlated to the temperature within the reactor. Figure 1.3 takes a closer look at this idea.

![Figure 1.3](image)

**Figure 1.3.** Schematic of a Nuclear Fuel Rod and the proposed thermoacoustic solution.

The cylindrically shaped object in figure 1.3 represents a typical nuclear fuel rod, which contains a nuclear fuel source toward the left of the schematic. A nuclear fuel rod happens to be a closed-closed, half-wavelength resonator as there is nothing else in the volume enclosed except for the nuclear fuel source. To facilitate the thermoacoustic process, a material known as a “stack” is inserted into the fuel rod. The stack used in this thesis is manufactured from a ceramic and consists of a regular array of parallel pores. These stacks are manufactured as the substrate for catalytic converters, found in most automotive exhaust systems, that convert noxious emissions into carbon dioxide, nitrogen and water vapor [10]. The stack facilitates the transfer of heat to the gas in the resonator in a way that enables the flow of some of that heat to produce sound when there is a temperature difference.
along the stack. For the proposed thermoacoustic sensor, the heat source will be the nuclear fuel, and through electromagnetic radiation (E.M.R.), this heat will reach the hot end of the stack. The ambient side of the stack is initially kept cool from the surrounding cooling fluid and this temperature gradient across the stack will produce acoustic oscillations. When the acoustic wave is generated, a phenomena known as acoustic streaming[11] will occur on the other side of the stack, i.e., the ambient end (right of figure 1.3). This acoustic streaming is a convective jet of gas which will circulate hot gas away from the heat source (nuclear fuel) and along the walls of the engine and then into the surrounding cooling fluid, as depicted by the arrows in figure 1.3. This will reduce the temperature of the ambient end of the stack, maintaining the temperature gradient needed for sustained acoustic oscillations.

There is a simple relationship between the frequency of the sound, the speed of the sound within the resonator, and the temperature of the gas within the resonator. Utilizing these relationships, it is possible to establish a clear, accurate correlation of the temperature within the nuclear fuel rod to the frequency of the sustained oscillation. This frequency is propagated by sound radiation through the cooling fluid in the reactor and monitored at some distance away using a hydrophone or another wireless technique.

In the bigger picture, this solution is simple and synergetic with the harsh operation conditions of the nuclear reactor. There are no physical moving parts required for the device, there are no heat exchangers required for operation, no in-pile cabling requirements and the sensor is self-powered as it uses the nuclear fuel for its heat source. Integrating such sensor systems along with the existing nuclear reactor instrumentation can prove to be a significant benefit for the nuclear industry.

1.3 Thermoacoustic Theory

1.3.1 A Brief History

The earliest mention of the conversion of heat to sound was the Kibitsunokama[12], a large cauldron used in many Japanese rituals that produced a sound thermoac-
It is supposed that the rice cooked in the Kibitsunokama provided a similar function to a porous stack. The ritual was mentioned in a Buddhist monk’s diary[13] in 1568 and was described in a story published in 1776[14]. The first record in the scientific literature of the thermoacoustic generation of sound appears in 1777 by Byron Higgins, in an experiment where acoustic oscillations in a large pipe were excited by placing a hydrogen flame at a suitable location[15]. A popular extension of this work is the Rijke tube[16] [17]; often used as classroom demonstrations.

Over 200 years ago, glassblowers also encountered the thermoacoustic effect when they placed a hot glass bulb near a cool glass tubular system and a sound was sometimes emitted from the tip. Sondhauss quantitatively investigated this phenomena and created the Sondhauss tube[18]. It was not until 1896 that John W. Strutt (Lord Rayleigh)[19] provided a qualitative explanation of the process that converted heat to sound in the Sondhauss tube:

“If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged.”

Almost a century later, Nikolas Rott introduced the term “thermoacoustics”, asserting that “its meaning is rather self-explanatory.”[20] He published a series of papers[20] which created a detailed theoretical framework that could produce a unified quantitative description of thermoacoustic phenomena in Taconis tubes as well as the Taconis effect[21] and explicitly calculate the behaviour of the Sondhauss tube[22]. In the 1980s, Los Alamos National Laboratory began the development of efficient production of standing acoustic waves in sealed resonators by the thermoacoustic process. By 1988, the field of thermoacoustic energy had advanced to a point where thermoacoustic engines were being designed to try to compete with traditional heat engines[23]. A detailed explanation of the standing-wave thermoacoustic engine can be found in the excellent publication by G.W.Swift[24]. While most research today is dedicated to improving the efficiency of these thermoacoustic engines, the research in this thesis is different in that high efficiency is not a primary goal. Greater focus is emphasized on exploiting the signal sensing and heat transfer mechanisms.
1.3.2 Standing Wave Engines

Heat engines can be functionally categorized in two ways: (i) prime movers and (ii) heat pumps. In the first, the engine produces work through a heat flow from a hot reservoir to a cold reservoir. The latter is the reverse process in which mechanical work is absorbed and used to extract heat from a cold reservoir and deliver it to a hot reservoir. These processes are shown in figure 1.4[25]. For this thesis, the use of the term “engine” will refer to the prime mover category.

![Prime mover and refrigerator energy flow diagrams](image)

**Figure 1.4.** Prime mover and refrigerator energy flow diagrams [25].

In a thermoacoustic engine, the work output is the sound that will be generated within the engine. The sound produced from heat flow from a hot reservoir to a cold reservoir is facilitated by the stack material which is placed inside the engine. The stack material used in this thesis was developed by Corning Environmental Products [10] for use as a catalytic substrate for automotive exhaust systems in automobiles and shown in figure 1.5.

Another distinction also needs to be made, since in the field of thermoacoustics, there is a further subdivision within the class of engines. This distinction is based upon the phasing of the working fluid’s (gas) pressure and velocity. For pressure and velocity nearly 90° out-of-phase, the engine is called a stack-based (standing wave) engine, and for pressure and velocity that are in phase, the engine is called
a regenerator-based (travelling-wave) engine. While the stack-based engine suffers from a reduction in performance because of the irreversible process of heat transport across a non-zero temperature difference\[11\], the simplicity of such an engine makes it attractive for incorporation within a fuel rod that can be inserted into a nuclear reactor when efficiency is not an important design consideration.

To understand the operation of the thermoacoustic standing wave engine, a half-wavelength, closed-closed resonator, with air as the working gas, will be considered. Firstly, it is important to know how the change in gas temperature is created by an adiabatic change in gas pressure. Manipulating the adiabatic equation of state and the Ideal Gas Law, equation 1.1 is found \[26\], where $\gamma = \frac{c_p}{c_v}$ is the polytropic coefficient, $p_1$ is the acoustic pressure, $p_m$ is the mean pressure, $T_1$ is the oscillating temperature and $T_m$ is the mean temperature.

$$\frac{T_1}{T_m} = \frac{\gamma - 1}{\gamma} \frac{p_1}{p_m} \tag{1.1}$$

Secondly, the existence of a thermal penetration depth, $\delta_\kappa$ needs to be introduced. The defining equation of the thermal penetration depth is given in equation 1.2, where $\rho$ is the density of the gas, $\kappa$ is the thermal conductivity of the gas,
\(c_p\) is the isobaric specific heat per unit mass and \(\omega\) is the angular frequency of the acoustic wave. It represents the characteristic length that characterizes the thickness of an oscillatory thermal boundary layer [27] defined by the distance heat can diffuse in one acoustic cycle. The boundary layer is the transition length from pressure oscillations being adiabatic to isothermal. Successful operation of the thermoacoustic standing-wave engine requires a deliberately imperfect thermal contact with the gas and the stack, through a spacing of a few \(\delta_\kappa\) [11].

\[
\delta_\kappa = \sqrt{\frac{2\kappa}{\rho c_p \omega}}
\]  

(1.2)

A simple and accurate approximation at low amplitudes to the motion of the gas is a simple harmonic motion. A half-wavelength resonator contains several modes, but it becomes easier to focus on the lowest mode of vibration and visualize the standing wave as a central mass of gas that oscillates between two springs on either ends of the resonator. Figure 1.6 shows the half-wavelength resonator and a four step cycle of the gas motion within the stack of the engine.

In the first step of the cycle, the gas parcel moves toward the closed end (closer to the location of the hot heat exchanger, left in case of figure 1.6) increasing the pressure by \(p_1\). Equation 1.1 indicates that this must increase the temperature of the gas by \(T_1\) and fig 1.6 designates the temperature as increasing from \(T_o\) to \(T_{++}\). Since the heat is applied to this (left) end of the stack, the stack is at a higher temperature, \(T_{+++}\). This provides the conditions necessary for step 2, as the gas being a thermal penetration depth away from the stack will absorb heat from the stack and increase its temperature further to \(T_{+++}\). In step 2, since the gas is at a constant high pressure, this increase in gas temperature causes an increase in volume of the parcel of the gas. Consequently, work \((p\Delta V)\) is done to the gas by the flow of heat from the stack to the gas. This corresponds directly to the “heat being given to the moment of greatest condensation” as Rayleigh theorized.
In the third step, the gas moves toward the other closed end and is decompressed adiabatically. The corresponding lowering of temperature is shown in fig 1.6, from $T_{+++}$ to $T_+$. With the gas at a higher temperature than the stack and at the thermal penetration depth from the stack’s surface, step four shows the heat transferred from the gas to the stack. Since this removal of heat from the gas occurs at a constant low pressure, the volume of the parcel is decreased. Hence, work is $(p\Delta V)$ is done to the gas and this corresponds to the other part of Rayleigh’s theory, where “heat being taken from it at the moment of greatest rarefaction”.

From this four step cycle, the properly phased heating with compression and expansion of the gas and the work being done on the gas, the “vibration is encouraged”. In steady state operation, the work input per cycle is equal to the sum of the
work absorbed by dissipative mechanisms (viscous and thermal losses in the stack and resonator walls) and the acoustic energy produced. To maintain sustained oscillations within the resonator, each gas parcel must be at a position where the stack is at a different temperature from the adiabatic temperature change of the gas. That fact leads to a concept of a critical temperature gradient for the acoustic oscillations to be maintained. A simple expression for this critical gradient is given in equation 1.3[26], where $k$ is the acoustic wavenumber. For a closed-closed resonator, since the wavelength of the fundamental resonance is two times the length ($L$) of the resonator, $k = \frac{\pi}{L}$. This expression, however, neglects complications that are introduced by the viscosity of the gas, the partial occlusion of the resonator’s cross-sectional area by the stack material, and the assumption that the available heat capacity per unit area of the solid stack material overwhelms the available heat capacity per unit area of the gas that is in contact with the stack [26]. A more detailed expression is discussed in [23].

\[
(\nabla T_m)_{crit} = \frac{\partial T_m}{\partial x} \approx (\gamma - 1)kT_m\cot(kx)
\] (1.3)

It is important to emphasize how synergistic this thermoacoustic engine design is with the nuclear application. The heat from the nuclear fuel and the surrounding cooling fluid initially establishes the necessary temperature gradient for sustained acoustic oscillations. The heat from the nuclear fuel is deposited on the hot end of the stack through electromagnetic radiation. The temperature gradient across the stack is maintained from the acoustic streaming that continually removes heat from the ambient end of the stack. As a result, this thermoacoustic sensor will function without any hot or cold heat exchangers. Additionally, there are no physical moving parts, nor are there any electrical cables required inside the fuel rod for this sensor.
Without having (or wanting!) any nuclear raw material, laboratory “simulations” of the process of inserting a thermoacoustic engine into a nuclear reactor had to be designed. The idea here was to fully instrument the resonator with sensors, mainly pressure and temperature, and then submerge this instrumented resonator into a calorimeter filled with water. With knowledge of the pressure and temperature of the sustained acoustic oscillations, it is possible to understand the temperature-frequency dependence as well as the effect of acoustic streaming on the heat transfer of the system. This chapter will describe the instrumentation and the equipment used to accomplish the laboratory experiments.

2.1 Thermoacoustic Resonator

Idaho National Laboratories manufactured a resonator which is of the same dimensions and material as that of a nuclear fuel rod. It is made from Nitronic 60 Stainless Steel. Figure 2.1 shows a cross-sectional scale drawing of the resonator with all the dimensions in inches. The resonator is built with a 1-8 UNC screw nut on one end and a flange at the other end. The screw nut, compresses a grafoil gasket, which ensures a pressure tight seal at high temperatures. The flange on the opposite end allows for the attachment of different instrumentation plates to measure characteristics of the resonator, as well provide information such as pressure and temperature during operation. The stack in this resonator is placed closer to the end with the screw nut (hot end).
Initial testing of the resonator to generate thermoacoustics failed to reach onset of thermoacoustic oscillations. One of the main reasons for this was that the heat generated on the hot end of the stack was not flowing into the gas, but rather into the walls of the resonator. As a remedy, the wall of the resonator was thinned in the vicinity where the stack is located as shown in figure 2.2. The wall thickness was reduced by one quarter of its original thickness for a length of 1” along the resonator.

![Figure 2.1. Schematic of the resonator built from INL. All dimensions in inches.](image1)

![Figure 2.2. Thinned wall of the resonator.](image2)

### 2.2 Instrumentation

#### 2.2.1 Instrumentation Plates

To prepare the resonator for measurement, instrumentation plates had to be built. The first instrumentation plate that was built was to measure the Quality Factor
(Q-Factor) of the empty resonator without any thermoacoustics. The second instrumentation plate built was to measure pressure, temperature and provide the ability to fill the resonator with different gases, if necessary.

### 2.2.1.1 Q-Factor Plate

Figure 2.3 shows the Q-Factor Plate. It was manufactured from PVC and has 6 bolt circles on the outer region that line up exactly with the flange of the resonator to create an air tight seal. On the front face of the plate is a mini electrodynamic driver manufactured by Matsushito (Model EAS1P103A), which is used to excite the resonator. At the side of the plate, an electret microphone, Panasonic WM-62GT301A, was inserted to measure the response of the resonator. An HP 3562A Digital Signal Analyzer (DSA) is used to send a sinusoidal sweep signal to the speaker and measure the response from the microphone. The DSA uses a pole-zero fit function to find the Q-Factor of the different resonances within the resonator.

![Q-Factor instrumentation plate. At the 10 o’ clock position, there is the small electret mic. A hole was drilled through the PVC to provide a path for the airflow to reach the mic.](image)
2.2.1.2 Sensor Instrumentation Plate

Similar to the Q-Factor Plate, a sensor instrumentation plate was built, manufactured from brass. This plate was designed to measure acoustic and static pressures as well as mean temperatures along the length of the resonator. It was also designed to allow the resonator to be filled with different gases or to be pressurized. For pressure measurement, an Endevco Pressure Transducer model 8530C100 was used. This is a miniature, high sensitivity piezoresistive pressure transducer and has a range up to 100 psia (≈ 69 kPa). The factory calibration is 1.60 mV/psi.

The microphone was connected to a Stanford Research Systems amplifier (model SR560) with a gain of 100 and a band-pass filter from 300 Hz to 3 kHz. For temperature measurement, three Type-E thermocouples and an integrated circuit, current output temperature sensor, AD 592, were used. The sensitivity of the AD 592 is 1µA/K and has an operating range from -25°C to 105°C. A Schrader valve was also included to insert different gases or pressurize the resonator. Figure 2.4 shows the layout of the instrumentation plate. An O-Ring, size -116 (AS568A-116) also goes around the elevated part of the plate, to ensure a leak free seal with the resonator.

A thermocouple array was also built which facilitated the positioning of the three Type-E thermocouples anywhere along the resonator. Three legs were soldered to a stiff wire in two different positions and the thermocouple wire was wound around the stiff wire to the desired location, as shown in figure 2.5. This stiff wire was inserted into the resonator until it came into contact with the cold end of the stack. The placement of the thermocouples was driven by what the particular experiment was trying to accomplish. In most experiments, one of the thermocouples measured the temperature of the hot end of the stack and another measured the cold side of the stack. The third thermocouple was often used to probe different parts of the resonator, mostly the center or the hot duct end of the resonator.

Since the resonator would be submerged in water during the experiment, this instrumentation plate needed to be protected from any water that could potentially damage the sensors or their wires. Consequently a PVC housing had to be built around the plate, which also provided a path for the wires from the sensors to the environment without becoming wet. This PVC enclosure is shown in fig. 2.6.
Figure 2.4. Sensor Instrumentation Plate. At the 12 o’clock position, the AD592 sensor measures the gas temperature at the ambient end of the resonator. Going clockwise, it is followed by three feedthrough pairs, which are for three thermocouples that can measure temperatures along the resonator and stack. At the 6 o’clock position lies the Endevo Pressure Sensor which measures the static and acoustic pressure of the gas oscillations, it is conveniently located at a pressure antinode. Finally, the dark hole is the other side of the Schrader valve used to fill the resonator with a different gas or pressurize if necessary. The pressurization option was useful also for leak testing.

Figure 2.5. Thermocouple array to be inserted into the resonator.
2.2.1.3 The “Killer”

To make heat transfer measurements with and without the presence of an acoustic oscillation in the resonator, a mechanism had to be devised that could suppress the acoustics when desired. The sensor instrumentation plate that was previously described, included a Schrader valve. The original intent of the Schrader valve was to have the capability to pressurize the resonator or fill it with different gases, if necessary. Although the Schrader valve’s only practical purpose was to check for gas leaks during experimentation, it turned out that it was an appropriate device for fully suppressing the acoustic oscillations as desired. When the stem of the Schrader valve is depressed, this would essentially change the boundary condition on this end of the resonator from a rigid boundary to a pressure release boundary and introduce additional dissipation. Since the thermoacoustic engine was designed to operate in a closed-closed, half wavelength resonator, the same operating conditions fail to reach onset with a different set of boundary conditions. The acoustic dissipation will also lower the quality factor of the resonator and contribute to the suppression of the acoustic oscillation.

To automatically kill the acoustic oscillations while the resonator was submerged in water, a linear actuator, with a narrow shaft was used to depress the valve. This was wired to a simple switch that could be controlled from the outside.
to either depress or release the valve. A PVC housing was built around this to prevent water from damaging the actuator or shorting the wires. The mechanism is shown in figure 2.7.

Figure 2.7. The “Killer” mechanism is used to change the boundary conditions on the thermoacoustic engine, hence suppressing the acoustic oscillations. The black object is the linear actuator with the narrow shaft that is moving to depress the Schrader valve. The movement of this shaft is controlled externally via a switchbox, when the resonator is submerged.

2.2.2 Simulating the Nuclear Fuel

In the absence of nuclear fuel, a strategy for applying heat in the same way as the nuclear fuel had to be devised. For the experiments that were performed, two methods of heating were employed, one used an indirect heater, where heat would be radiated onto the stack and the other, a direct heater where the heater was positioned against the stack.

2.2.2.1 The Indirect Heater

In the nuclear reactor, it is the radiation heat from the fuel rod that is hypothesized to create the temperature gradient across the stack required to produce sustained acoustic oscillations. The indirect heater was designed such that it would mimic this same function. To make the heater, narrow slits were cut into a block of 100 cells/in² Celcor® ceramic stack material. In these slits, NiCr wire of 0.032” diameter were seated in serpentine windings. This NiCr wire was then connected to a high temperature Nickel lead cable capable of withstanding up to 700°C
To concentrate this heat into the resonator and reduce heat loss to the ambient environment, a fire-brick furnace was built around the Celcor® block. Fig 2.8 shows the indirect heater glowing red when a current is applied. The entire furnace was placed on top of the lid of the calorimeter, directly above the resonator as shown in fig 2.9.

**Figure 2.8.** Indirect heater, with a fire-brick furnace housing and glowing red when a current is applied.

### 2.2.2.2 The Direct Heater

To provide finer control of acoustic amplitudes there was also a need for a direct heater, one that rests up against the stack. The input electrical power into the indirect heater was between 200W to 400W, while the input power into the direct heater was between 15W to 35W. The direct heater is used to focus on heat transfer measurements at lower powers, which provided more accuracy and was less dangerous. The actual heating element of the direct heater was also NiCr wire, but of 0.0256" diameter. To get the heater up against the stack, a hole had to be drilled in the screw nut, into which a ceramic tube would be placed. This ceramic tube (Omega Engineering Model TRA-11614-6) had two smaller holes, which allowed for copper leads to connect to the NiCr heater. Figure 2.10 shows the ceramic tube with the NiCr heater glowing red when a current is applied.
Figure 2.9. Fire-brick furnace on top of the fire-brick lid of the calorimeter, with the protruding resonator nut. The Indirect Heater (see figure 2.8) is lowered into the furnace during operation in this mode and rests on the lower shelf that surrounds the nut.

Figure 2.10. (Foreground) Direct NiCr Heater against the stack and glowing red. It is connected via copper leads which run through the ceramic tube. (Background) The resonator into which the direct heater and stack will be placed.
2.2.3 The Calorimeter

The fuel rods in a nuclear reactor or spent-fuel pool are surrounded by water or another cooling fluid. Hence, it was necessary that the resonator be submerged in a similar way. A 5-gallon thermally insulated beverage cooler (figure 2.11) was used as the vessel to hold distilled water into which the resonator would be submerged. The water was filled up to just below the thinned area of the resonator. This served as the calorimeter for subsequent experiments.

![Figure 2.11](image)

*Figure 2.11.* Thermally Insulated Beverage Cooler which was converted into a Calorimeter. Note the leak tight fitting where the wires exit.

The water spout of the beverage cooler had to be removed and was replaced with another leak tight fitting (McMaster Carr Part Number: 50775K335) that allowed a $\frac{3}{8}$" Tygon® tubing to pass from the PVC housing of the resonator to the outside. This encased all the wires from the sensor instrumentation plate,
protecting them from the water inside the calorimeter. The lid of the beverage cooler also needed to be replaced to allow additional instrumentation and a support for the resonator. Figure 2.12 shows one half of the lid constructed and holding the resonator under the direct heater configuration. It consists of two pieces of styrofoam, with PVC plates on the outer ends. The sandwich was held together by 6-32 machine screws. A Celcor® block was placed in the lower piece of styrofoam to hold the resonator at the screw nut and to prevent the PVC and styrofoam from melting. Also shown in figure 2.12 is one half of a PVC tube into which two holes were drilled to provide binding posts for electrical power to the direct heater. Behind this PVC tube, there is a hole drilled in the PVC-styrofoam sandwich (cannot be seen) that provides the insertion point for a thermistor which was used to measure the water temperature. The thermistor used was manufactured by Logan Enterprises, Model No 4150-1 1/8-6-72-TH55033-RPS. In the other half of the lid that is placed to seal the top of the calorimeter, there is another hole into which an electric motor with a paddle is inserted to stir the water and keep it at thermal equilibrium.

Figure 2.12. Half of the Calorimeter lid used to hold the resonator with the direct heater (ceramic tube into resonator).
Another lid also had to be made to withstand the high temperatures of the indirect heater which ranged up to 900°C. This lid was simply made from fire-brick in two halves to allow positioning of the resonator as shown in figure 2.13. Two holes were also drilled in the fire-brick again for a motor and the thermistor. The indirect heater (see figure 2.8) was placed directly on top of this fire-brick lid, right above the screw nut of the resonator.

![Fire-Brick Lid for the Indirect Heating Configuration.](image)

**Figure 2.13.** Fire-Brick Lid for the Indirect Heating Configuration. On the top right hand corner is the thermistor (S/N: Rasta) used for measuring water temperature. On the left hand side is a small DC electric motor to stir the water and eliminate any temperature gradients.

### 2.2.4 Data Acquisition

The HP 34970A Data Acquisition and Control Unit with the 34901A 20-channel multiplexer card was used to acquire all the signals in the system. Signals were then fed to a software package from National Instruments, called LabVIEW™. LabVIEW™ was run on an HP Laptop Computer with Windows 7 and 4GB of RAM. Table 2.1 shows the different signals measured and the associated channels on the HP 34070A. A LabVIEW™ program was written to communicate with
the HP 34970A and automatically record all the data at an adjustable sampling interval, usually every two minutes. The LabVIEW™ program also communicated with the DSA 3562A to retrieve frequency and amplitude data from the Endevco pressure sensor. A screen shot of the LabVIEW™ program is also shown in figure 2.14.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Signal</th>
<th>Function/Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AC Voltage</td>
<td>Endevco Mic (Acoustic Pressure)</td>
</tr>
<tr>
<td>2</td>
<td>TC2</td>
<td>Stack Hot End (Type-E)</td>
</tr>
<tr>
<td>3</td>
<td>TC3</td>
<td>Stack Cold End (Type-E)</td>
</tr>
<tr>
<td>4</td>
<td>TC4</td>
<td>Temp at Center of Res (Type-E)</td>
</tr>
<tr>
<td>5</td>
<td>TC5</td>
<td>Screw Nut Temp (Type-K)</td>
</tr>
<tr>
<td>8</td>
<td>TC8</td>
<td>Ambient Air (Type-E)</td>
</tr>
<tr>
<td>9</td>
<td>Thermistor</td>
<td>Water Bath Temp.</td>
</tr>
<tr>
<td>10</td>
<td>AC Voltage</td>
<td>Variac</td>
</tr>
<tr>
<td>11</td>
<td>AC Voltage</td>
<td>Variax 0.103 Ω current sense</td>
</tr>
<tr>
<td>12</td>
<td>DC Voltage</td>
<td>Endevco Mic (DC Pressure)</td>
</tr>
<tr>
<td>13</td>
<td>Frequency</td>
<td>Frequency of Acoustic Oscillation in Resonator</td>
</tr>
<tr>
<td>21</td>
<td>DC Current</td>
<td>AD 592 Gas Temp Sensor</td>
</tr>
</tbody>
</table>

Table 2.1. Signals read by HP34970A.
Figure 2.14. Screen Shot of the LabVIEW Program to acquire the signal information.
Chapter 3

Modelling With DeltaEC

3.1 DeltaEC Basics

The software package, Design Environment for Low-amplitude Thermoacoustic Energy Conversion (DELTAEC) is designed to solve the one-dimensional wave equation to calculate spatial dependence of acoustic pressure and velocity [28], particularly useful to thermoacoustic engines. The program was originally written by Dr. Bill Ward and Dr. Gregory Swift of Los Alamos National Laboratories. The user is able to design the engine by a series concatenation of what DELTAEC refers to as “segments”. These segments include simple geometries such as ducts or cones, stack geometries, transducers (speakers), heat exchangers and compliances. Conventional acoustic boundary conditions of geometry and impedance can be imposed as well as temperature and thermal power in thermoacoustic systems [28]. The user can also choose the working fluid and a pre-programmed option to include gases and liquids.

With a complete model constructed from the desired segments, the user will go through a process of setting Guess and Target vectors, which DELTAEC then uses to converge to a solution via implementation of a shooting method. The Guess vector, as the name suggests, contains values of particular variables the user is trying to find. The Targets can be particular boundary conditions or desired parameter values in a particular segment. For instance, the user can guess what the volume velocity will be for an acoustic pressure of 1 Pa and a frequency of 100 Hz, when the Target is set to infinite complex impedance at the end of the
model (closed end). DELTAEC can sometimes be quite frustrating for the novice thermoacoustician because if the Guess vector is not set in the vicinity of the actual solution, the model may not converge. Additionally, making big steps in parameter changes can cause the model to diverge quite easily. Some strategies to avoid this problem would be to check the heat fluxes in heat exchangers and work flux along the stack to assure energy conservation and to check the acoustic variables and temperatures to ensure they make physical sense [25] and agree with experimental data, if available. Another useful option in DELTAEC is to create incremental plots. This allows the user to vary a particular parameter and observe the effect of this change on other variables. It is possible to increment two parameters at once in nested loops. This feature is valuable when trying to optimize a design for a parameter such as stack length, position or pore size.

### 3.2 Thermoacoustic Engine Model

Figure 3.1 is the constructed DELTAEC model of the thermoacoustic engine with the dimensions of the resonator built by INL for this project. Before inserting a stack into the actual resonator, DELTAEC was used to guide the choice of stack size (i.e., cells/in²), its length, and its position in the resonator usually specified as the distance from the hot boundary. The incremental plot feature was used to accomplish this. To gain a better understanding of how DELTAEC is used to create a model and optimize a particular design, the different parts of the model in figure 3.1 will be explained.

As discussed in the previous section, the model consists of a number of segments, which are highlighted in purple tabs in figure 3.1. A visualization of the model is also helpful and a good cross check to see if there are any anomalies with the geometry. Figure 3.2 shows DELTAEC’s schematic drawing of the model.

In figure 3.1, it should be noted that the “RPN” segment was used multiple times. This is not a physical part of the geometry but rather is a segment that is used to make calculations of various quantities, such as gas displacements and speed of sound probes in various parts of the model. It makes the calculations using a Reverse Polish Notation [29], hence the name “RPN”. The other segments that were used will now be explained [30]:

```
Figure 3.1. Fuel-rod thermoacoustic engine DELTAEC model, with a few segments “open” to illustrate parameter values.

**SURFACE:** A surface does not have any volume, it is usually placed at the end of a **DUCT**, before a **HARDEEND**. It absorbs acoustic power by thermal relaxation on the surface. The area input for this was the cross-sectional area of the resonator $2.4 \times 10^2$ m$^2$.

**DUCT:** As the name suggests, this is essentially a pipe of any cross section,
Figure 3.2. Schematic of DELTAEC thermoacoustic engine model. The numbers in this drawing correspond to the number of the segments in figure 3.1. The labels “G” and “T” represent the guesses and targets defined in the model. “RPN” refers to segments where Reverse Polish Notation was used.

whose dimensions are provided by the user. In this model, the duct was used to represent the tubular sections of the resonator. Note that one continuous duct was not used because the stack needed to be included and a different type of duct was used (explained below) on the ambient end of the resonator. The length of this duct was the hot section of the resonator up to the hot end of the stack.

HX: Although there were no heat exchangers in the physical thermoacoustic engine, heat exchangers were included here as the hot part and cold part of the stack to allow for a heat input and removal from the model in DELTAEC. The length of each of these heat exchangers were 1 mm and will be included in the total length of the stack in the physical experiment. A length of 1mm for the HX segments was chosen since it is short in comparison to the actual stack length and will have minimal physical impact on the results of the model.

STKRECT: This segment represents the stack material. There are different types of stack segments to represent different pore geometries, however STKRECT have square (or rectangular) pores which accurately represented the Celcor® material used for the experiments. The length of this duct was the section of the resonator up to the hot end of the stack. The parameters “aa” and “bb” are half the pore width and breadth respectively, but are equal for square pores (in this case). “GasA/A” represents the volumetric porosity measure of the material, i.e. the ratio of the area of the gas to the area of the entire cell. “L-Plate” is half the thickness of a cell wall of the stack. For an 1,100 cells/in² stack with porosity of 74%, the thickness of the cell wall = 99.6 μm, hence L-Plate = 49.8 μm. For these
same parameters, \( aa = bb = 165 \, \mu m \).

**STKDUCT:** This segment represents the section of the resonator after the stack (ambient end). In a **STKDUCT**, the mean temperature along the duct is not constant. In the physical system, if there was an ambient heat exchanger, a reasonable approximation would be to assume that the temperature along the entire duct was constant and then a normal **DUCT** segment could be used. However, since this is not the case, it was necessary to understand the changes in temperature along this duct as this would influence the frequency of the acoustic oscillations. All the input parameters for this segment are the same as a **DUCT** except that it also requires the thickness of the wall of the resonator (“WallA” variable).

**VOLUME:** As the name suggests, this segment is used to calculate the volume of a section of the model. This is useful since it is somewhat tedious to calculate the effective volume of the resonator with the porous stack material in it.

**HARDEND:** This segment provides a rigid boundary condition at the end of the model. The complex impedance of this segment is set to be infinite (or inverse of the complex impedance is zero) and is a target to be met by the model, ensuring that the model would represent a closed-closed resonator system.

Observing figure 3.1 again, under each expanded segment, there are lower-case and upper-case alphabetical lettering for different variables. The lower-case letterings represent quantities that are inputs in that particular segment, for instance, 3a in segment 3 is the cross sectional area of the duct. The upper-case lettering represents output variables within that segment, so 3A in segment 3 represents the acoustic pressure magnitude in that segment. The meanings of the other variables can be found in the *User Manual* of DELTAEC[30].

Additionally, the Guesses and Targets can be identified by the parameter which has a yellow highlight to its left that says ”Gues” or ”Targ” as shown in figure 3.3 In this model, the guesses were frequency of the oscillation(1b), the initial temperature at the hot end (1c) and acoustic pressure (1d), which matched the targets of infinite complex impedance (17a, 17b) and complete dissipation of heat.
(18a). The other guess-target pair are segments 8a and 8b, which guess the heat that leaves the ambient end of the stack matched to the temperature of that part of the stack.

![Figure 3.3. A guess-target pair in DELTAEC.](image)

Referring to the earlier discussion about checks on the model and understanding the results, one useful feature we can exploit would be the ability to plot certain variables calculated by DELTAEC. The spatial distribution of acoustic pressure and velocity is well known for a closed-closed resonator, so checking this to make sure it agrees with the theory will build confidence in the model. Figure 3.4 shows the plot from DELTAEC which shows the acoustic pressure, velocity and mean temperature along the resonator. Note that there is some irregularity in the plots in the area where the stack is located.

### 3.3 Stack Optimization

The optimization of the stack was done using a combination of theory, DELTAEC and simple experiments. This optimization dictated the choice of the stack size, its position from the end of the resonator and its length. While efficiency was not a priority in the design, an effort was still made to get the loudest sound in the resonator for the smallest temperature difference. Even if the design was not fully optimized for efficiency, the fact that the thermoacoustic effect is present will provide the necessary results to draw significant conclusions.
Figure 3.4. Acoustic pressure, volume velocity and mean temperature from the DeltaEC model. Note that this is similar to the pressure and velocity distribution in a resonator of uniform temperature and uniform cross-section operating of its fundamental half-wavelength resonance.

3.3.1 Stack Size

The total power of a standing wave thermoacoustic engine is directly proportional to the imaginary part of the negative of a spatial-average function, $f_k$, which is dependent on the hydraulic radius, $r_h$ (this is the ratio of area of the wetted area to the perimeter of a cell in a stack) and the thermal penetration depth, $\delta_\kappa$ of the stack. This dependence is shown in figure 3.5 [11] and for rectangular/square pores, there is a negative minimum of this function when $\frac{r_h}{\delta_\kappa}$ is approximately 1. Consequently, the choice of the stack size was driven by this criterion.

For the half-wavelength resonator of length, $L = 0.215 \text{ m}$ and an ambient sound speed of 345 m/s, the fundamental resonant frequency is 802 Hz. Although the sound speed in the thermoacoustic resonator will be higher (due to higher temperatures), and hence the frequency as well, the frequency of 800 Hz will still be close enough to establish an appropriate choice for the stack size. At 800 Hz, $\delta_\kappa$ in air = 94.6 $\mu$m from equation 1.2.
Figure 3.5. Spatial-average Rott function, $f$, for different geometries. The ratio $\frac{r_h}{\delta_n}$ yields the spatial-average function $f_k$. \[11\]

For square pores, $r_h$ is simply one-quarter of the length of the wall of one cell of the stack. Table 3.1 shows some different values of $\frac{r_h}{\delta_n}$ that were calculated for different stack sizes. The porosity of the stacks used is 0.74. The length of each cell was calculated from the known number of cells per inch and multiplying by the square root of the porosity (This neglects the cell wall thickness in the first approximation).

<table>
<thead>
<tr>
<th>Sample (cells/in)</th>
<th>Length (cells/in^2)</th>
<th>Length (µm)</th>
<th>$r_h$ (µm)</th>
<th>$\frac{r_h}{\delta_n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>400</td>
<td>1,092</td>
<td>273</td>
<td>2.89</td>
</tr>
<tr>
<td>33</td>
<td>1,089</td>
<td>662</td>
<td>166</td>
<td>1.75</td>
</tr>
<tr>
<td>58</td>
<td>3,364</td>
<td>377</td>
<td>94</td>
<td>1.00</td>
</tr>
<tr>
<td>24</td>
<td>576</td>
<td>910</td>
<td>228</td>
<td>2.41</td>
</tr>
</tbody>
</table>

Table 3.1. Stack sizes and their associated ratios of hydraulic radius to thermal penetration depth at 800 Hz for air at atmospheric pressure.
From figure 3.5 and table 3.1, it is clear that the appropriate choice for the stack size is 3,364 cells/in$^2$. However, with such a high density of cells, the walls of a stack with this size were very thin and quite brittle, breaking very easily while working with it. The next best choice for the stack size was 1,089 $\approx$ 1,100 cells/in$^2$ as it resulted in a value of $\frac{r_h}{\lambda}$ relatively close to 1 and still maintained a fair amount of structural integrity.

### 3.3.2 Stack Location

To reduce viscous dissipation of acoustic power, it is recommended that the magnitude of the local acoustic impedance be significantly larger than $\frac{c}{A}$ since that creates the condition where the acoustic velocity is small [11]. In a closed-closed resonator, there are velocity nodes at the ends of the resonator, hence placing the stack closer to the velocity node seems sensible. However, since the acoustic power is proportional to the quantity of the imaginary part of the product of pressure and complex conjugate of the volume velocity, placing the stack exactly at the velocity node will result in zero net power [11]. The compromise that is recommended by Swift between high power (with a high velocity) and a high efficiency (with low velocity), is to place the stack a distance of $\frac{\lambda}{20}$ from the nearest velocity node (or pressure antinode)[11], where $\lambda$ is the wavelength, which in this case corresponds to twice the resonator length (0.215 m) = 0.430 m. Hence the recommended location for placement of the stack was 21.5 mm from the hot end of the resonator. DELTAEC was also used to guide this decision and a similar location was recommended. However, experimentally with the stack at this position, the resonator did not reach onset. This was because a large quantity of the heat was being lost to the walls of the resonator and not directed down into the gas. This was one of the drawbacks of using DELTAEC for this part of the model since it did not take this wall loss into consideration. Alternatively, through some trial and error, the stack location that allowed the resonator to reach onset was found to be at a distance of 36.7 mm from the hot end.
3.3.3 Stack Length

There is some optimized critical gradient across the stack that is required for the thermoacoustic engine to reach onset, and it is directly dependent on the length of the stack. If the length of the stack is too short, then there will be too much heat conduction from the hot end of the stack to the ambient end of the stack, and the critical temperature gradient will not be sufficient to generate acoustic pressure oscillations. On the other hand, if the stack was too long, this will increase the viscous dissipation of acoustic power produced and it would become more difficult to get the resonator to reach onset. An experiment to physically change the stack length is tedious, so DELTAEC’s convenient incremental plot feature was utilized. The “length” parameter of the segment, STKRECT was set as the dependent variable and the acoustic pressure was observed. The stack length would be chosen based on the maximum acoustic pressure generated. Figure 3.6 shows the result from DELTAEC’s incremental plot.

![Optimization of Stack Length](image)

**Figure 3.6.** Acoustic pressure vs. stack length. The peak of the curve shows the length of the stack which gave the largest acoustic pressure.

From the graph, it is seen that the optimal stack length is 8.5 mm. Since in the DELTAEC model the lengths of the two heat exchangers are included in the stack
length, the overall stack length was chosen to be 10.5 mm. Table 3.2 summarizes the parameters of the stack used in the subsequent experiments in this thesis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stack Size</td>
<td>1,100 cells/in$^2$</td>
</tr>
<tr>
<td>Distance of Hot End from End of Resonator</td>
<td>36.7mm</td>
</tr>
<tr>
<td>Stack Length</td>
<td>10.5mm</td>
</tr>
<tr>
<td>Stack Type</td>
<td>Corning Celcor®</td>
</tr>
</tbody>
</table>

**Table 3.2.** Stack parameters used in experiments.
Chapter 4

Radiative Heat Transfer

4.1 Introduction

The nuclear fission process that takes place within a nuclear reactor generates large quantities of energy at high temperatures and requires precise control. The magnitude of the potential hazards that can occur in a nuclear reactor is so severe that including additional new components to the nuclear reactor will have to be extensively validated before being implemented. Hence, even a simple system such as the proposed thermoacoustic engine will present a challenge to be allowed to be installed inside a test reactor.

The principle of the thermoacoustic engine relies on some way to introduce and remove heat from opposite ends of the stack to maintain the temperature gradient. In a nuclear reactor, the high operating temperatures can be introduced to the stack through radiative heat transfer. This eliminates the need for a physical high temperature heat exchanger. In the same way that the sun is able to radiate its heat to the earth from over 100 million kilometers, the heat generated by the fission process in the nuclear fuel rods will be able to radiate to the hot end of the stack. Additionally, the cooling fluid surrounding the engine, as well as the acoustic streaming developed from the sustained acoustic oscillation, eliminates the need for a physical cold heat exchanger.

Therefore, the thermoacoustic engine needed to monitor temperature in a fuel rod only requires a bare minimum form, consisting of only the resonator tube containing the heat source (fuel) and a stack within that tube. There are no moving
parts or electrical cabling requirements for this system which would interfere with the nuclear process in the fuel rod, hence there is no introduction of additional safety hazards by adapting a nuclear fuel rod into a thermoacoustic engine. This section explores the idea of the radiation of heat from the nuclear fuel to the hot end of the stack more in depth, describes the experimental method and the results of the laboratory simulated operation.

4.2 Radiative Transfer Experiment

In section 2.2.2.1, the indirect heating mechanism was described and was used for the experiments in this section. The experiment to prove the radiative heat transfer was very simple: different electrical powers would be applied to the indirect heater and the acoustic pressure, along with the various temperatures would be monitored to observe any indications of the onset of the thermoacoustic oscillations. In this experiment, the three thermocouples inside the resonator measured: \((i)\) the temperature of the hot duct (between the resonator nut and hot end of the stack), \((ii)\) the hot end of the stack and \((iii)\) the ambient end of the stack as shown in figure 4.1.

![Figure 4.1. Location of the three type-E thermocouples and microphone for the radiative heat transfer experiment.](image)
4.3 Indirect Heater Measurements

After applying an input electrical power of 287W for approximately 1 hour, the thermoacoustic engine reached onset as shown by the rise of acoustic pressure in figure 4.2. While this was a very promising initial result, it is important to confirm that it is indeed radiative heat transfer responsible for moving heat from the indirect heater to the hot end of the stack.

![Graph showing temperature and acoustic pressure over time](image)

**Figure 4.2.** Onset of the thermoacoustic engine via indirect heating method at 287W.

The indirect system was allowed to run at two different input electrical powers: 287 W, 233 W and 233 W (without acoustics). Figure 4.3 shows the time history of the temperature of the nut over the entire operating duration. It should also be noted that in these experiments, there was no motor to keep the water in thermal equilibrium. At 233 W, the system was just above onset, but because the amplitude of the acoustic streaming (section 6) was low, it is possible that not enough heat was being removed from the cold end of the stack. As a result,
the temperature gradient across the stack eventually decreased below the critical temperature gradient for onset and the system operated at the same input electrical power of 233 W but without acoustic oscillations. Table 4.1 shows the average temperatures and acoustic amplitudes for each of the operating points. For the analyses that will follow, the cross sectional area of the resonator and the stack is 2.4 cm$^2$ based on an internal diameter of 1.75 cm. The distance from the nut to the hot end of the stack is 3.8 cm.

![Nut Equilibration](image)

**Figure 4.3.** Time history of the temperature of the nut at the end of the resonator for the entire measurement. In the first hour and a half, there is an initial increase in the temperature of the nut. During 1.5 to 2.5 hours shows a steady temperature at 287 W. 3.5 to 5 hours was the time period where there was operation at 233 W (with acoustics). From 5 to 7 hours, the system operated at 233 W without acoustics, after which the input electrical power was turned down to zero, and the temperature of the nut quickly falls.

Observation of Table 4.1 shows that in general the hot duct temperatures are roughly the average of the nut and hot stack temperatures, which suggests some thermal equilibrium between the nut and the hot stack. Before analyzing the radiative heat transfer, it can be ruled out that heat transfer from the nut to the stack is through conduction. Assuming that the gas is conductively transported from the gas at the nut, $T_{nut}$ to the hot side of the stack, $T_{hs}$, then from table 4.1 at 273 Pa, $T_{nut} = 739^\circ C$ and $T_{hs} = 535^\circ C$, so $\Delta T = 204^\circ C$. The thermal conductivity of air at the temperature of the hot duct ($602^\circ C + 273.15 = 875.15$
\[
\begin{array}{ccccccc}
\Pi_{elec} & AcousticPressure & Q_{net} & Nut & HotDuct & HotStack & Amb.Stack \\
(\pm 2 W) & (\pm 45 Pa_{rms}) & (\pm 10 W) & (\pm 0.1 ^\circ C) & (\pm 0.1 ^\circ C) & (\pm 0.1 ^\circ C) & (\pm 0.1 ^\circ C) \\
233 & 0 & 51.5 & 738 & 608 & 544 & 133 \\
233 & 273 & 55.5 & 739 & 602 & 535 & 115 \\
287 & 1,220 & 48.2 & 851 & 666 & 595 & 105 \\
\end{array}
\]

**Table 4.1.** Temperatures, acoustic pressures and electrical input heat powers for the three operating points in the indirect heating system. The temperatures were all measured using Type-E thermocouples, except for the nut, for which a Type-K thermocouple was used. \(Q_{net}\) is the net power going into the water. The acoustic pressure is the pressure of the acoustic wave generated within the resonator. The uncertainty of the acoustic pressure is the relative error associated with fluctuations in its amplitude during the time the device was operating at a particular electrical input power.

\(\kappa = 6.14 \times 10^{-2} \text{W/m}^{-1} \text{K}^{-1}\). Applying Fourier’s Law of thermal conduction (equation 4.1), where \(Q_{\text{cond}}\) is the heat power input, \(A\) is the cross sectional area and \(l_h\) is the distance from the nut to the hot stack, the \(Q_{\text{cond}} = 79.1\ \text{mW}\). From table 4.1, the power entering the water = 55.5 W, hence it is not possible that thermal conduction is responsible for the heat transfer.

\[
Q_{\text{cond}} = \kappa \frac{A}{l_h} \Delta T \quad (4.1)
\]

Radiative heat transfer is the more likely candidate for the heat transfer from the nut to the hot end of the stack. It is governed by equation 4.2, which is the Stefan-Boltzmann Radiation Law. A theoretical discussion of this equation will be presented, following by the estimation of this radiative quantity.

\[
Q_{rad} = \sigma AF (\epsilon T^4_{\text{nut}} - \alpha T^4_{\text{hs}}) \quad (4.2)
\]

In this law, \(\sigma\) consists of a number of well-known physical constants which yield \(\sigma = 5.6697 \times 10^{-8} \ \text{W/m}^2\text{-K}^4\). The other parameters in this equation are fairly difficult to accurately calculate. For instance, \(\epsilon\) and \(\alpha\) are the emissivities of the stainless steel resonator parts and the Celcor\textsuperscript{®} stack respectively. Those values range between 0 to 1 and depend upon surface condition of the materials and the
wavelength of the electromagnetic radiation[31]. Additionally, the product, $AF$, represents a surface area, $A$, and a “look angle” correction factor, $F$, known as the “shape factor”. While an accurate calculation cannot be done, the important thing to note about the Stefan-Boltzmann Law is that the power radiated is proportional to the fourth power of the temperatures of the nut and hot stack. Hence for high operating temperatures (such as that in a nuclear reactor), it can be expected that there will be a significant amount of heat transfer by radiation.

The fact that the resonator is an electrical conductor and can act as an electromagnetic waveguide needs to be taken into consideration for the look angle coefficient, $F$ in equation 4.1. The number of electromagnetic modes, $N$ in the hot duct volume, $V = AL = 9.1 \times 10^{-6} m^3$, is a function of the ratio of the volume to the cube of the electromagnetic wavelength, as given in equation 4.3.

$$N \approx \frac{4\pi V}{3\lambda^3} \quad (4.3)$$

The Wein Displacement Law (equation 4.4) gives a simple expression to calculate the wavelength of the electromagnetic radiation at the peak of a black-body/Plank distribution. At the nut temperature, $T_{nut} = 739^\circ C = 1,012 K$, $\lambda_{max} \approx 3 \mu m$.

$$(\lambda T)_{max} = 2.89 \text{ mm.K} \quad (4.4)$$

From equations 4.3 and 4.4, the number of modes are over 1.4 trillion, which allows us to treat the electromagnetic energy as being diffuse and ignore any influence of the waveguide effects in determining the look-angle coefficient, $F$.

To make a quantitative estimate of the plausibility of the electromagnetic radiation from the nut to the hot stack, several assumptions have to be made. An effective radiating area, $A_{eff}$ can be created by incorporating the constants $\epsilon$ and $\alpha$, as well as the shape factor, $F$. If black body radiation is assumed, then $\epsilon$ and $\alpha = 1$, so $A_{eff} = \epsilon F A$. Equation 4.2 can be modified to 4.5 where it is possible to find an effective area based on the temperatures of the resonator and decide if
this is reasonable.

\[ Q_{rad} = \sigma A_{eff} (T_{nut}^4 - T_{hs}^4) \]  

(4.5)

Table 4.1 shows the heat flux, \( Q_{net} \) into the water. Some of this heat travels along the resonator walls, \( Q_{res} \) and must be subtracted from \( Q_{net} \) to get the effective radiated heat, \( Q_{rad} \) that travels from the nut to the hot stack. \( Q_{res} \) was calculated using equation 4.1, but with \( \Delta T = T_{hs} - T_{water} \), \( \kappa = 18.5 \text{ Wm}^{-1}\text{K}^{-1} \), \( A = 1.73 \times 10^{-5} \text{ m}^2 \) and \( l_h = 0.0254\text{m} \). \( l_h \) in this case represents the length of the resonator which was thinned to reduce the heat leak along the walls of the resonator.

Table 4.2 accumulates these values and shows the calculated effective areas, \( A_{eff} \) for the different operating points. The main error associated with this calculation is the uncertainty in the net power going into the water, which is dependent on the rate of increase of the water temperature. This measurement was affected by not having a motor in the water to reduce any potential thermal gradients. As a worst case scenario, it was assumed that the thermal gradients that existed in the water was \( \pm 2^\circ \text{C} \), which translates into the rate of change of the water temperature having an uncertainty within 40\%, and hence the net power into the water being known to \( \pm 20 \text{ W} \).

The surface area of the stack was \(2.4 \text{cm}^2\) and the surface area of the nut plus the resonator in the duct region is \(23.3 \text{cm}^2\), hence the average result of \(13.3 \pm 5.4 \text{ cm}^2\) for the 233W (with acoustics) operating point seems quite plausible, assuming \( F = 1\) and the emissivities are in the range \( \epsilon = \alpha = 0.5 \pm 0.2 \).

<table>
<thead>
<tr>
<th>( \Pi_{elec} )</th>
<th>Acoustic Pressure</th>
<th>( Q_{net} )</th>
<th>Water</th>
<th>( Q_{res} )</th>
<th>( T_{nut}^4 - T_{hs}^4 )</th>
<th>( (\epsilon A)_{nut} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pm 2 \text{ W} )</td>
<td>( \pm 45 \text{Pa rms} )</td>
<td>( \pm 20 \text{W} )</td>
<td>( \pm 2^\circ \text{C} )</td>
<td>( \pm 0.1 \text{W} )</td>
<td>( \pm 0.1 \text{K}^4 )</td>
<td>( \pm 5.4 \text{cm}^2 )</td>
</tr>
<tr>
<td>233</td>
<td>0</td>
<td>51.5</td>
<td>47</td>
<td>6.3</td>
<td>5.97 x 10^{11}</td>
<td>13.4</td>
</tr>
<tr>
<td>233</td>
<td>273</td>
<td>55.5</td>
<td>38</td>
<td>8.8</td>
<td>6.21 x 10^{11}</td>
<td>13.3</td>
</tr>
<tr>
<td>287</td>
<td>1,220</td>
<td>48.2</td>
<td>26</td>
<td>10.4</td>
<td>1.03 x 10^{12}</td>
<td>6.5</td>
</tr>
</tbody>
</table>

**Table 4.2.** Calculating the effective areas of the nut from the simplified Stefan-Boltzmann Law (equation 4.5) at the different operating points. The \( Q_{rad} = Q_{net} - Q_{res} \), so that it does not take into consideration the heat that was conducted along the resonator walls.
Chapter 5

Thermometry

5.1 The Frequency-Temperature Invariant

For ideal gases, there exists a simple relationship between the pressure-independent speed of sound in a gas, \( c \), and its absolute (Kelvin) temperature, \( T \), as shown in equation 5.1. In this equation, \( R = 8.314471 \text{ J/mole-K} \) is the Universal Gas constant, \( \gamma = \frac{c_p}{c_v} \) is the polytropic coefficient, which is the ratio of the specific heat at constant pressure to that at constant volume = 1.403 for air \([32]\) and \( M \) is the mean molecular weight of the dry air = 0.02897 kg/mol \([32]\). Equation 5.1 is so well established that it is currently the best means to determine the value of \( R \) (and of Boltzmann’s constant) \([33]\).

\[
c = \sqrt{\frac{\gamma RT}{M}}
\]  
(5.1)

In a rigid-walled cylindrical resonator of constant cross-section that is rigidly-terminated at both ends, the fundamental plane-wave mode of oscillation, \( f \), corresponds to one half-wavelength, \( \frac{\lambda}{2} \), fitting within the overall length, \( L \), of the resonator (i.e., \( f = \frac{c}{2L} \)). For this case of both uniform temperature and uniform cross-section, equation 5.1 can be re-arranged to produce an invariant ratio that relates this fundamental resonance frequency of the sound, \( f \), and the absolute temperature, \( T \), since \( \gamma, R, \) and \( M \) are constants for any ideal gas (equation 5.2).
While this equation is quite simple, additional complexity in its application to the fuel-rod thermoacoustic resonator arises from the fact that the thermoacoustic resonator does not have a constant temperature throughout and the cross-sectional area of the duct changes (slightly) in the region where the stack partially occludes the resonator. A substantial temperature gradient across the stack of approximately 400°C to 450°C is required for operation, and also since there are no hot or cold heat exchangers, the temperature is less spatially uniform in the hot duct side of the stack or the ambient temperature duct on the other side of the stack. For equation 5.2 to be applied in this more complicated situation, some effective sound speed \( c_{\text{eff}} \) must be defined, which is determined by an effective temperature \( T_{\text{eff}} \) of the gas at some location within the resonator.

The thermoacoustic oscillation can be thought of as averaging the spatially varying temperature in the resonator, thus determining its resonance frequency and the dependence of that frequency on an effective temperature. Hence it is expected that there is no absolute temperature measurement that can be made within the gas that will give the correlation to the frequency according to equation 5.2. Alternatively, a more complex model will have to be developed to arrive at this averaged temperature that the frequency is “measuring”.

5.2 Thermometry Experiment

To understand the relationship between the temperature and frequency within the fuel-rod resonator, it was decided to run the system using the indirect heater. In this configuration, there would be five available temperatures: three Type-E thermocouples, one AD 592 temperature sensor, and one Type-K thermocouple. The three Type-E thermocouples were placed in the hot end of the stack \( (T_H) \), the ambient end of the stack \( (T_C) \) and at the middle of the resonator \( (T_M) \). The AD 592 would measure the temperature at the ambient-temperature closed end of the resonator \( (T_E) \). The Type-K thermocouple would measure the temperature

\[
\frac{f}{\sqrt{T}} = \frac{1}{2L} \sqrt{\frac{\gamma k}{M}}
\]  

(5.2)
of the nut of the resonator \( (T_N) \), which was assumed to be the same temperature of the gas directly at that end. The frequency is also measured on the ambient end (at the location of the AD592) via the microphone on the instrumentation plate as described in section 2.2.1.2. This configuration is shown in figure 5.1. Data was acquired from the experiment every two minutes and included these five temperatures as well as the frequency of the sound.

![Diagram of temperature and pressure sensors](image)

**Figure 5.1.** Location of temperature and pressure sensors for temperature-frequency experiments.

Figure 5.2 shows a plot of the measured frequency-temperature invariant from equation 5.2 using the middle thermocouple temperature. If the resonator were to be modelled as a simple mass at the centre, oscillating between two springs on either closed ends (see the following section), then it would be expected that the invariant will exist in relation to this temperature at the centre of the resonator. However, figure 5.2 shows that the measured “invariant” quantity is actually varying! The
dotted black line in figure 5.2 shows how the theoretical invariant quantity would be expected to appear. This confirms the previous discussion that equation 5.2 is in reference to some effective temperature that determines an effective speed of sound and hence the frequency. This effective temperature will not be any absolute measured temperature in the resonator.

**Figure 5.2.** An ideal representation of the frequency invariant from equation 5.2 (dotted black lines). The frequency invariant using the measured frequency of the resonator and the Type-E thermocouple situated at the middle of the resonator, $T_M$ (red dots). It is clear that since this quantity is varying, the fundamental frequency of the resonator is not being produced from $T_M$.

### 5.3 Theoretical Model

In this section, two different models will be analyzed to characterize the temperature-frequency relationship in this thermoacoustic sensor. The first model is a simple lumped element model, which is not expected to approximate the behaviour of
the device very well. However, this model will provide some fundamental concepts that can be applied to the second model that is more accurate - a transfer matrix network.

### 5.3.0.1 Lumped Element Model

One way to understand the oscillations within the closed-closed resonator is to use “lumped elements”. These lumped elements are small compared to the wavelength of sound and represent simple mechanical or electrical topologies which can be used to analyze an acoustical system (or vice-versa). The most fundamental lumped element model that can be applied to a closed-closed tube consists of an acoustical inertance (mass/inductor) and an acoustical compliance (spring/capacitor). In figure 5.3, the resonator can be split into three sections: two compliances on either ends and an inertance at the centre. The inertance can be viewed as a mass of gas which is in simple harmonic motion, bouncing back and forth between the two compliances or springs.

![Figure 5.3. Lumped element representation of a resonator. At the top of the diagram, the typical acoustic pressure and volume velocity profiles for a closed-closed thermoacoustic resonator is shown (from DELTAEC). Note the slight discontinuities in the region of the stack. The lumped element model is shown at the bottom, where the inertance (mass), $L_{acs}$ is at the centre and two compliances (springs), $C_{acs}$ on either side.](image)
The acoustical compliance can be expressed in terms of the volume of the segment, \( V_c \), the mean pressure of the segment \( p_m \) and the polytropic coefficient, \( \gamma \), as shown in equation 5.3.

\[
C_{acs} = \frac{V_c}{\gamma p_m}
\]  

(5.3)

Similarly, there is a relation for the acoustical inertance to the density of the gas in that segment, \( \rho_m \), the length of the segment, \( x_{inert} \), and the cross-sectional area of the segment, \( A \), as shown in equation 5.4.

\[
L_{acs} = \frac{\rho_m x_{inert}}{A}
\]  

(5.4)

Utilizing these two elements, it is possible to calculate a natural frequency, \( f_o \), from equation 5.5, based on the fact that imaginary parts of the impedance go to zero at resonance.

\[
f_o = \frac{1}{2\pi \sqrt{L_{acs} C_{acs}}}
\]  

(5.5)

\( C_{acs} \) is not affected by temperature changes because only the mean gas pressure determines the gas compliance near the rigid ends of the resonator. On the other hand, \( L_{acs} \) is density dependent and in turn temperature dependent. Since the static pressure is spatially uniform within the resonator, the local temperature of the gas determines its local mass density. Near the center of the resonator, where the gas acoustic velocities are the largest for the fundamental half-wavelength resonance mode, the density of the gas becomes important in determining the resonance frequency. Consequently, the temperature at the location of the inertance element will determine the effective speed of sound, \( c_{eff} = 2Lf \), and consequently the fundamental frequency. This proposed lumped element model suggests that the temperature at the middle of the resonator (see figure 5.3) should be the defining temperature for the frequency of the sound that is generated.

To apply this lumped element model, the length of the compliance and iner-
tance segments must be chosen. A common choice would be equal lengths for all segments, i.e., one-third of the total length of the resonator. However, another method demonstrates an improved accuracy through using one-quarter of the total length for the compliance segments and one-half of the total length for the inertance segment. This can be shown through the development of the following equations, where the closed-closed resonator is treated as two closed-open resonators. In figure 5.4 the imaginary line separating the compliance from the inertance section is located at \(d\), and is varied to minimize the resulting frequency. \(L\) is the total length of the resonator and \(L/2\) is the length of one closed-open resonator.

Equation 5.5 can be re-arranged along with equations 5.4 and 5.3 to find the fundamental resonance in terms of the volume of the compliance section, \(V_c\), the cross sectional area of the tube, \(A\), and the length of the inertance section, \(x_{\text{inert}}\). Equation 5.6 is the result and utilizes the angular frequency, \(\omega = 2\pi f\).

\[
\omega^2 = c^2 \frac{A}{V_c \cdot x_{\text{inert}}} = c^2 \frac{A}{Ad \cdot (\frac{L}{2} - d)}
\]  
(5.6)
Re-arranging again, gives a new expression (equation 5.7) in terms of the length of the tube, \( L \), and the distance of the imaginary line, \( d \), separating the compliance and inertance.

\[
\frac{c^2}{\omega^2} = \frac{Ld}{2} - d^2
\]  

(5.7)

To solve for \( d \) such that equation 5.7 is maximized, it is differentiated with respect to \( d \) and set equal to zero. This results in a value of \( d = \frac{L}{4} \), representing the minimal energy solution that can be achieved using this lumped element model; one-quarter length segments for compliances and one-half length for the inertance. Substituting \( d = \frac{L}{4} \) into equation 5.6 and taking the ratio to the standing wave solution for a closed-closed resonator \( (f = \frac{c}{2L}) \), results in a factor of 1.27. Using this model, the frequencies of the resonator were calculated from the middle thermocouple temperature. Figure 5.5 shows these modelled frequencies in comparison to the measured results.

### 5.3.0.2 Transfer Matrix Model

In addition to the overestimation of the frequency obtained in figure 5.5, the lumped element model, while very simple, makes one big assumption: the temperature is constant along the resonator. The inertance of the gas was only determined by a single temperature (at the centre of the resonator), which was assumed to be constant along the length of the tube. However, figure 5.6 shows temperature measurements along the resonator while in operation from the thermocouples and AD 592 temperature sensor. This clearly shows that there is a variable temperature profile along the entire length of the resonator that can be represented by an exponential fit (from the hot end of the stack to the ambient rigid end). Consequently, the mass of the lumped element model did not take into consideration these different temperatures and calculated an inaccurate fundamental frequency for the resonator as evident in figure 5.5.

The method of using transfer matrices to model the temperature effects on the fundamental frequency of the resonator offers a more accurate solution. Instead of using three lumped elements for the entire resonator, a concatenation of
Figure 5.5. Measured and lumped element frequency as a function of the temperature of the middle of the resonator. The actual lumped element model frequencies were divided by a factor of 1.27 to compensate for the overestimation by the lumped element model with a single inertance flanked by two compliances.

these inertances and compliances will be utilized (figure 5.7), which will take into consideration varying parameters along the duct (temperature in this application).

The lumped element that will be used will be comprised of two compliances on either ends with a single mass in the centre. Each of the compliances in this case will be one half of the compliance of any segment. The model is shown in figure 5.8. A transfer matrix will be developed for this system which relates the pressure, $p_2$ and volume velocity, $U_2$ on one fixed end to the pressure, $p_1$ and volume velocity, $U_1$, on the other fixed end. The matrices will then be concatenated to represent the entire resonator as a network of transfer matrices [34].
Figure 5.6. Typical temperature profile of the fuel-rod resonator as measured by the Type-K thermocouple, three Type-E thermocouples and AD 592 sensor (blue dots). The red line shows an exponential fit that was used to approximate the temperature distribution in the resonator from the hot stack to the ambient end.

Observing the series element alone (figure 5.9), a simple transfer matrix, $T_s$, can be used to related $p_1$ and $U_1$ to $p_2$ and $U_2$ through equation 5.8.

\[
\begin{align*}
p_2 &= p_1 + U_1 Z_2 \\
U_2 &= U_1 \\
T_s &= \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix}
\end{align*}
\]  

(5.8)

Similarly, a transfer matrix, $T_p$, can be developed for the element in parallel (figure 5.10 and equation 5.9).
Figure 5.7. Concatenation of several lumped element segments (inertances and compliances) to represent the acoustic variables within the resonator. Acoustic pressure and volume velocity profiles taken from DeltaEC.

\[ Z_2 = \frac{\rho \ell_{\text{seg}}}{A} \]

\[ Z_1 = \frac{A \ell_{\text{seg}}}{2 \gamma p_m} \]

\[ Z_1 = \frac{A \ell_{\text{seg}}}{2 \gamma p_m} \]

Figure 5.8. The lumped element segment that will be used to build the transfer matrix to represent the resonator. In such a segment, the compliances are divided by two since each compliance is “shared” as demonstrated by comparison to the lumped element representation in figure 5.7.

\[ p_2 = p_1 \]

\[ U_2 = p_1 \left( \frac{1}{Z_1} \right) + U_1 \]

\[ T_p = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_1} & 1 \end{bmatrix} \]
Figure 5.9. The series element (inertance) of the lumped element that relates acoustic pressure and volume velocity on either sides of the resonator.

Figure 5.10. The parallel element (compliance) of the lumped element that relates acoustic pressure and volume velocity on either sides of the resonator.

In the lumped element segment (figure 5.8), there are two parallel components and one series component, hence a transfer matrix, $T$, for the entire network can be created with these individual series and parallel transfer matrices, $T_s$ and $T_p$. Equation 5.10 shows the resulting transfer matrix for the network.

$$ T = T_p\cdot T_s\cdot T_p = \begin{bmatrix} 1 + \frac{Z_2}{Z_1} & Z_2 \\ \frac{2}{Z_1} + \frac{Z_2}{Z_1} & 1 + \frac{Z_2}{Z_1} \end{bmatrix} $$ (5.10)

With a transfer matrix for the lumped element network established, it is now possible to split the duct into several “slices” and apply this transfer matrix for each of these slices. Each slice will have a different inertance value in accordance with the variation in temperature along the resonator and the transfer matrix will reflect these changes. The total transfer matrix for the resonator, $T_{res}$, will then become a multiplication of the transfer matrices of each of these individual ($n$) slices as in equation 5.11.
\[ T_{\text{res}} = T_1 \cdot T_2 \cdot T_3 \cdot \ldots \cdot T_n \]

\[
\begin{bmatrix}
  p_2 \\
  U_2
\end{bmatrix} = T_{\text{res}} \begin{bmatrix}
  p_1 \\
  U_1
\end{bmatrix}
\]

Finally, to make the total transfer matrix, \( T_{\text{res}} \), be completely representative of the fuel-rod resonator, the boundary conditions for a closed-closed tube need to be applied. For such a configuration, on either end of the resonator, the acoustic pressures \( (p_1 \text{ and } p_2) \) are a local maximum and the volume velocities \( (U_1 \text{ and } U_2) \) are zero as shown in figures 5.3 and 5.7. In equation 5.12, the boundary condition, \( U_1 = 0 \) is initially applied. This results in a useful expression, \( \frac{U_2}{p_1} \), in terms of only the total transfer matrix of the resonator. To satisfy the second boundary condition, \( U_2 = 0 \), this expression can simply be solved when it is equal to zero. Recall that all the impedances within the transfer matrix are a function of frequency, hence a digital frequency sweep can be injected into the last equation of equation 5.12 and the first zero crossing will correspond to the fundamental frequency of the tube.

\[
\begin{bmatrix}
  p_2 \\
  U_2
\end{bmatrix} = T_{\text{res}} \begin{bmatrix}
  p_1 \\
  0
\end{bmatrix}
\]

\[
\begin{bmatrix}
  p_2 \\
  p_1 \\
  U_2 \\
  p_1
\end{bmatrix} = T_{\text{res}} \begin{bmatrix}
  1 \\
  0
\end{bmatrix}
\]

\[
\frac{U_2}{p_1} = \begin{bmatrix}
  0 & 1
\end{bmatrix} T_{\text{res}} \begin{bmatrix}
  1 \\
  0
\end{bmatrix}
\]

Figure 5.11 shows how the resonator was segmented. A total of 31 slices were used to describe the model. The first slice in figure 5.11 was representative of the hot duct area from the hot end of the screw nut to the hot end of the stack of length 36.4 mm. Since this hot duct is considered to be a rigid end, the gas is largely compliant and hence the temperature change in this portion of the resonator will not influence frequency change with temperature. Consequently one slice was used to represent this region with a temperature that is the average of the
screw nut and hot stack temperatures. For the remaining section of the tube the exponential temperature profile (figure 5.6) was applied. The region of the stack was split into 10 slices, each of length 1.06 mm ($l_{\text{segstk}}$), since the stack length was 10.6 mm for these experiments. In addition to the temperature profile being applied to each of these segments, the formulae for the inertance and compliance were modified, such that equations 5.3 and 5.4 become equations 5.13 and 5.14.

$GasA/A$ represents the porosity of the stack and for the 1,100 cell/in$^2$ used in the experiment, $GasA/A = 0.59$. Since the thermal contact in the stack occurs within the thermal penetration depth, the pressure oscillations are assumed to be in between adiabatic and isothermal, hence the use of an averaged polytropic coefficient, $\frac{\gamma + 1}{2}$ in equation 5.13.

\begin{equation}
C_{\text{acsstk}} = \frac{GasAVc}{\frac{\gamma + 1}{2}p_m} \tag{5.13}
\end{equation}

\begin{equation}
L_{\text{acsstk}} = \frac{p_ml_{\text{segstk}}}{GasA} \tag{5.14}
\end{equation}
The remaining 20 slices of the resonator were dedicated to the ambient end, i.e., from the ambient end of the stack to the cooler end of the resonator. The total length of this ambient region was 168 mm and each slice was of length 8.4 mm (5% of the duct length).

As previously discussed, in figure 5.6 an exponential variation in temperature from the hot end to the stack to the ambient fixed end of the resonator was a reasonable approximation by eye to the temperature profile of the resonator while in operation. For each time interval that the data in the system was acquired (every two minutes), a new exponential fit was created to represent the temperature profile at that specific time. This fit serves as the input to the transfer matrix model evaluated at every time interval. Using this model, the transfer matrix network for the entire resonator was calculated, and then solved satisfying the boundary conditions outlined in equation 5.12 resulting in a modelled fundamental frequency for the resonator. Figure 5.12 shows the modelled and measured frequencies as a function of $T_M$. The Matlab code for the process can be found in Appendix B.

![Figure 5.12. Measured and T-Network modelled frequencies as a function of the temperature of the middle of the resonator.](image-url)
The first thing to notice from figure 5.12 is that the T-Network approach is a highly improved model in comparison to the single lumped element model in section 5.3.0.1. Whereas in the single lumped element model, there was a discrepancy from the measured frequencies by 20%, the T-Network model is within 1% of the measured frequencies. Although the initial part of the graph does not seem to correlate well by eye, it is only a difference of 1%.

In practice, if this thermoacoustic fuel-rod resonator were to be used as a temperature sensor, the only output information that would be available to an operator would be the frequency. Utilizing equation 5.2, this frequency can be used to calculate a corresponding temperature somewhere along the length of the resonator. From figure 5.2, it is known that this temperature will not be the temperature of the middle of the resonator, but rather an effective temperature along the resonator. Figure 5.13 shows this effective temperature from the measured frequencies, along with the temperature calculated from the T-Network model, the temperature at the centre of the resonator and the water temperature. Observing the temperature profile of the resonator in figure 5.6, it can be seen that the measured effective temperature lies between $T_C$ and $T_{M}$, but closer to $T_M$.

The effective temperatures obtained from the T-Network model were within $\pm 8^\circ C$ of the measured effective temperatures. The difference between the measured and modelled values could have stemmed from a number of possibilities. Clearly, one of the main approximations in the model was the exponential fit and with only four thermocouple measurements, it is quite likely that there would have been a number of outliers. Additionally, this model did not represent any azimuthal temperature variations. It was assumed that within any slice the temperature was the same from the walls of the resonator to the centre of the resonator. This is not a necessarily bad assumption since it will be seen in chapter 6, that there is an improved thermal contact between the gas within the resonator and the water in the presence of acoustic streaming (figure 6.4).

Instead of focusing on assumptions and limitations in the model to explain the differences, previous experiments have led to some alternative speculations. These experiments to correlate the temperature profile and the frequency have been carried out multiple times. While all of these experiments will not be presented, there were some unusual results in certain experiments that should be noted. In
Figure 5.13. The effective temperature from the measured frequency ($T_{\text{eff}}$, red), the temperature at the centre of the resonator ($T_M$, purple), the water temperature ($T_{\text{water}}$, blue) and the T-Network modelled temperature ($T_{\text{-net}}$, green) during an indirect run of the resonator. After 1.7 hours, the system began to lose acoustics due to a gas leak; note the divergence of the middle gas temperature from the water temperature indicating reduction of thermal contact.

In particular, an identical experiment carried out before the experiment presented in this section, operated such that its effective temperature was significantly below $T_M$ for the first thirty minutes. This is not physically possible if the speed of sound within the resonator is based only on the temperature within the resonator. It is believed that in the initial stages of operation, there was some mass (possibly from burning of the graffoil gasket at the nut end) which “polluted” the measurement. This could explain the changes in the speed of sound within the resonator. If any of this mass had not been fully burnt out, then it could also affect slightly the presented measurement, creating discrepancies from the model.

Returning attention to figure 5.13, there are also some subtle but important facts to be addressed. After approximately one hour into the experiment, the thermoacoustic fuel rod resonator went into onset and there were acoustic oscillations
present. This shows up as a small initial decrease in the effective temperature obtained from the measured frequency (red dots in figure 5.13) before steadily increasing. This can be attributed to the acoustic streaming that is removing heat from the gas within the resonator to the surrounding water (see section 6).

At about an additional forty minutes later (1.7 hour point in figure 5.13) the amplitude of the acoustic oscillations began to decrease. This was most likely due to a gas leak in the fuel rod resonator, whose rate increased at high temperatures and introduced increased dissipation. This demonstrates that the ability to detect the effective temperature using this device, is amplitude independent.

It also shows that the effective temperature of the gas within resonator has some dependence on the velocity of the acoustic streaming. As will be explained in section 6, the velocity of the acoustic streaming is proportional to the square of the acoustic pressure. Hence, as the amplitude of the acoustic oscillation decreased, so too would the velocity of the acoustic streaming, resulting in less heat being removed from the gas in the resonator to the surrounding water. The result is seen as the increasing of the effective temperature in figure 5.13. This is also supported by the increase in $T_M$ in figure 5.13 (purple dots) towards the end of the experiment.

### 5.4 Technical Specifications and Future Considerations

From these measurements and models, it can be concluded that using a fuel rod resonator adapted to a thermoacoustic engine is a feasible way of obtaining an effective temperature inside a nuclear fuel rod. However, the discussion is not complete until some estimation of the technical characteristics, advantages, disadvantages and potential future works are acknowledged.

Using equation 5.2, the sensitivity of the thermoacoustic sensor in this thesis can be evaluated. Squaring and re-arranging this equation, an expression of the ratio of temperature (in Kelvin) to the square of frequency can be found. This results in the units of the sensitivity of the sensor to be $\text{K/Hz}^2$. Using the values of $\gamma$, $\mathcal{R}$, $M$ and the length of the resonator as 0.215m, the sensitivity of the sensor
is 0.459 mK/Hz\(^2\). In equation 5.2, it is assumed that there is no error associated with \(\gamma\), \(\Re\) or \(M\), since they are all well-known constants. The major error in the sensitivity calculation therefore, is the uncertainty in the length of the resonator. While there were tolerances in the length of the manufacture of the fuel rod resonator, the bigger uncertainty arises from the fact that the stack occludes the cross sectional area of the resonator, and there exists some effective length of the resonator. This effective length was approximated using DELTAEC and is taken as the uncertainty in the length of the resonator. This results in the sensitivity, 0.459 mK/Hz\(^2\) of the sensor to be valid within 5%.

Another specification of this sensor that can be quantified is its absolute accuracy. Although the sensitivity of the device is valid within 5%, the uncertainty in the absolute temperature measurement is also a function of the uncertainty in the measured frequency. In the experiments described in this section, the frequencies were measured to \(\pm 0.01\) Hz, resulting in very small uncertainty in this measurement. Hence, the effective temperature is considered to be accurate within 5%. However, this measurement was done using a high sensitivity piezoresistive microphone which is not available at the site of the nuclear reactor. Naturally, this leads to one of the other important concerns which the future work of this technology would address: effective and reliable methods to measure the frequency of the sound propagated throughout the nuclear reactor. Current ideas which have been put forth include wireless hydrophones and robots to measure the sound. In guiding the decision of a sensor to measure the frequency, it would also be useful to characterize the background noise in an existing reactor. Additional signal processing may have to be done to the measured sound in the reactor to extract the frequency of the sound produced by the thermoacoustic sensor. The uncertainties of all these methods will all have to be considered in their contribution to the overall accuracy of the sensor. The power requirements for the devices measuring the frequency will also have to be explored, especially for emergency situations.

One of the more difficult parameters to evaluate from this sensor is its operating temperature range. The equipment used for the experiments was limited in the capability to reach temperatures beyond 1000°C. However, the Celcor\textregistered stack material can operate up to a temperature of 1200°C and withstand spikes up to 1400°C[35], which provides a physical limit for the thermoacoustic sensor in this
thesis. However, other porous media that can be used as a stack may provide higher operating temperatures. One example is reticulated vitreous carbon, which has a temperature limitation of 3500°C in an oxygen-free atmosphere [36].

In addition to this thermoacoustic sensor having the capability to measure an effective temperature, it is not inconceivable to take this idea one step further and obtain the entire temperature profile of the resonator from the measured frequency. Recall that an exponential fit was used to approximate the temperature distribution from the hot end of the stack to the ambient rigid end of the resonator. From the model, each effective temperature has an associated exponential fit. Hence, if a look-up table were to be created for each effective temperature measured, the temperature profile of the resonator from the hot end of the stack to the ambient rigid end can be retrieved. This would require additional calibration experiments that can be explored in the future work from this project.

Another area worth exploring in the future work is the ability to use this technology to probe different parts of the nuclear reactor, in particular the surrounding cooling fluid or graphite capsules used to house fuel stacks in gas-cooled reactors (see Appendix C). The effective temperature of the fuel rod resonator can be representative of the temperature of the surrounding cooling fluid. In chapter 6, acoustic streaming will be discussed and it will be shown that there is an improved thermal contact between the gas within the resonator and the surrounding cooling fluid when there are acoustic pressure oscillations present. Consequently, it may be possible to infer the surrounding cooling fluid’s temperature from the effective temperature of the gas within the fuel rod. Additionally, the high temperatures of graphite capsules in nuclear reactors can be another source of heat to drive a thermoacoustic engine into onset. Preliminary experiments and analysis on this application is discussed further in Appendix C.
Heat Transfer

Although this thesis is focused on using the thermoacoustic engine as a temperature sensor, for the sake of completeness, it is also worth dedicating some of the effort into understanding the heat transfer within the system. Figure 6.1 shows the different paths through which heat is transferred from the heat source (nuclear fuel) to the hot and ambient ends of the stack, into the water and ultimately into the surrounding environment.

As previously discussed, the dominant method of heat transfer from the heat source to the hot end of the stack is electromagnetic radiation, $\dot{Q}_{\text{rad}}$. $\dot{Q}_{\text{resw}}$ is the conductive heat flow from the heat source that entirely bypasses the gas, and alternatively flows through the walls of the resonator and into the water. The heat from the hot end of the stack can be transported to the ambient end of the stack through a conductive path, $\dot{Q}_{\text{ha}}$. Another conductive heat flow path also exists from the ambient end of the stack to the walls of the resonator and into the water, $\dot{Q}_{\text{aw}}$.

With the thermoacoustic effect present within the resonator, there are two primary effects that contribute to the heat transfer: (i) enhanced thermal contact between the acoustically-oscillating gas and the walls of the resonator due to acoustically-driven streaming ($\dot{Q}_{\text{sd}}$ in figure 6.1) [37] [38], and (ii) enthalpy transport along the stack due to the “bucket brigade” effect [39] which acoustically transports heat through the stack. The total power flowing through the stack, which includes $\dot{Q}_{\text{ha}}$ is denoted by $\dot{H}_2$ [11], hence in figure 6.1, this enthalpy transport from the hot end of the stack to the ambient end of the stack due to
thermoacoustics alone is $\dot{H}_2 - \dot{Q}_{ha}$.

The remaining heat flow paths in figure 6.1, $\dot{Q}_{henv}$ and $\dot{Q}_{wenv}$ are the heat losses from the heat source and the water to the ambient environment. The overall heat flow from the gas in the resonator to the water was measured but it was not possible to isolate the various paths completely. For example, there was no instrumentation included to directly measure the streaming velocity. Because of this limitation, the influence of the thermoacoustic effects can only be estimated.

**Figure 6.1.** Various paths of heat flow within the thermoacoustic fuel rod engine, the surrounding water and the ambient environemnt. The paths $\dot{Q}_{sd}$ and $\dot{H}_2 - \dot{Q}_{ha}$ exist only when the thermoacoustic effect is present.
6.1 Acoustic Streaming

In thermoacoustic devices, the acoustic streaming can be either a loss mechanism by depositing heat in undesirable areas within the device or an essential heat transfer method [11]. The latter applies to this thermoacoustic fuel rod resonator as acoustic streaming works in favour of the continued operation of the fuel rod resonator by transferring heat from the ambient end of the stack to the walls of the resonator and into the surrounding cooling fluid. This establishes better thermal contact between the gas within the resonator and the surrounding cooling fluid when the acoustic oscillation (hence acoustic streaming) is present in the resonator. Consequently, the temperature gradient across the stack can be maintained and the fuel rod resonator can continue its operation without the need for a physical ambient temperature heat exchanger.

6.1.1 Acoustic Streaming Theory

Lord Rayleigh provided the initial quantitative explanation for acoustic streaming that was supported by a standing wave in 1883 [40]. The time-averaged, second-order steady flow in the axial direction $\langle u_2 \rangle$ and in the transverse (radial) direction $\langle v_2 \rangle$ were calculated by Rayleigh from the amplitude of the first-order acoustical velocity $u_1$ for a standing wave of wavelength $\lambda$ in a gas with sound speed $c$, where $x$ is the position along the axis of the resonator and $r$ is the radial distance from the axis in a toroidal streaming cell of length $L_t$ in equations 6.1 and 6.2. It is important to note that the acoustic streaming velocities are directly proportional to the square of the acoustic velocities, hence proportional to the square of the acoustic pressure. Figure 6.2 shows the flow pattern of the acoustic streaming calculated from the following equations:

$$\langle u_2 \rangle = \frac{3}{8} \frac{u_1^2}{c} (1 - \frac{2r^2}{R^2}) \sin \left( \frac{\pi x}{L_t} \right)$$ (6.1)

$$\langle v_2 \rangle = \frac{3}{8} \frac{u_1^2}{c \lambda} \frac{2\pi r}{(1 - \frac{r^2}{R^2}) \cos \left( \frac{\pi x}{L_t} \right)}$$ (6.2)
Rayleigh’s theory was extended by Nikolas Rott [42] to include (i) the acoustic temperature fluctuations $T_1$ caused by the acoustic pressure, $p_1$, $(T_1/T_m) = [(\gamma-1)/\gamma](p_1/p_m)$; (ii) the thermal boundary layer with thickness $\delta_k = (2\kappa/\rho c_p \omega)^{\frac{1}{2}}$, where $\kappa$ is the thermal conductivity of the gas, $\rho$ is the gas density, $c_p$ is the gas specific heat at constant pressure, the polytropic coefficient $\gamma = c_p/c_v$, and $\omega = 2\pi f$; (iii) the variation of mean temperature $T_m$ with respect to the axial coordinate $x$; and (iv) the dependence of viscosity $\mu$ and thermal conductivity $\kappa$ on temperature, assuming the form for the viscosity of $\mu(T) \propto T^\beta$, where $\beta$ is a constant (typically about 0.7) that depends on the properties of the fluid [41]. These considerations modify the axial component of the time-averaged streaming velocity from the Rayleigh result of equation 6.1. The Prandtl number, $P_r = \mu c_p / \kappa = (\delta_v / \delta_k)^2$, is the dimensionless ratio of the fluid’s ability to transmit viscous shear to its ability to transfer heat by thermal conduction (e.g., for molasses $P_r$ is very large and for mercury $P_r$ is very small and in air $P_r$ is approximately 0.7).

$$\langle u_2 \rangle = (1 + \alpha_1) \frac{3 u_1^2}{8 c} \frac{2 \pi r}{\lambda} (1 - \frac{r^2}{R^2}) \cos \left( \frac{\pi x}{L_t} \right)$$  \hspace{1cm} (6.3)

$$\alpha_1 = \frac{2}{3} (1 - \beta)(1 - \gamma) \frac{\sqrt{P_r}}{1 + P_r}$$  \hspace{1cm} (6.4)
The acoustically-driven streaming flow outside of the viscous boundary [43] layer was studied by Thompson and Atchley [44] using laser Doppler anemometry and a dimensionless parameter they designated as the “nonlinear Reyleigh-number”, $R_e$, defined in equation 6.5. They also identified two cases corresponding to slow streaming when $R_e << 1$, as described by Rott in equations 6.3 and 6.4 and others [45], as well as non-linear streaming when $R_e \geq 1$.

$$R_e = 2\left(\frac{u_1}{c}\right)^2\left(\frac{R}{\delta_e}\right)^2$$

(6.5)

Figure 6.3 shows the measured time averaged velocity profiles for $R_e = 5.7$ and $R_e = 19$. These graphs clearly show that there is a significant breakdown of the streaming velocity equation 6.3 at high amplitudes. However, this disagreement is mainly concentrated near the center of the tube and it can be expected that the simple theory will hold for higher amplitudes closer to the walls of the resonator. This is the area where the acoustic streaming will be important for the device described in this thesis, because the streaming velocity in this region is what controls the heat transfer from the gas to the resonator walls and into the surrounding cooling fluid.

Figure 6.3. Measurement of the normalized time-averaged axial streaming velocity $\langle u_2 \rangle / M^2 c_o = \langle u_2 \rangle c_o / u_1^2$ as a function of radial position in the tube using laser Doppler anemometry in a tube with radius $R = 23$ mm at $f = 310$ Hz, in air with the thermodynamic sound speed $c_o$. The figure on the left shows the streaming velocity when $R_e = 5.7$ and on the right, shows the streaming velocity when $R_e = 19$. While there is a departure from the classical theory in the right figure, there remains fairly reasonable agreement closer to the walls of the resonator where heat transfer is significant [44].
6.1.2 Measured Impact of Streaming Driven Convection

Using the “killer” mechanism it was possible to perform an experiment to understand the influence of the acoustically driven streaming on the gas in the resonator and the water. The fuel rod resonator was operated in the direct heating mode. An electrical power of 26 W was applied to the heater and the resonator was allowed to reach onset. The device was allowed to run for several hours while the thermoacoustic oscillations were present to establish a temporal steady state. The acoustic oscillations were then suppressed using the “killer” and allowed to establish a new temporal steady state. The temperatures of the gas at the centre of the resonator and the surrounding water were acquired every two minutes over the course of the experiment. Figure 6.4 shows these temperatures from this experiment with the acoustic oscillations being suppressed twice. It is evident from figure 6.4 that the gas at the centre of the resonator dropped to a lower temperature, closer to the temperature of the water. This is suggestive of an improved thermal contact between the gas in the resonator and the surrounding water when the streaming driven convection is present.

**Figure 6.4.** Temperatures of the gas and water with acoustic oscillations turned on and off operating at 26 W electrical power input to the heater. The time when the gas temperature drops closer to the water temperature are the times when the acoustic oscillations were present.
6.2 Thermoacoustic Enthalpy Flux

The increased enthalpy flow attributed to the thermoacoustic standing wave in the stack can be understood by examination of the boundary-layer approximation as shown in equation 6.6 \[11\].

\[
\dot{H}_2 = \frac{A \delta_k}{4 \tau_h} \frac{|p_1||U_1|}{\Lambda} \left[ \frac{1 + \sqrt{\sigma + \delta}}{1 + \sqrt{\sigma}} - 1 - \sqrt{\sigma + \delta} \right] - (A \kappa + A_{solid} \kappa_{solid}) \frac{dT_m}{dx} 
\]

(6.6)

Although equation 6.6 seems quite complex, thinking of it in two parts can be helpful. The first term, which includes all the terms in the square brackets represents the thermoacoustic contributions to the total power or enthalpy flow in the stack (\(\dot{H}_2 - \dot{Q}_{ha}\) in figure 6.1). The second part of the equation represents the conductive thermal effects without any thermoacoustics (\(\dot{Q}_{ha}\) in figure 6.1). It is also important to recognize that the thermoacoustic term is proportional to \(|p_1||U_1|\), hence quadratic in the acoustic amplitude.

Rather than attempt a solution to 6.6 or any other algebraic expression for enthalpy flow, DELTAEC can be used to obtain \(\dot{H}_2\), since it calculates equation 6.6 when a model is solved. The DELTAEC model from Chapter 3 that preserves the physical geometry, was adjusted to produce a standing-wave acoustic pressure of \(p_1 = 1.85kPa = \sqrt{2}p_{rms} = \sqrt{2} \cdot 1.31kPa_{rms}\). This acoustic pressure was chosen because an experiment was also performed (following section) that operated at the same acoustic pressure.

The conductive term, \(\dot{Q}_{ha} = (A \kappa + A_{solid} \kappa_{solid}) \frac{dT_m}{dx}\) can then be calculated manually (since these constants are known). It is then possible to estimate how much additional heat is transferred from the hot end of the stack to the ambient end of the stack as a result of the enthalpy transport. \(\kappa\) is the thermal conductivity of the gas (air) and is taken as the average thermal conductivity from the hot and cold ends of the stack and is \(3.79 \times 10^{-2} W/m^oC\). \(\kappa_{solid}\) is the thermal conductivity of the Celcor® ceramic, which is \(2.5 W/m^oC\). \(A\) and \(A_{solid}\) are the cross-sectional areas of the gas within the stack and of the solid stack material respectively.

DELTAEC calculates a parameter, \(GasA/A\), which is essentially the porous area
of the stack. For 1100 cells/in², $GasA/A$ is 0.59. Hence, 0.41 represents the solid area of the stack and multiplying this by the cross sectional area, $A$ (240 mm²), of the resonator, $A_{\text{solid}}$ is 98.4 mm². $\frac{dT_m}{dx}$ is the temperature gradient across the stack. From this model, the stack temperatures were 317°C and 72°C at the hot end of the stack and ambient end of the stack respectively. Since the effective stack length is 8.5 mm in the model (excluding the two 1.0 mm thick sections that are treated as the two heat exchangers), $\frac{dT_m}{dx}$ is equal to $2.88 \times 10^4 \, ^\circ C/m$. Figure 6.5 shows the “state variable plot” that provides the values of the acoustic pressure amplitude, $p_1$, the acoustic volume velocity, $U_1$, the mean temperature $T_m$, the enthalpy flow $\dot{H}_2$, and acoustic power flow $\dot{E}_2$, at all locations within the model of the fuel-rod resonator.

In this model, the total enthalpy flux $\dot{H}_2$ through the stack is approximately 9.33 W and the conductive term in equation 6.6 provides 7.35 W of the heat flow, corresponding to an acoustically heat flow increase of about 1.98 W, or 21%.

Figure 6.5. The “state variable” plot generated by the DELTAECD model of the fuel-rod resonator that is scaled (approximately) in the horizontal direction to match the schematic view above. The heat input and temperature have been adjusted so that the acoustic pressure amplitude $p_1 = 1.85 \, \text{kPa}$ at the ambient-temperature rigid termination (HARDEND, Seg. #17) matches that measured for the highest amplitude result in Table 6.1: $p_1 = \sqrt{2} \cdot p_{\text{rms}} = \sqrt{2} \cdot 1.31 \, \text{kPa}_{\text{rms}}$. 
Further confirmation that the heat flow path, $\dot{H}_2 - \dot{Q}_{ha}$ (figure 6.1), appears when the thermoacoustic effect is present is shown in figure 6.6. This is a measurement of the temperature at the location of the ambient end of the stack from the same experiment described in section 6.1.2, where the acoustic oscillations are intentionally suppressed and reactivated. It shows an increase in temperature when the acoustic oscillation is present. Hence, it is indicative of the enthalpy transport that moves heat from the hot end of the stack to the cold end of the stack.

**Figure 6.6.** Temperature of the ambient end of the stack with and without acoustic oscillations demonstrating the enhancement of heat flux through the stack due to thermoacoustic effects.

### 6.3 Calorimetric Heat Transfer Measurements

#### 6.3.1 Thermal Resistance Model

Figure 6.7 is an additional simple thermal model that is used to understand the way in which the heat is distributed throughout the system. This model groups some
of the heat flow paths from figure 6.1 and represents them as thermal resistances through which heat will be allowed to flow. Figure 6.7 shows the calorimeter with the resonator’s ambient end submerged in water and the various thermal resistances. There are two paths for heat flow to the surrounding room-temperature air: $R_{\text{solid}}$ and $R_{\text{leak}}$. $R_{\text{solid}}$ is the path where heat will flow from the resonator through the lid of the calorimeter and the resonator nut to the surrounding air. $R_{\text{leak}}$ is the path of heat flow directly from the water through the walls of the calorimeter. These resistances are representative of the $\dot{Q}_{\text{henv}}$ and $\dot{Q}_{\text{wenv}}$ from figure 6.1.

In figure 6.7, $R_{\text{na}}$ is the path of heat flow from the hot gas within the resonator to the water when there is no acoustic oscillation. This groups $\dot{Q}_{\text{ha}}, \dot{Q}_{\text{aw}}, \dot{Q}_{\text{resw}}$ and $\dot{Q}_{\text{rad}}$ from figure 6.1. $R_{\text{ac}}$ is the additional path created when there is acoustic streaming and represents the contribution of $H_2 - \dot{Q}_{\text{ha}}$ and $\dot{Q}_{\text{sd}}$ from figure 6.1.

$R_{\text{na}}$ and $R_{\text{ac}}$ can be treated as resistors in parallel, so that when $R_{\text{ac}}$ exists, it will drop the total thermal resistance of this parallel combination, hence allowing more heat flow from the gas to the water for a given gas-to-water temperature difference. $\Pi_{\text{elec}}$ is the electrical power input to the NiCr heater element.

### 6.3.2 Calorimeter Heat Leak Measurement

To make sense of thermoacoustic heat transfer measurements, it is essential to determine the heat leak from the water to the ambient environment, $\dot{Q}_{\text{wenv}}$. $\dot{Q}_{\text{wenv}}$ is determined by $R_{\text{leak}}$ and the temperature difference between the water and the air. A small cartridge heater (Omega Engineering Model CSH-201200 cartridge heater, 3/8” diameter, 1.5” long, 200 W at 110 V) was placed into the calorimeter through the compression fitting that replaced the spigot and submerged in the distilled water. Electrical power of 60 W was applied to the heater for several hours. When the heater was turned off, the cooling history of the water was recorded, as shown in Figure 4(a). Also shown in Fig. 4(a) is an exponential fit as suggested by Newton’s Law of Cooling. Figure 4(b) shows the calculated heat leak based on the rate of cooling. The constant slope in Fig 4(b) indicates a temperature independent value for $R_{\text{leak}}$.

The equation for the cool-down model of the temperature of the water, $T_{H_2O}$, is
Figure 6.7. Schematic representation of the Thermal Resistance Model for measurement of the enhanced heat transfer due to thermoacoustically-induced streaming. Streaming is represented by the oblong flow paths within the resonator. Heat leaves the calorimeter either by traveling up the resonator through the insulated lid and nut, represented by $R_{\text{solid}}$, or through the insulation provided by the calorimeter represented by $R_{\text{leak}}$. Heat that can leave the fuel-rod resonator and enter the water in the absence of a thermoacoustically-driven standing wave is represented by $R_{\text{na}}$. When there is a standing sound wave within the resonator, the acoustically-driven streaming increases the heat transfer from the gas in the resonator to the water through the path represented by the additional heat flow through the thermal resistance $R_{\text{ac}}$.

given in equation 6.7. Using this equation, the rate-of-change of the temperature of the water could be calculated for each data sample. Knowing the thermal mass and the specific heat capacity of distilled water, $\dot{Q}_{\text{env}}$ was calculated and plotted as a function of water temperature in Figure 4(b). This heat leak model, represented by $R_{\text{leak}}$, was used to calculate the heat leak in subsequent experiments. In equation 6.7, all temperatures are in °C and the time, $t$ is in seconds. Since the thermocouple measuring the ambient air temperature was the largest source of error, measuring to a precision of 0.1°C, this resulted in the heat leak being accurate to ± 0.005W.
Figure 6.8. (a) The cool-down history of the water over a 40 hour period (blue) and the exponential fit to that data (red) that is represented in equation 6.7. (b) The associated heat leak $\dot{Q}_{\text{wenv}}$ from the natural cool-down of the water. The constant value of the slope demonstrates that $R_{\text{leak}}$ (see figure 6.7) is a constant.
\[ T_{H_2O} = (21.92 \pm 0.007) + (7.7461 \pm 0.0066)e^{\frac{-1}{(143600 \pm 220)}} \] (6.7)

### 6.3.3 Measurements with and without acoustic oscillations

Following these initial calibration experiments, the thermoacoustic resonator was put into operation. Electrical power was applied to the direct heater, which established the sustained thermoacoustic oscillations. The resonator was allowed to operate for a few hours to establish the steady-state operating point. The oscillation was then suppressed using the remote “killer” mechanism (as described in Section 2.2.1.3), and the resonator was left with the same heat input for several hours so that it would reach a different steady state. The net heat, \( Q_{net} \), was calculated through the rate-of-change of temperature of the water during the two steady state periods with corrections for the well-characterized \( \dot{Q}_{wenv} \). This process was performed at the four different operating points shown in table 6.1. All of the results in table 6.1 were taken with the resonator filled with air very close to atmospheric pressure or just slightly above. The thermoacoustic resonance frequencies ranged from 815 Hz to 890 Hz, depending upon the power levels and temperatures. The rate of change of the water temperature contributed to the largest error in these experiments such that \( Q_{net} \) is calculated to \( \pm 0.1 \) W. The associated error with all the other measured variables are shown in table 6.1.

The first feature worthy of note in Table 6.1 is that \( Q_{net} \) is always smaller compared to the measured \( Q_{net} \) when the acoustic oscillations are present in the resonator. The parallel combination of the thermal resistances of \( R_{na} \) and \( R_{ac} \) is also evidence of this. When the acoustic oscillation is present, the total parallel resistance decreases, suggesting that \( R_{ac} \) is introduced as an additional parallel thermal resistance created when there is acoustic streaming. It should be noted that at the lowest operating electrical power of 17.5 W, the difference of \( Q_{net} \) with and without the acoustic oscillation is within the 0.1 W error to which \( Q_{net} \) can be calculated. Hence, it cannot be confirmed that there is an increase in the net heat into the water at very low acoustic amplitudes, below and including 600 Pa.
Table 6.1. Four different electrical powers applied to the heater with and without acoustics. The acoustic pressure is zero when there is no acoustics. $R_{na||RaC}$ is the parallel combination of the thermal resistances from the gas within the resonator to the surrounding distilled water. The enhancement is the percentage increase of heat that is transported when acoustic streaming is present to that transported in the absence of a thermoacoustically-driven standing wave. The uncertainty of the acoustic pressure is the relative error associated with fluctuations in its amplitude during the time the device was operating at a particular electrical input power.

Referring again to figure 6.1, can help make sense of these results. Without any thermoacoustic oscillations, all the heat flows except $\dot{H}_2 - \dot{Q}_{ha}$ and $\dot{Q}_{sd}$ are active. If the heat being transferred from the hot end of the stack to the ambient end of the stack is assumed to be small by $\dot{Q}_{ha}$, then the temperature of the hot end of the stack will not change significantly and will reach a thermal equilibrium with the heat source (provided that the heat source is a steady temperature). However, when the acoustic oscillations are present, $\dot{H}_2 - \dot{Q}_{ha}$ and $\dot{Q}_{sd}$ are introduced and begins to transport heat throughout the system. $\dot{H}_2 - \dot{Q}_{ha}$ continuously drives more heat from the hot end of the stack to the ambient end of the stack, lowering the temperature at the hot end of the stack and increasing the temperature of the ambient end of the stack (figure 6.6). Hence, there is a greater temperature difference between the heat source and the hot end of the stack. From chapter 4, the Stefan-Boltzmann Law (equation 4.2) states that the electromagnetic radiation from the heat source to the hot end of the stack is proportional to the fourth power of this temperature difference, and consequently, $\dot{Q}_{rad}$ will increase. At the same time, the heat that is being accumulated on the ambient stack is continuously being
removed by the acoustically-driven streaming convection, $\dot{Q}_{sd}$, and deposited onto the walls of the resonator and finally into the water. Unfortunately, no experiments were done with the activation and suppression of acoustic oscillations using the indirect heater, hence the claims on $\dot{Q}_{rad}$ and the temperature of the hot end of the stack cannot be confirmed.

Another interesting result from table 6.1 is the dependence of the percentage increase in the net heat transferred into the water (with acoustic oscillations), on the acoustic pressure. Figure 6.9 shows a quadratic relationship between the two variables. While there are only three useful points on this graph (recall the lowest amplitude was within the error limits), it seems reasonable that the increase in net heat should be quadratic with the acoustic pressure since the thermoacoustic effects are also quadratic with acoustic pressure. The thermoacoustic enthalpy transport is proportional to the product of pressure and volume velocity (equation 6.6), hence the square of the acoustic pressure. The acoustically-driven streaming velocity is proportional to the square of the acoustic velocity (equation 6.3) and in turn also proportional to the square of the acoustic pressure.

![Graph](https://example.com/graph.png)

**Figure 6.9.** Fractional heat transfer as a function of acoustic pressure amplitude using the values taken from Table 6.1. Both the thermoacoustic enthalpy transport and the streaming velocity are directly proportional to the square of the acoustic pressure.
These results should not be misinterpreted to think that more power can be obtained from a heat source (nuclear fuel) with a finite amount of energy. Rather it demonstrates the importance of the acoustic streaming and enthalpy flux flow to keep this thermoacoustic fuel rod device in operation. These two thermoacoustic effects work in favour of the device and removes heat energy to maintain the critical temperature gradient for the continued functioning of this device as a sensor. It is also believed that if more work can be done by the sensor, then it can operate at higher amplitudes, removing larger quantities of heat. This has the potential to reduce the effective temperature in the ambient section of the fuel rod (figure 6.4), which can be useful for the insertion of additional instrumentation within the fuel rods. However, exploring such a benefit will call into question the efficiency of thermoacoustic engines, which was not addressed in this thesis.
Chapter 7

Conclusion

The thermoacoustic fuel rod sensor developed in this research has introduced a novel technique for monitoring the temperature within the core of a nuclear reactor. It essentially uses the heat from the nuclear fuel to generate sustained acoustic oscillations, whose frequency will be indicative of the temperature within a nuclear fuel rod. Adapting a nuclear fuel rod into this type of thermoacoustic sensor simply requires the insertion of a porous material (stack). This sensor has demonstrated a synergy with the elevated temperatures that exist within the nuclear reactor using materials that have only minimal responsibility to high energy particle fluxes.

Electromagnetic radiation is responsible for transporting enough heat from the nuclear fuel to the hot end of the stack to establish the required temperature gradient across the stack for the generation of acoustic pressure oscillations. This eliminates the need for any physical hot heat exchangers and hence no external electrical power is required to drive the sensor. The thermoacoustic pressure oscillations are of sufficient amplitude to create acoustic streaming that acts as an acoustic gas pump removing heat from the ambient end of the stack, and transferring heat from the gas to the walls of the resonator and finally to the surrounding cooling fluid. This maintains the temperature gradient across the stack to keep the system in continuous operation. This also eliminates the need for a physical cold heat exchanger on this side of the stack. Hence, inserting a stack of the proper length and at the proper position is the only modification required to convert the fuel rod into a thermoacoustic sensor. Additionally, since it is only the gas that moves within the fuel rod through pressure oscillations, there are no physical mov-
ing parts to fatigue or fail. Furthermore, there are no electrical cables needed inside the fuel rod for successful operation of this sensor.

When the sensor is in operation, the sound waves that are radiated from the fuel rod resonator will propagate through the surrounding cooling fluid. It was confirmed that the frequency of these oscillations is directly correlated an effective temperature within the fuel rod resonator. For this half-wavelength thermoacoustic resonator, the effective temperature is more representative of the ambient section in the fuel rod (from the ambient end of the stack to the ambient rigid of the fuel rod), than the temperature of the nuclear fuel. In such a design, utilizing a look up table of exponential fits that correlate to the effective temperature, may allow for the extraction of the temperature profile of the entire fuel rod from the hot end of the stack to the ambient rigid end. However, this is an idea that is yet to be fully investigated. It is also worthy to note that a different resonator design could emphasize the temperature in a different location as shown in Appendix C.

In this thesis, with a fuel rod of approximately 0.215 m long, the sensitivity of the sensor is 0.459 mK/Hz² and valid within 5%. The accuracy of the thermoacoustic sensor is dependent on how well the effective length of the fuel rod is known and the accuracy with which the frequency of the sound is measured. For this fuel rod sensor, since there is very low uncertainty in measuring the frequency, the effective temperatures are accurate to within 5%. Future work will explore various methods to measure the frequency of the sound that is propagated throughout the reactor. However, current ideas that have been put forth are hydrophones and remotely controlled robots. The method of measuring the frequency outside the reactor vessel will likely be able to operate on some reserve power supply such as a battery for this sensor to provide information in emergency situations, such as Fukushima where the main electrical power was lost.

The operating temperature range of this thermoacoustic sensor is one of the main limitations that can be encountered when using this device. The upper limiting temperature of the Celcor® ceramic stack material is between 1200°C to 1400°C [35], and will be the maximum temperature at which this sensor can function properly using this type of stack material. However, other porous media may provide higher operating temperatures. For instance, reticulated vitreous carbon has a temperature limitation of 3500°C in an oxygen-free atmosphere [36].
The effects of the acoustic streaming on this device is also worth acknowledging. Due to this forced convective motion of gas, heat is continually removed from the ambient end of the stack and maintains the critical thermal gradient across the stack for continuous operation of this device. It was also shown that this acoustic streaming reduced the temperature difference between the middle of the fuel rod and the surrounding water. Since it is has been demonstrated that the amount of heat removed is quadratic with acoustic pressure amplitude, high amplitude acoustic oscillations may help in lowering the temperature at the centre of the fuel rod even further.

While this thermoacoustic fuel rod sensor has been designed to measure the temperature within the gas of the fuel rod or the surrounding cooling fluid, it is possible to utilize the same technology to measure temperature in other parts of the reactor, such as a graphite capsule (see Appendix C) or infer the temperature of the surrounding cooling fluid. The frequency and the amplitude of the acoustic oscillations can also provide additional information to use as other types of sensors. For instance, at the Idaho National Laboratories, experiments have been conducted to use the thermoacoustic effect to measure fuel porosity, molecular mass of the gas within the fuel rod and to track fission gases.

As the implementation of nuclear power is growing globally, it is important that the mechanisms and sensors employed to ensure that a nuclear reactor is in its region of safe operation be continuously refined and revisited. The thermoacoustic fuel rod sensor is representative of this motive and demonstrates an exciting, alternative system to monitor the status within a nuclear reactor, particularly useful in emergency situations.
Appendix A

Miniature Thermoacoustic Demonstration Resonator

Prior to the manufacture of the fuel-rod resonators, a miniature thermoacoustic resonator was built to determine if it was possible to create the required conditions for sustained pressure oscillations in such a small device. Figure A.2 shows a picture of the thermoacoustic resonator. It was built from a glass test tube of length 70 mm. A celcor stack of 1,100 cells per square inch was used at a length of 7.5 mm. The stack was placed 17.5 mm from the closed end of test tube. A NiCr wire of 0.012” diameter was used to heat the stack. The wire was wound in a serpentine pattern using the pin arrangement as shown in figure A.1. The thermoacoustic resonator was driven of 6-AA batteries as seen in figure A.2.

Figure A.1. NiCr wire glowing red that is wound around a pin pattern on a block of machineable ceramic.

A DELTAE C model was also constructed and the schematic drawing is shown in figure A.3. This model was constructed in the same way as the model used for
the fuel-rod resonator described in chapter 3.

Figure A.2. Miniature thermoacoustic engine.

Figure A.3. Schematic drawing from the DELTAEC used to model the miniature thermoacoustic resonator. It consists of 2 ducts before and after the stack and heat exchangers. At the end is a “Pistbranch” and “Hardend” segments which were used to simulate the radiation load on the open end and to allow the volume velocity to oscillate at the open end of the resonator.

In addition to the model, a B&K 1” microphone was used to measure the sound emitted from the resonator. Figure A shows the resulting spectrogram. Figure A.4(a) uses a record length of 0.02 seconds, whereas figure A.4(b) uses a 1 second record length and focuses on the fundamental frequency. From these graphs, the fundamental frequency is found to be approximately 1,131 Hz and is at a sound pressure level of 90 dB at 30 cm away from the mouth of the thermoacoustic resonator. In addition to the fundamental frequency, there is a strong third harmonic
from the fact that it is an open-closed resonator. The weak second harmonic is a result of the shock wave distortion of the fundamental frequency.

![Spectrogram of the miniature thermoacoustic resonator: (a) uses a shorter record length and gives better temporal resolution. Strong fundamental and third harmonics for the resonator can be seen, (b) uses a longer record length and zooms on the fundamental frequency of approximately 1,131 Hz and a sound level of 90 dB at 30cm away from the mouth of the resonator.](image)

**Figure A.4.** Spectrogram of the miniature thermoacoustic resonator: (a) uses a shorter record length and gives better temporal resolution. Strong fundamental and third harmonics for the resonator can be seen, (b) uses a longer record length and zooms on the fundamental frequency of approximately 1,131 Hz and a sound level of 90 dB at 30cm away from the mouth of the resonator.
Appendix B

Matlab Code for Transfer Matrix Model

clc
clear all
close all

%T networks for a duct of varying temperature
%Closed-Closed fuel rod resonator from INL

ThotK = 795.52 + 273.15; %temp in kelvin
TnutK = 938.17 + 273.15; %nut temp
TcoldK = 349.23 + 273.15; %cold stack temp

Tamb = 26.85; %temp in oC
TambK = 300; %temp in K
gamma = 1.4; %gamma for air
R = 8.3144621;
M = 0.02897; %molar mass air 0.02897
pm = 101000; %dc pressure (Pa)
camb = sqrt((gamma*R*TambK)/M); %ambient sound speed
rhoamb = (pm*M)/(R*TambK); %ambient density

A = 2.398e-4; % cross sect. area of tube (m^2)
Leff = 0.215; % length of tube

% splitting model into different segments
% 1st segment is the hot duct area
% 2nd segment is the stack area
% remainder of the duct will be split in 20 segments.

Nsegcold = 20; % # segments in the cold duct
nsegstk = 10;
nseghot = 1;
% # segments to model (n hot, n stk, n cold segments)
Nseg = nseghot + nsegstk + Nsegcold;
Leffcold = 0.168; % length of the cold duct
Leffhot = Leff-Leffcold; % Length of hot end including stack

Lhd = 0.0364; % length of hot duct
lseghot = Lhd/nseghot;

Lstk = 0.0106; % length of the stack
lsegstk = Lstk/nsegstk;

l = Leffcold/Nsegcold; % length of a segment in the cold duct
Tseghot = (ThotK+TnutK)/2; % temp in the segment 1 hot duct
Tsegstk = (ThotK+TcoldK)/2; % temp in the segment 2 stack

gasAoA = 0.59; % from deltaEC
% Radiation Impedance 
Zrad = 1;

% model parameters for exponential temp. 
% eqn: T = Tinf + To*exp(-t/Tau), t-time
% Tinf = 20.78904108;
% Tau = 0.012890966;
% To = 9928.716778;

Tinf = 32.29854213;
Tau = 0.014649935;
To= 8522.468946;

w = 2000:1:10000; % omega (frequencies)

Zacs = zeros(length(w),1); % predefine length of acoustic impedance

% this sweeps through frequencies looking for zeros in impedance
% see the transfer matrix for reference, finding when U2 = 0.

for n = 1:length(w)
    Tnew = [1 0; 0 1]; % define an identity matrix

    for m=1:1:Nseg
        if m <= nseghot
            rhoseg = (pm*M)/(R*Tseghot);
            L = (rhoseg*lseghot)/A;
% acoustic compliance, no change with temp.
C = (A*lseghot)/(2*gamma*pm);
elseif m <= (nseghot+nsegstk)
% get position at center of segment
x = (((((m-(nseghot))*1)) - (1/2)) + Lhd;
Tseg = Tinf + To*exp(-x/Tau);
% temp from model in kelvin
TsegK = Tseg + 273.15;
rhoseg = (pm*M)/(R*TsegK);
L = (rhoseg*1seghot)/(gasAoA*A);
% acoustic compliance, accommodate for stack volume
% change and gamma in between adiabatic and isothermal
C = (gasAoA*A*lseghot)/(2*((gamma+1)/2)*pm);
else
% get position at center of segment
x = (((((m-(nseghot+nsegstk))*1)) - (1/2)) + Leffhot;
Tseg = Tinf + To*exp(-x/Tau);
% temp from model in kelvin
TsegK = Tseg + 273.15;
rhoseg = (pm*M)/(R*TsegK);
L = (rhoseg*l)/A;
% acoustic compliance, no change with temp.
C = (A*l)/(2*gamma*pm);
end

Z1 = 1/(1j*w(n)*C);
Z2 = 1j*w(n)*L;

T = [(1+(Z2/Z1)) Z2; ((2/Z1)+(Z2/(Z1^2))) (1+(Z2/Z1))];
Tnew = Tnew*T;
end
Zacs(n) = [0 1]*(Tnew*[1;0]);
end

Zabs = abs(Zacs);
freqs = w ./ (2*pi);
[minval, locs] = min(Zabs);
format short g
for k = 1:length(locs)
    f(k) = w(locs(k)) / (2*pi);
end
f

% modelled frequency
The Graphite Simulation Experiment

In addition to being able to measure an averaged gas temperature within the nuclear fuel-rod resonator or the temperature of the surrounding cooling fluid, it is possible to use this same technology to “probe” other parts of a nuclear reactor and measure their temperature through the thermoacoustic effect. For instance, it is possible to measure the temperature of a graphite capsule that encloses multiple fuel stacks as shown in figure C.1.

Figure C.1. Schematic diagrams of a graphite capsule containing three fuel stacks [46].
Another fuel-rod resonator or actual graphite capsule was not available, hence a simpler experiment was designed that demonstrated a proof-of-concept. Figure C.2 shows the experiment in operation. The open-closed thermoacoustic laser used for educational purposes [47] was modified so that instead of the stack being nearer to the closed end, the stack was placed near to the open end of the test tube. A NiCr wire was used to apply heat to the hot end of the stack, which was able to create a sustained acoustic oscillation at the fundamental frequency. A heater tape was wrapped around the hot duct area to adjust the temperature within this region. A small electret microphone (Panasonic WM-62GT301A) was clipped at the open end to measure the frequency. A Type-E thermocouple was placed in centre of the hot duct to measure the temperature of that region.

![Figure C.2](image)

**Figure C.2.** Photograph of the acoustic laser configured to simulate one-half of a self-powered thermoacoustic engine that would measure temperature in a graphite capsule. Most of the resonator (test tube) is wrapped with half-inch thick silica insulation. Under the insulation and outside the glass test tube, there is a flexible electric heating tape to control the temperature of the gas inside the thermally-insulated portion the resonator. Near the open end of the resonator is an electrically-heated thermoacoustic stack. The hot end of the stack faces the closed end of the test tube and is glowing orange from the electrical power delivered by the power supply also visible in the photo that is providing 6.7 Vdc at 3.42 Adc (23 watts). The resonator is radiating high sound levels into the laboratory that are measured by the small electret microphone mounted on a stick that protrudes from the open end (lip) of the test tube resonator.
The idea behind this resonator setup is that the hot end can be inserted into the graphite and that temperature would drive the resonant frequency accordingly. In chapter 4, the thermoacoustic effect was generated via heat radiation, and since this graphite capsule reaches to temperatures above 1,000°C, it would be expected that it there would be enough heat to generate sustained pressure oscillations within the resonator.

The schematic from the DELTAE C model for this resonator is shown in figure C.3. The entire resonator was 20 cm long. A stack of length 0.9 cm and of size 600 cells/in² was placed 15.7 cm away from the rigid end of the resonator. The remaining ambient-temperature length to the mouth of the tube was 3.4 cm.

![Schematic diagram from the DELTAE C model of the thermoacoustic resonator used in the graphite simulation experiment.](image)

The state variable plot from DELTAE C is also shown in figure C.4. This shows the expected pressure and volume velocity distributions within an open-closed resonator as well as the typical temperature gradient across the stack. Note that the volume velocity is quite constant in the ambient end, closer to the mouth. This implies that the mass of gas at the end of the tube moves as an oscillating plug of air. This will be used to develop a model for the behaviour of the system.

Figure C.5 shows plots of both the fundamental frequency and the frequency-temperature invariant as a function of the temperature measured in the hot duct. The frequency invariant was calculated from equation 5.2, using this hot duct temperature.

In figure C.5(a), it is clear that there is a direct relationship between the temperature and the frequency of the sound generated. However, figure C.5(b) is indicative again that the temperature at the centre of the hot duct region is not the acoustically-averaged temperature sampled by the fundamental frequency. Again, due to the fact that for the thermoacoustic standing wave engine to oper-
Figure C.4. State variable plot for one operating condition of the DELTAEC model shown schematically in figure C.3. The horizontal axis is the position along the test tube (resonator) as measured from the closed end at $x = 0$. The black solid line is amplitude of the oscillating acoustic pressure $p_1$ in kPa and the dashed blue line is the amplitude of the oscillating volume velocity $U_1$ of the gas in m$^3$/sec $\cdot$ 1000 (i.e., milli-m$^3$/sec). The solid red line is the gas temperature $T_m$ in $^\circ$K/1000. The visible discontinuity in the volume velocity and the temperature occur at the stack location. For this standing wave, the volume velocity and the pressure are approximately 90$^\circ$ out-of-phase with $U_1$ ($x = 0$) = 0 at the closed end and $p_1$ ($x = 0.20$) = 0 at the open end.

However, it is possible to model the behaviour of this system making an acoustic analogy to the longitudinal vibrations of a mass loaded bar that is fixed on one end. From the state variable plot in figure C.4, it is possible to treat the resonator in two sections: (i) a hot section on the left side of the stack that supports the standing wave (bar fixed at one end) and (ii) an ambient temperature section of the resonator, near the open end, that behaves as a plug of oscillating gas mass (mass loaded end). From basic acoustics, the pressure distribution of the hot side can be expressed as in equation C.1, both spatially ($x$) and temporally ($t$). This equation also satisfies the boundary condition that $U_1$, the volume velocity on the rigid end is zero.
Figure C.5. (a) The fundamental frequency of the resonator as a function of the temperature in the hot duct. (b) The frequency-temperature invariant in relation to the temperature of the hot duct.
\[ p_1(x,t) = p_1 \cos(kx)e^{jwt} \quad (C.1) \]

If this lumped mass between the stack and the open end of the resonator is to move with the same velocity as the hot gas behind the stack, then a transcendental equation (same as that for a mass loaded fixed bar or string) can be derived [48] (equation C.2) that will calculate the fundamental frequency in terms of the ratio of the mass of gas in the ambient section, \( m \) to the mass of gas in the hot duct, \( m_{hd} \). The mass of gas in both cases can be derived from the mean density, \( \rho_m \), the cross sectional area, \( A \) (same for both sections) and the lengths of the associated section, \( x_{hd} \) and \( x_{amb} \) (hot duct and ambient length respectively). For the hot duct, \( \rho_m \) will be a function of temperature, whereas \( \rho_m \) on the ambient end will be evaluated at a temperature of 300K (room temperature). The solutions to this transcendental equation are shown in figure C.6.

\[ \cot(kL) = \frac{m}{m_{hd}}kL = \frac{\rho_m(300K)x_{amb}}{\rho_m(T)x_{hd}}kL \quad (C.2) \]

It is possible to simplify the calculation of the fundamental frequency, \( f_1 \) as a function of the hot duct temperature from C.2 through the introduction of an ambient sound speed, \( c_o = c(T = 300K) \) and use of the equation for the speed of sound (equation 5.1). This is shown in equation C.3.

\[ (kL) = \frac{\omega L}{c} = \frac{2\pi Lf_1}{c} = \frac{2\pi Lf_1}{c_o} \sqrt{\frac{300}{T}} \quad (C.3) \]

Hence, it is possible to get an expression for the invariant according to this model as shown in equation C.4.

\[ \frac{f_1}{\sqrt{T}} = \frac{c_o}{2\pi L\sqrt{300}}(kL) \quad (C.4) \]
Figure C.6. Graphical representation of equation C.2 for various mass ratios \( m/m_{hd} \) represented by straight lines of different slope. The intersections of those lines and the function \( \cot(kL) \) represent solutions. The intersection closest to the origin corresponds to the fundamental resonance frequency. For \( m/m_{hd} = 0 \), the intersection of \( \cot(kL) \) and the x-axis at \( \pi/2 \) corresponds to a quarter-wave resonance with no mass loading [48].

Using the range of hot duct temperatures from the measured data, the transcendental equation (equation C.2) can be solved for values of \( kL \) through a digital means or analytically using a Taylor series expansion, both of which have very good convergence. With values of \( kL \), equation C.4 was used to calculate the modelled frequency-temperature invariant. Figure C.7 shows how this model compares to the measured data. Both the measurements and the model show a smooth and systematic variation in the frequency-temperature invariant with temperature for a resonator where the mean pressure is constant but the change in the mean gas density has significant effects on the inertance of the gas. The approximation is also better at higher temperatures.

Although this frequency-temperature relationship is not a constant, as it would be for a simple resonator with uniform temperature, this analysis demonstrates that the variation in the “invariant” with temperature can be understood and modelled at various levels of approximation. The result is that it is possible to provide a reasonable and reliable one-to-one mapping of resonance frequency to
absolute temperature for any resonator geometry and/or temperature distribution.

**Figure C.7.** The frequency-temperature invariant measured data and the transcendental equation model as a function of the temperature of the hot duct. The frequency-temperature invariants were calculated with respect to the temperature of the hot duct.
Bibliography


