The Pennsylvania State University

The Graduate School

The Mary Jean and Frank P. Smeal College of Business Administration

ESSAYS ON TECHNOLOGY MARKETS

A Thesis in

Business Administration

by

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Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

May 2005
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ABSTRACT

This dissertation consists of two essays, each of which provides preliminary answers to different problems faced by firms. The first essay (Chapter 2) considers whether it is necessary for a firm to be first to enter a new market to attain market leadership and under what conditions it can attain market leadership if it fails to be the first mover. The second essay (Chapter 3) considers how buyers should structure their compensation schemes for reverse auction market makers, such that the buyer lowers its overall cost of procurement. The following abstracts provide additional detail about the two essays.

Abstract of Essay I: Determinants of Market Leadership in Competition Between Standards: The Roles of Network Externalities, Switching Costs, and Customer Preferences

Casual observation of the market outcomes of competition between standards in high-technology markets suggests that the first standard to enter the marketplace does not always win. Sometimes it loses and exits the marketplace; sometimes it survives as a follower in the marketplace. A natural question to ask therefore is: What factors determine which standard wins in the marketplace? How do these factors interact with one another? The present research answers these questions by modeling the competition between two standards, one of which enters the marketplace first. We model what we believe to be the key drivers of high-technology markets, namely, network externalities, customers’ preferences for competing standards, and the level of switching costs that customers face.
We model customer behavior as a stochastic process in a market with two competing standards. The pioneering standard, standard 1, enters the market at a certain point in time, and the second standard, standard 2, enters later. Customers arrive continuously into the market and choose a product made with either standard 1 or standard 2. The market shares of the two standards evolve stochastically as customers arrive and as they switch between the two standards. We construct the market share stochastic process by aggregating the customer-level stochastic process and then characterize the long-run behavior of the market share process.

Our analytical results suggest that the pioneering standard does not always win in high-technology markets. Instead, we find that a range of possible market outcomes arises, depending on the level of switching costs, degree of network externalities, and customer preferences. Our findings suggest that empirical studies that examine the outcome of competition between standards must account for the nature of the market structure.

Counter-intuitively, we find that standard 2 can sometimes overtake the pioneering standard, even if it is inferior. A customer’s purchase decision is dependent not only on the benefits the standard offers but also on the situational factors that surround the purchase decision. Therefore, a customer may purchase an inferior standard that offers lower utility because of situational constraints. Consequently, the inferior standard may become the market leader if enough customers encounter situational factors that make them purchase the inferior standard. Our findings suggest that some typical marketing strategy prescriptions for firms operating in high-technology markets, such as building an
early lead and exploiting positive feedback, must be contingent on the extent of the switching costs, the strength of the network effects in the market, and customer preferences.

Abstract of Essay II: Incentive-Compatible Compensation Schemes for Reverse Auction Market Makers

We investigate the nature of the incentive-compatible compensation scheme that a buyer must offer to a market maker when the buyer plans to hold an online reverse auction. When the buyer requires that the market maker qualify suppliers on its behalf and when the market maker’s effort is unobservable, the buyer faces a moral hazard problem. This problem arises because the market maker may shirk its responsibility by failing to exert any effort to qualify suppliers rigorously. Structuring appropriate compensation schemes represents one way to solve the moral hazard problem. We therefore use principal–agent and auction theory to build a model to investigate how to structure compensation schemes to solve the moral hazard problem.

Counter-intuitively, our findings suggest that, in some procurement situations, the buyer should structure the market maker’s compensation as a percentage of the contract value. Such compensation schemes offer an incentive to the market maker to increase the winning bid, which seems, at first glance, undesirable for the buyer. However, such a counter-intuitive compensation scheme is necessary when quality costs rise very quickly with poor quality and when there is significant variation in quality among suppliers. In this procurement situation, the buyer faces a high risk of incurring substantial costs associated with awarding business to a poor quality supplier. Therefore, the buyer wants
to identify and screen out low quality suppliers so they do not bid in the auction. Proper supplier qualification is consequently necessary so that the buyer identifies low quality suppliers correctly. But, the market maker will not qualify suppliers rigorously if it receives compensation based on the amount of savings the buyer obtains. The resulting lack of rigor in supplier qualification can lead to a low quality supplier being invited to participate in the auction, in which case the winning bid is likely to be low, which increases the market maker’s compensation. In contrast, offering the market maker compensation that is based on a percentage of the contract value encourages the market maker to qualify suppliers rigorously. Rigorous qualification ensures that the buyer eliminates the low quality suppliers from the auction, and hence, the winning bid is likely to be high, which will increase the market maker’s compensation. More generally, our analysis suggests that the nature of an incentive-compatible compensation scheme depends on the extent of quality variation among suppliers and how fast quality costs increase with poor quality.
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ACKNOWLEDGMENTS

I am extremely grateful to my thesis adviser, Professor Gary Lilien, for his support and guidance during my stay at Penn State as a graduate student. It has been my privilege to work with him, and I have learned a lot during the many interactions I have had with him over the years. I particularly appreciate the many times he read drafts of my essays and the valuable feedback he gave me on every occasion. I am sure that the insights he provided will help me for the rest of my academic career.

I also thank the rest of my committee—Professor Arvind Rangaswamy, Professor Gary Bolton, Professor Anthony Kwasnica, and Professor Jogesh Babu—for agreeing to be members. I believe my essays are much stronger in light of their valuable suggestions. I would like to thank Professor Arvind Rangaswamy in particular for supporting me in my final years at Penn State and for involving me in various interesting research projects.

Finally, I thank my wife, Lakshmi, for her support and understanding in the difficult times during the past four years. I am confident we will be able to navigate any future difficult periods in our life successfully with her by my side. I would also like to thank my in-laws, Mylavarapu Nagaraju and Mylavarapu Girija, for their support and encouragement. Last but not least, I thank my parents, Vadali Krishna Mohan and Vadali Kamakshi, for their love and for supporting me in my dreams of completing a doctorate in business administration.
1. Overview of Dissertation

This dissertation consists of two essays, each of which addresses different business problems faced by firms. The next two paragraphs provide an overview of the two essays.

In Chapter 2, we consider the problem of entry timing in high-technology markets, which are often characterized by competition between standards (e.g., VHS versus Beta in the context of VCRs). When a firm develops a new standard, a fundamental decision is whether to launch the standard immediately or delay the launch with the hope of improving the standard even further. At stake is the advantage that a first mover may obtain in most markets. Delaying entry into a new market may result in a competing standard being the first mover in the market, which could mean that the delaying firm loses first-mover advantages. However, empirical studies in the context of non-technological markets show that being the first mover does not guarantee market leadership. Anecdotal evidence from technological markets also suggests that first-mover advantages may not hold even in these markets. In light of these studies, Chapter 2 considers how the benefits offered by the standards in the marketplace, the extent of switching costs a consumer incurs when switching between standards, and entry timing affect the outcome of competition between standards.

In Chapter 3, we consider a buyer that plans to procure materials through a reverse auction. Procuring materials by means of a reverse auction is beneficial to the buyer because it can access suppliers on a global basis. However, assessing whether the suppliers are capable of meeting the buyer’s procurement requirements is not easy. In
such situations, the buyer appoints a market maker who qualifies the suppliers on behalf of the buyer. In return for its qualification services, the buyer offers the market maker compensation. The problem we address in the second essay is the design of the market maker’s compensation, such that the market maker is motivated to qualify suppliers rigorously. A compensation scheme that does not motivate the market maker in this way may result in unqualified suppliers participating in the auction, which is undesirable for the buyer. Chapter 3 provides preliminary answers to the buyer’s problem of structuring appropriate compensation schemes to market makers so that the buyer obtains desirable auction outcomes.
2. Determination of Market Leadership in Competition Between Standards: The Roles of Network Externalities, Switching Costs, and Customer Preferences

2.1 Introduction

Whether first-mover advantages exist has received considerable academic attention in marketing. Current evidence from empirical studies is mixed, with several studies finding support for first-mover advantages (e.g., Bowman and Gatignon 1996; Brown and Lattin 1994; Kalyanaram and Urban 1992; Robinson 1988; Robinson and Fornell 1985; Robinson and Min 2002; Urban et al. 1986) and others finding evidence of follower advantages (e.g., Golder and Tellis 1993; Lieberman and Montgomery 1998; Lilien and Yoon 1990; Shankar, Carpenter, and Krishnamurthi 1998). However, most of the above research does not consider the issue of first-mover advantages in the context of markets with positive network externalities (cf. Shankar and Bayus 2003; Srinivasan, Lilien, and Rangaswamy 2004). In a market with positive network externalities, such as a technology market in which incompatible standards compete, the utility a customer derives from a standard increases with the number of other customers who use the same standard. For example, the utility that a customer obtains from using a Windows computer depends on how many other customers use Windows computers, because the more customers that use Windows, the more software that becomes available for that platform. Given the presence of network externalities, is it better to be a first mover in a technology market? Consider the following quotes from practice that illustrate the lack of consensus on the presence of first-mover advantages in technology markets:

In the networked market, whoever gets a small, early advantage in a market may soon have a large, insuperable edge of increasing returns to scale due to the networked effects (Fallow 1999).
There are loads of examples in networked markets, where being first doesn’t get you much. Apple Computer’s Macintosh was the first to offer a graphical user interface. Microsoft’s Windows was second. Bob Metcalfe started 3Com to sell the first Ethernet networking standards. Cisco came along later (Maney 2001).

Table 2.1 illustrates the fate of various standards in different technology markets. As it suggests, the pioneering standard does not always go on to dominate the market. In other words, being the first mover in a competition between standards does not seem to guarantee market leadership. If this is true, then what other factors might determine the outcome of competition between standards? How do these factors interact with one another to determine which standard wins the market? In this research, we develop a stochastic model to answer these questions, and our answer suggests that first-mover advantage depends on the extent of customer\(^1\) switching costs, the strength of network externalities, and the preferences of customers for the standards in the market.

\(^1\) We use the word “customer” to refer to both business customers and consumers.
## Table 2.1: First Movers and Current Market Leaders in Markets with Network Externalities

<table>
<thead>
<tr>
<th>Market Need</th>
<th>First Standard</th>
<th>Later Standards*</th>
<th>Outcome of Competition</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Movie playback, such as VCRs¹</td>
<td>Beta</td>
<td>VHS</td>
<td>Beta: Lost</td>
<td>Pioneer lost.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>VHS: Leader</td>
<td></td>
</tr>
<tr>
<td>Color television²</td>
<td>NTSC</td>
<td>SECAM, PAL</td>
<td>All three coexist.</td>
<td></td>
</tr>
<tr>
<td>Videogames before the early 1980s</td>
<td>Magnavox</td>
<td>Atari</td>
<td>Magnavox: Survivor Atari: Leader</td>
<td>Pioneer has reasonable market share but is not the dominant standard.</td>
</tr>
<tr>
<td>Videogames since the mid-1980s</td>
<td>Nintendo</td>
<td>Sega, PlayStation, Xbox</td>
<td>Nintendo: Survivor Sega: Lost PlayStation: Leader Xbox: Too early to tell</td>
<td>Pioneer has reasonable market share but is not the dominant standard.</td>
</tr>
<tr>
<td>Internet access: 56 kbps modems³, ⁴</td>
<td>X2</td>
<td>K56Flex, V92</td>
<td>X2, K56Flex: Lost V92: Leader</td>
<td>Pioneer lost.</td>
</tr>
<tr>
<td>Web browsers⁵</td>
<td>Netscape</td>
<td>Internet Explorer</td>
<td>Netscape: Survivor Internet Explorer: Leader</td>
<td>Pioneer is a fringe player.</td>
</tr>
<tr>
<td>Desktop databases⁶</td>
<td>dBase</td>
<td>FoxPro, Access</td>
<td>dBase: Lost FoxPro: Survivor Access: Leader</td>
<td>Pioneer lost.</td>
</tr>
</tbody>
</table>

Notes:
1. Ampex launched a VCR in 1963, but the actual standards that competed in the marketplace were Beta and VHS.
2. Color television standards competition is across countries rather than within a country.
3. kbps refers to the speed at which a modem transmits data.
4. The history of modem standards dates back to 1960s, when a 300 bps modem was launched. However, there was no standard battle per se at that time. With technological advances, modems with higher speeds—such as 1200 bps and 2400 bps—started replacing modems with lower speeds. Technically, the faster modems were of a different standard than lower speed modems, but we chose to highlight the battle among X2, K56Flex, and V92 because all three standards were for the same speed, 56 kbps.
5. Many earlier browsers were launched starting from Lynx in 1989. However, we focus on the competition between Netscape and Internet Explorer because competition between these two for market dominance is well known.
6. Many other database products were launched, but we list only the prominent ones.

*Later standards are listed in chronological order of entry into the market.
We consider two standards that enter the market at different times. We call the standard that enters first the “first mover” and the standard that enters later the “second mover,” for ease of exposition. The two standards compete in the market for a fixed number of potential customers who arrive into the market continuously. A customer’s choice of a standard is modeled in a probabilistic manner; so that the standard that offers the customer higher utility will be chosen with a higher probability. As a first step, we focus primarily on understanding market outcomes as a result of demand-side dynamics. We do not focus on firm-level strategic variables, such as product pricing or advertising, because the primary impact of these variables is on brand market shares rather than the market shares of the standards the firm uses. However, the market shares of standards depends on the market shares of the brands that use that standard, and therefore, firm-level marketing actions can have a second-order effect on market shares. In light of this indirect impact of marketing mix variables on the market shares of standards, extending our present research to include firm-level strategic actions represents a promising avenue for further research.

Our theoretical analysis leads us to conclude that in high-technology markets, the first mover need not win in all situations. Intuition suggests that a first mover should have an advantage over followers, even if customers can switch between the standards. A standard with a temporary monopoly will have more customers by the time the other standard enters the market, and with positive network externalities, it should be more attractive to customers than a smaller network would be. Therefore, we would expect the larger network to dominate in the long run. However, our model suggests that the outcome of competition between two standards in a networked market is not intuitive.
Rather surprisingly, we find that the second mover has a chance to overtake the first mover, even if it is a *moderately* inferior standard, a finding that is consistent with anecdotal observations that inferior standards sometimes win in networked markets, despite their late entrance. Our model predicts that an inferior second mover can overtake the first mover if the second mover manages to obtain a market share that is greater than a “tipping point market share.” When the second mover obtains a sufficiently high market share, it compensates for its inferiority and thereby offers higher overall utility compared with that of the first mover. Note that when the second mover enters the market, the first mover already has an installed base of customers; hence, at that time, the first mover offers higher overall utility than the second mover. However, because of situational factors that surround the purchase decision, customers might not necessarily purchase a product made with the first mover’s standard. Therefore, if enough customers face situational constraints that make them purchase the second mover’s standard, the second mover eventually can obtain a sufficient market share that compensates for its inferiority and allows it to overtake the first mover. More generally, our results show that the outcome of competition between standards depends on the extent of switching costs, customer preferences, and the strength of network effects in the market.

The rest of this chapter is organized as follows: In Section 2.2, we discuss how markets with positive network externalities differ. On the basis of this discussion, we develop our stochastic model in Section 2.3. We then present our method of analysis in Section 2.4 and present and discuss our key analytical results in Section 2.5. Finally, in Section 2.6, we conclude with a discussion of some limitations and future research directions.
2.2 Markets with Positive Network Externalities

Positive network externalities\(^2\) exist when the value of a standard to a customer increases as the total number of other customers who buy the same or a compatible standard increases (Katz and Shapiro 1986). We hereafter call markets with network externalities “networked markets” and positive network externalities “network externalities” for ease of exposition.

Networked markets can be viewed generically as a network that consists of nodes, with links connecting some of the nodes (Economides 1997). Economides and White (1994) classify networks as either two-way or one-way. Consider a telephone network as an example of a two-way network.

A telephone network can be represented by a somewhat simplified topology, as we show in Figure 2.1.

**Figure 2.1: Simplified Topology of a Telephone Network**

Nodes A–G represent customers, and node S represents the telephone company that offers switching services to customers. The telephone network is an example of a two-way network.

\(^2\) Network externalities can also be negative, as in the case of costs of congestion, but we focus on the more common positive network externalities.
way network, because calls can be made from either A to B or B to A. The two paths are also identical in the sense that it does not matter whether A talks to B or B talks to A; in either case, a conversation takes place. Other examples of two-way networks include railroads, roads, and other forms of telecommunication networks.

In contrast, one-way networks are those networks in which the paths A→B and B→A are not identical or one of the paths is not meaningful. For example, consider the same network topology as in Figure 2.1 but interpret node S as a television broadcasting station and the remaining nodes as customers who receive broadcast programming. Using this interpretation, the path from a customer to node S is not meaningful. Other examples of one-way networks include ATM networks and pagers.

A feature of two-way networks that arises from the network structure is the presence of direct network externalities. Consider the example of the telephone network. As more customers join the network in Figure 2.1 the number of calls that a customer can make increases and consequently the utility that a customer gets from a two-way network increases as the number of customers on the network increases. Because the increase in utility depends directly on the number of customers using the network, two-way networks have direct network externalities.

In contrast, one-way networks have indirect network externalities, because the value to a customer of a standard depends on the number and variety of complementary goods available for that standard. For example, in the case of television broadcasting, the value of a television to a customer depends on the amount and quality of broadcast programming available, which in turn depends on the number of televisions sold.
Therefore, in one-way networks, the utility that a customer gets from a standard is *indirectly* dependent on the number of customers who buy compatible standards.

Note that the utility a customer receives from a networked standard can consist of a stand-alone component plus a network-related component or solely a network-related component. For example, the value of a fax machine is zero if no one else adopts a fax machine. As more customers use compatible fax machines, the value to the customer increases, because he or she can send faxes to more people. However, a fax machine has no stand-alone value; that is, it is worthless if the consumer is the only person in the world to have a fax machine. In contrast, a standard such as a word processing package has stand-alone value along with network-related value because a customer can use word processing software as a stand-alone system even if no other customer in the world has it. In addition, it has network-related value because the benefits of the software increase as more customers adopt compatible software, which increases the customer’s ability to share files and collaborate with others.

We further subdivide two-way and one-way networks according to the level of switching costs between incompatible networks. Most existing research in economics in the context of network externalities assumes the presence of “infinite” switching costs between incompatible networks, which implies that customers do not switch between the alternatives in the market. However, it is more realistic to consider switching costs as ranging from nonexistent to very high. Therefore, we suggest that the level of switching costs is another factor that differentiates one networked market from another. In Table 2.2, we summarize our suggested taxonomy of markets with network externalities.
Table 2.2: Taxonomy of Networked Markets

<table>
<thead>
<tr>
<th>Switching Costs</th>
<th>Two-Way</th>
<th>One-Way</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>Fax machines</td>
<td>VHS versus Beta, software + hardware</td>
</tr>
<tr>
<td>Low</td>
<td>Online games, music sharing services, community forum sites</td>
<td>Browsers</td>
</tr>
</tbody>
</table>

We now turn to the development of an analytical model that captures the preceding structural characteristics of networked markets. Our model adapts an existing stochastic model from sociology (Weidlich and Haag 1973). The original idea of adapting models from sociology to study markets in which positive network externalities exist along with customer switching behavior appears in Arthur (1994). We extend the original model in two directions. First, we incorporate switching costs into the model, and second, we assume that the two standards enter at different points in time and analyze how the entry timing of the second mover affects the market outcome. We believe these extensions make the model more realistic.

2.3 The Model

Consider two incompatible standards in a networked market. We denote the two standards by the indices 1 and 2, where standard 1 is designated as the first mover in the market. We denote the time of entry of standards 1 and 2 by $t_1$ and $t_2$, respectively, where $t_2 > t_1$. We simplify the demand side by assuming that the total number of potential customers, denoted by $N$, is a constant. That is, we assume that the potential market size does not increase with time. We also do not model firm-level strategic actions explicitly because they are unlikely to have a significant impact when standards
are unsponsored, that is, when no firm can claim ownership of that standard. For example, the modem standard that became the market leader (i.e., V92) was not owned by any single firm but rather was developed by an industry-wide consortium. In situations in which standards are sponsored by rival firms, understanding what happens to the long-run market outcomes in the absence of strategic behavior is a necessary first step to building models that incorporate strategic behavior.

In terms of the timing of events in our model, standards 1 and 2 enter the market at times $t_1$ and $t_2$, respectively. After standard 1 enters the market, customers make purchase decisions at exponentially distributed inter-arrival times. After standard 2 enters the market, new customers continue to arrive into the market. In addition, existing customers consider switching to the rival standard, and the time between inter-purchase decisions is distributed exponentially. Eventually, all customers enter the market. We now model these events in detail.

A customer’s decision to purchase a product with standard 1 or standard 2 depends on many factors, not all of which we can account for in our model. Specifically, our model assumes that a customer’s decision whether to purchase a product made with standard 1 or 2 depends only on the benefits offered by the standard, not on any situational factors that surround the purchase situation. For example, we neglect the availability of products in retail outlets and the income levels of consumers, which may influence the purchase decision. Similar to discrete choice theory (Ben-Akiva and Lerman 1985), we model customer purchase behavior in a probabilistic manner because we do not account for all factors associated with the purchase decision.
In light of the preceding, we model the utility that a customer receives from purchasing standard \( i = 1, 2 \) as \( U_i = V_i + \epsilon \), where \( V_i \) is the deterministic component of the utility and \( \epsilon \) is a random error associated with unobserved factors that affect the purchase decision and assumed to be distributed as a Type-I extreme value random variable (McFadden 1974). We assume that \( V_i = \delta_i + k n_i \), where \( \delta_i \) represents the intrinsic preferences of the customer for that standard and equals the stand-alone benefits delivered by standard \( i \), \( n_i \) is the number of customers using standard \( i \), and \( k \) is the strength of network externality. The second term, \( k n_i \), represents the network-related benefit. Note that our results do not change qualitatively if we model the utility function as \( V_i = \delta_i + k n_i \), a formulation that would be appropriate if products made with these standards did not have stand-alone benefits.

This utility specification implies several things about customer choices. First, when the pioneering standard is the only standard in the marketplace, the probability that a new customer will buy the pioneering standard is given by \( \frac{\exp(V_1)}{1 + \exp(V_1)} \), where \( V_1 \) is the deterministic utility that the customer gets from that standard. Second, when both standards are present in the market, the probability that a new customer chooses standard 1 is given by \( \frac{\exp(V_1)}{1 + \exp(V_1) + \exp(V_2)} \), and the probability that he or she chooses standard 2 is \( \frac{\exp(V_2)}{1 + \exp(V_1) + \exp(V_2)} \), where \( V_1, V_2 \) is the deterministic utility that the customer gets from standards 1 and 2, respectively. Third, consider a customer who needs to repurchase a product and contemplates switching between standards, and let \( \gamma \) be the cost of
switching between the two standards. Our utility specification implies that the probability that the customer will switch from standard 1 to standard 2 is given by

\[ p_{12} = \frac{\exp(V_2 - V_1 - \gamma)}{1 + \exp(V_2 - V_1 - \gamma)}, \]

where \( V_1, V_2 \) is the deterministic utility from standards 1 and 2.

Similarly, we define

\[ p_{21} = \frac{\exp(V_1 - V_2 - \gamma)}{1 + \exp(V_1 - V_2 - \gamma)}. \]

We assume that customers enter the market according to a Poisson process, namely, the inter-arrival times of customers follow an exponential distribution with parameter \( \beta \). We also assume that all potential customers eventually enter the market and no customers leave the market after entering it. Once a customer enters the market and chooses a standard, he or she will make a repurchase decision at some future point in time. We assume that inter-purchase times follow an exponential distribution with parameter \( \lambda \). Prior marketing research indicates that either an exponential or an erlang distribution is a good model for inter-purchase timing (Ehrenberg 1959; Hauser and Wisniewski 1982; Jeuland, Bass, and Wright 1980). We use the simpler negative exponential form here for analytical tractability. The last assumption we make, for technical reasons, is that in a small time interval \([t, t + \Delta t]\), only one of the following events will occur: (1) a new customer arrives into the market or (2) a customer considers switching. We provide a list of all substantive and technical assumptions in Appendix 2.7.1.

2.4 Analysis

The model we outlined in Section 2.3 is a first-order Markovian model. There are two approaches to analyze Markovian models: use game theory and characterize the equilibrium outcomes or analyze the model as a stochastic process and characterize its
long-run behavior. Because we do not consider firm-level actions, we chose to analyze the model as a stochastic process.

2.4.1 Overview of Analysis Approach

One method to analyze a stochastic model is to find the stationary transition probability matrix of an individual customer and then equate the stationary probability of a customer purchasing a particular standard with the market share for that standard. However, this method assumes that all customers switch at the same time. Only because of this assumption can we justify the conclusion that the long-run transition probability of purchase of a particular standard by a typical customer is equivalent to the market share of the standard.

There is work in marketing literature relaxes the assumption of simultaneous switching by customers. For example, Jeuland, Bass, and Wright (1980) assume that inter-purchase times are erlang distributed of order $r$. They then develop an integrated choice–purchase timing model. A more generalized model developed by Hauser and Wisniewski (1982) models the process of customer response as a semi-Markov model, which means that purchase timing and standard selection are interdependent. However, the presence of network externalities implies that a customer’s decision depends on what other customers decide to do, a phenomenon that models such as Hauser and Wisniewski’s do not capture. If we model the number of customers waiting to enter the market and the number using each standard as a stochastic process, we can account for the interdependence of customer decisions in markets with network externalities in a natural way. For example, let $Y_i(t)$ denote the “state” of a customer at time $t$. Then,

$$Y_i(t) = 0 \text{ if the customer has yet to arrive in the market,}$$
\( Y_i(t) = 1 \) if the customer uses a product that uses standard 1, and
\( Y_i(t) = 2 \) if the customer uses a product that uses standard 2.

Let
\[ n_o(t) \text{ be the number of customers yet to enter the market at time } t, \]
\[ n_1(t) \text{ be the number of customers using products made with standard 1 at time } t, \text{ and} \]
\[ n_2(t) \text{ be the number of customers using products made with standard 2 at time } t. \]

Therefore,
\[
\begin{align*}
n_o(t) &= \sum_{i=1}^{N} I[Y_i(t) = 0] \\
n_1(t) &= \sum_{i=1}^{N} I[Y_i(t) = 1] \\
n_2(t) &= \sum_{i=1}^{N} I[Y_i(t) = 2]
\end{align*}
\]
where \( I[.] \) is the indicator function.

Because we have modeled customer arrivals and customer switching as a stochastic process, \( \{ Y_i(t); t \geq 0 \} \) is a stochastic process, and it follows that \( \{ n_o(t), n_1(t), n_2(t); t \geq 0 \} \) also is a stochastic process. Modeling the number of customers yet to arrive and the number of customers using each standard (i.e., \( n_o(t), n_1(t), n_2(t) \)) as a stochastic process enables us to associate a probability distribution with various possible market outcomes. We then can investigate the long-run character of this probability distribution to assess how the long-run market outcomes depend on factors such as switching costs, the strength of the network externality, and customer preferences. We call the \( \{ n_o(t), n_1(t), n_2(t); t \geq 0 \} \) stochastic process the market share stochastic process.
for ease of exposition, and we suppress the dependence of \( n_a(t), n_1(t), n_2(t) \) on time \( t \) for notational convenience.

There are two ways we can model the market share stochastic process (Weidlich and Haag 1973). One approach, known as the Langevin approach, models \( n_a, n_1, n_2 \) as a deterministic process and then adds an error term that makes the process stochastic. The second approach models \( n_a, n_1, n_2 \) directly as a stochastic process using the notion of an ensemble. We adopt the ensemble approach, which relies on “aggregating” the individual customer stochastic process, for analytical tractability.

We first define an ensemble of possible worlds, each of which consists of \( N \) customers and two standards. Informally, an ensemble is the set of all possible paths that a world can take from the same starting conditions. Because the total number of potential customers is constant, the state of a world at any time \( t \) is defined fully by the variables \( n_2 \) and \( n_0 \). Therefore, the possible paths of the ensemble consist of all possible paths of \( n_2 \) and \( n_0 \) over time, and the actual path that a world takes will be one of the paths in this ensemble. We define \( r(n_2, n_0; t) \) as the probability of finding an ensemble in state \( \{n_2, n_0\} \) at time \( t \). Our goal in subsequent sections is to relate this probability to the individual transition probabilities and then attempt to characterize the stationary distribution:

\[
 r^*(n_2, n_0) = \lim_{t \to \infty} r(n_2, n_0; t). \tag{C2.2}
\]

\footnote{Note that we can define \( n_1 \) in terms of \( n_2 \) and \( n_0 \) as \( n_1 = N - n_2 - n_0 \).}
2.4.2 Analysis Strategy

Our goal after constructing the market share stochastic process is to find the stationary transition probability for the process. The lemma below simplifies our analysis considerably.

*Lemma 1:* The only states with a positive stationary probability for the market share stochastic process are those in which \( n_0 = 0 \). Also, states in which \( n_0 = 0 \) form a closed communicating class; that is, the market share stochastic process cannot reach a state \( n_0 > 0 \) from a state \( n_0 = 0 \), and the process can go from any state \( \{n_2, n_0 = 0\} \) to any other state \( \{n_2, n_0 = 0\} \).

Proof: See Appendix 2.7.2.

Intuitively, Lemma 1 makes sense: In the long run, all customers will enter the market, and hence, there is zero probability of finding the market share process in a state in which \( n_0 > 0 \). Because no customers exit the market and customers switch from one standard to another, the process can attain any state \( \{n_2, n_0 = 0\} \) from any state \( \{n_2, n_0 = 0\} \).

Lemma 1 also implies that the stationary distribution of the market share stochastic process can be analyzed by examining how the process evolves from the time when all customers enter the market, namely, the long-run behavior of \( r(n_2, n_0 = 0; t) \). Instead of characterizing the actual form of the stationary distribution, we focus our attention on the maxima of the stationary distribution, which represent the most probable long-run outcomes. We show that this focus is sufficient as long as the market size is
very large, which is a reasonable assumption in our context. Therefore, we divide our
remaining analysis into the following steps:

(a) In Section 2.4.3, we relate \( r(n_2, n_0 = 0; t) \) to the individual transition probabilities,

establish an expression for the corresponding stationary distribution, and designate
the necessary and sufficient conditions for a point to be a maximum of the stationary
distribution.

(b) In Section 2.5, we establish the conditions in which the stationary distribution has one
maximum or two maxima and characterize the maxima in terms of which standard
dominates. (Note: We prove that the stationary distribution cannot have more than
two maxima.)

From this analysis, we determine that the second standard can dominate the market. We
also analyze the impact of delaying entry on the chance that the second standard will
dominate. We now turn to aggregating the individual stochastic processes.

2.4.3 Market Share Process: “Aggregating” Individual Customer Processes

For notational convenience, we henceforth denote \( r^*(n_2, n_0 = 0) \) as \( r^*(n_2) \) and
\( r(n_2, n_0 = 0; t) \) as \( r(n_2; t) \). Using standard stochastic arguments, we derive the following
stochastic differential equation, which describes how the probability distribution,
\( r(n_2; t) \), evolves over time (see Appendix 2.7.3):

\[
\frac{\partial r(n_2; t)}{\partial t} = \left[ w_1(n_2 + 1) r(n_2 + 1; t) - w_1(n_2) r(n_2; t) \right] + \left[ w_1(n_2 - 1) r(n_2 - 1; t) - w_1(n_2) r(n_2; t) \right],
\]

(C2.3)

where

\[
w_1(n_2) = (N - n_2) \, p_{12}(n_2) \lambda
\]

\[
w_1(n_2) = n_2 \, p_{21}(n_2) \lambda
\]
\( w_\uparrow(n_2) \) is the probability that the system moves from \( n_2 \rightarrow n_2 + 1 \) at time \( t \), and \\
\( w_\downarrow(n_2) \) is the probability that the system moves from \( n_2 \rightarrow n_2 - 1 \) at time \( t \).

Equation C2.3 represents a discrete space (\( n \) can only take on integer values), continuous time differential equation. We transform Equation C2.3 into a continuous space, continuous time differential equation for mathematical convenience by introducing the appropriate transformations (see Appendix 2.7.4):

\[
\frac{\partial R(x_2; t)}{\partial t} = \frac{\partial}{\partial x_2} \left[ K(x_2) R(x_2; t) \right] + \frac{1}{2N} \frac{\partial^2}{\partial x_2^2} \left[ Q(x_2) R(x_2; t) \right],
\]

(C2.4)

where

\[
K(x_2) = W_\uparrow(x_2) - W_\downarrow(x_2),
\]

\[
Q(x_2) = W_\uparrow(x_2) + W_\downarrow(x_2),
\]

\[
W_\uparrow(x_2) = (1-x_2) p_{12}(x_2) \lambda, \text{ and}
\]

\[
W_\downarrow(x_2) = x_2 p_{21}(x_2) \lambda.
\]

\( W_\uparrow(x_2) \) and \( W_\downarrow(x_2) \) are the continuous versions of the transition probabilities \( w_\uparrow(n_2) \) and \( w_\downarrow(n_2) \), which we defined previously. \( W_\uparrow(x_2) \) is the transition probability per unit time for the state variable \( x_2 \) to move from \( x_2 \) to \( x_2 + dx_2 \). Similarly, \( W_\downarrow(x_2) \) is the transition probability per unit time for the state variable \( x_2 \) to go from \( x_2 \) to \( x_2 - dx_2 \). The stationary distribution for \( R(x_2; t) \) (denoted by \( R^*(x_2) \)) is given by the following differential equation (see Appendix 2.7.5):

\[
\frac{1}{N} \frac{\partial \left[ Q(x_2) R^*(x_2) \right]}{\partial x_2} = K(x_2) R^*(x_2).
\]

(C2.5)
We avoid solving this differential equation explicitly by using two features of the stationary distribution. First, the maximum of the stationary distribution defines the most probable stationary point that the system will reach. Second, the market share stochastic process can be approximated by a deterministic process that converges to the maximum of the stationary distribution (for proof, see Proposition 2 in Appendix 2.7.7). Note that this approximation increases in accuracy as the market size increases. Therefore, for all practical purposes, the maximum of the stationary distribution defines the long-run outcomes. Therefore, the necessary and sufficient condition for a point to be a maximum of the stationary distribution is

$$K(x_m) = 0 \text{ and } \left. \frac{\partial K(x_m)}{\partial x} \right|_{x = x_m} < 0,$$

provided $\frac{Q'(x_m)}{N} \ll 1$ and $\frac{Q'(x_m)}{N} << 1$ (see Appendix 2.7.5).

Having completed part (a) of the analysis strategy we outlined in Section 2.4.2, we now turn to part (b); that is, we characterize the conditions in which the stationary distribution has one maximum or two maxima, as well as the set of starting states from which the process converges to a particular maximum when there are two maxima.

### 2.5 Analytical Results

We divide part (b) of our analysis into three cases according to the intrinsic preferences of customers for standards 1 and 2. In Section 2.5.1, we characterize the long-run nature of the market share stochastic process when customers are indifferent between the two standards, or when $\delta_1 = \delta_2$. In Section 2.5.2, we consider the case when standard 2 is inferior to standard 1, that is, when $\delta_2 < \delta_1$; finally, in Section 2.5.3, we consider the case when standard 2 is superior to standard 1, so that $\delta_2 > \delta_1$. In Section
2.5.4, we present our model predictions for a case in which network effects are nonexistent, which serves as a “face validity” check for our model. Finally, we summarize and discuss our results in Section 2.5.5.

2.5.1 Customers Indifferent Between the Two Standards

Proposition 1:

(a) When \( \delta_1 = \delta_2 \), the stationary distribution has two maxima if the strength of the network externality \( k > \frac{1+\exp(-\gamma)}{N} \).

(b) The market shares of the two standards are asymmetric for either maximum; that is, in one case, standard 2 dominates the market, and in the other, standard 1 dominates.

Proof: See Appendix 2.7.6.

Figure 2.2 illustrates the stationary distribution for a parameter set that satisfies the conditions of Proposition 1. As it indicates, the stationary distribution has two maxima. In one maximum, the market share of standard 2 is greater than the market share of standard 1; in the other, the market share of standard 2 is less than the market share of standard 1. In other words, in one maximum, the first mover (standard 1) manages to retain market leadership, whereas in the other, standard 1 survives but loses market leadership to the non-pioneering standard (standard 2).
The presence of two maxima suggests that the stationary distribution depends on the state of the process when all customers enter the market. Proposition 2 establishes the set of states that lead to standard 1, rather than standard 2, dominating.

**Proposition 2:**

(a) If the market share of standard 2 \( x_2 \) is greater than 0.5 when all customers enter the market, standard 2 dominates. Otherwise, standard 1 dominates the market.

(b) The probability of standard 2 attaining the critical market share of 0.5 when all customers enter the market decreases as the introduction of the standard is delayed.

**Proof:**

See appendix 2.7.7

Proposition 2(a) implies that standard 1 becomes the dominant player in the market if it has a greater market share than standard 2 when all customers enter the market; otherwise, standard 2 becomes the dominant player. Proposition 2(b) implies that the likelihood of first-mover advantage increases as the entry time of the second standard is delayed.
The intuition behind Proposition 2 is as follows: The customer’s decision to switch or not involves two factors: (1) the attractiveness of a particular standard (i.e., $\delta_i$) and (2) the benefits of that standard due to network externalities. When both networks are equally attractive, the network with the higher market share offers greater benefits to customers and therefore will tend to dominate the market. However, the results are different when customers prefer one standard over the other.

### 2.5.2 Customers Prefer Standard 1 to Standard 2

*Proposition 3:* When $\delta_1 > \delta_2$, the stationary distribution has only one maximum if the strength of the network externality $k > \frac{1+\exp(-\gamma)}{N}$ and $\delta_1 > h_1(\delta_2, k, N, \gamma)$. The stationary distribution has two maxima if $\delta_1 < h_1(\delta_2, k, N, \gamma)$.

**Proof:** See Appendix 2.7.8.

Figures 2.3 and 2.4 illustrate the stationary distribution for a parameter set that satisfies the conditions of Proposition 3. When standard 1 is sufficiently superior to standard 2, the stationary distribution has a single maximum, at which standard 1 dominates the market, which suggests the presence of a first-mover advantage. The presence of a first-mover advantage makes intuitive sense because not only has standard 1 entered the market first but it also is far superior to standard 2.
Figure 2.3: Standard 1 Dominates Because It Is Sufficiently Superior

![Stationary Distribution](image)

Figure 2.4: Standard 2 Has a Chance to Dominate the Market

![Stationary Distribution](image)

In contrast, when standard 1 is moderately superior to standard 2, the stationary distribution has two maxima, so sometimes standard 2 manages to overtake standard 1, an outcome that seems surprising because standard 2 enters the market later and also is inferior to standard 1. To understand how an inferior standard can dominate, despite entering the market later, we must identify the conditions in which the market share
stochastic process converges to make standard 2 dominant in the long run, as Proposition 4 establishes.

*Proposition 4:*

(a) If the market share of standard 2 is greater than a critical value $x_{2c}$ when all customers enter the market, standard 2 dominates. Otherwise, standard 1 dominates the market. Here, $x_{2c} > 0.5$ and decreases as $\delta_2$ increases.

(b) The probability of standard 2 attaining the critical market share of $x_{2c}$ when all customers enter the market decreases as the introduction of that standard is delayed.

Proof: See Appendix 2.7.9.

Proposition 4 implies that standard 2 successfully can overtake standard 1, despite entering late with an inferior standard, if it can obtain a market share greater than some critical value when all customers enter the market. By obtaining this market share, standard 2 compensates for its inferiority and ensures that its value to customers is greater than that of standard 1. In addition, the critical market share required for standard 2 to overtake standard 1 decreases as its entry into the market is delayed. The practical question of how a non-pioneering, inferior standard might obtain more market share than the pioneering standard is an issue we discuss in Section 2.5.5. Finally, Proposition 4(b) suggests that the probability of standard 2 reaching the critical market share decreases as its entry is delayed, a finding that makes intuitive sense.
2.5.3 Customers Prefer Standard 2 to Standard 1

Proposition 5:

When $\delta_2 > \delta_1$, the stationary distribution has only one maximum if the strength of the network externality $k > \frac{1+\exp(-\gamma)}{N}$ and $\delta_2 > h_2(\delta_1, k, N, \gamma)$. The stationary distribution has two maxima if $\delta_2 < h_2(\delta_1, k, N, \gamma)$.

Proof: Omitted because the proof is similar to that for Proposition 3.

Proposition 5, in a sense, represents the converse of Proposition 3. Similar to Figures 2.3 and 2.4, Figures 2.5 and 2.6 illustrate the stationary distribution for a parameter set that satisfies the conditions of Proposition 3. Proposition 5 indicates that when standard 2 is sufficiently superior, it tends to be the dominant player, even if it enters the market after standard 1, thus suggesting that there is no disadvantage to entering late in these circumstances. The mechanism by which standard 2 overtakes standard 1 involves introducing a far superior standard compared with standard 1. This observation suggests that it may be in the best interest of a firm with non-pioneering technology to delay entry and improve its standard sufficiently before entering a networked market.
In contrast, when standard 2 is moderately superior to standard 1, the stationary distribution has two maxima, which suggests that sometimes standard 2 manages to overtake standard 1. The observations we made in the context of Proposition 3 hold even here, the sole difference being that standard 2 is the better standard now. Proposition 6 establishes the conditions in which the market share stochastic process converges to make standard 2 dominant.
Proposition 6:

a) If the market share of standard 2 $x_2$ is greater than a critical value $x_{2c}$ when all customers enter the market, standard 2 dominates. Otherwise, standard 1 dominates the market. Here, $x_{2c} < 0.5$ and decreases as $\delta_2$ increases.

b) The probability of standard 2 attaining the critical market share of $x_{2c}$ when all customers enter the market decreases as its introduction is delayed in the market.

Proof: Omitted because the proof is similar to that for Proposition 4.

The observations we made in the context of Proposition 4 continue to hold here. Proposition 6 implies that standard 2 can become the market leader if and only if it has a sufficient lead over standard 1 at the time that all customers enter the market; otherwise, standard 1 becomes the dominant player. However, the critical market share required to overtake standard 1 is lower when standard 2 is moderately superior than when standard 2 is moderately inferior. That is, $x_{2c} < 0.5$ when standard 2 is moderately superior, but $x_{2c} > 0.5$ when standard 2 is moderately inferior. When standard 2 is moderately inferior, it needs a greater market share to overcome its standard-related disadvantages, so the critical market share it needs to overtake standard 1 is higher. Just as in the case of Proposition 4(b), Proposition 6(b) suggests that the probability of standard 2 reaching the critical market share decreases as its introduction is delayed, a finding that makes intuitive sense.

2.5.4 Non-Networked Markets: “Face Validity” of our Model

Proposition 7: There exists only one maximum for the stationary distribution $R^*(x_2)$ if $k \leq \frac{1+\exp(-\gamma)}{N}$. Under these parameter conditions, if $\delta_1 = \delta_2$, both standards get a 50%
market share, but if \( \delta_1 \neq \delta_2 \), the standard that enjoys a higher preference among customers obtains a higher market share. Moreover, the higher the preference difference between \( \delta_1 \) and \( \delta_2 \), the greater is the market share difference between the two standards.

Proof: See Appendix 2.7.10.

We include Proposition 7 only for completeness, as \( k \leq \frac{1 + \exp(-\gamma)}{N} \) essentially implies that that \( k = 0 \) (\( \because N >> 1 \)). Therefore, Proposition 7 characterizes the long-run market outcomes in a situation in which positive network externalities are not significant and illustrates that our model has face validity. Essentially, the proposition predicts that in non-networked markets, market shares reflect which standard is better. Figures 2.7 and 2.8 reinforce this prediction.

**Figure 2.7: In Non-Networked Markets, Both Standards Get Equal Market Shares when Consumers Are Indifferent**
2.5.5 Summary of Propositions and Discussion

We summarize Propositions 1–7 in the stylized Figure 2.9. The shaded region corresponds to Proposition 7, the area in which the strength of the network externality is close to 0. Therefore, the shaded region represents a market structure without positive network externalities, which is not the focus of our present research. The remaining regions correspond to Propositions 1–6, and the curves represent the demarcation curves \( h_1(\delta_2, k, N, \gamma) \) and \( h_2(\delta_1, k, N, \gamma) \), respectively (see Propositions 3 and 5).

**Figure 2.9: Stylized Summary of our Propositions**
The counterintuitive finding that emerges from our analysis is that an inferior standard can be the market leader, despite entering late. As indicated by our propositions (Propositions 2, 4, and 6), standard 2 can overtake standard 1 if it obtains a critical market share. By obtaining this critical market share, standard 2 offers higher network benefits and compensates for its inferiority. However, when standard 2 enters the market, the probability that a new customer will choose standard 2 is lower than the probability that he or she will choose standard 1. Similarly, the probability that an existing customer of standard 1 will switch to standard 2 is lower than the probability that an existing customer of standard 2 will switch to standard 1. If a customer chooses standard 2, despite the lower utility it offers, situational factors must counteract the lower utility standard 2 offers. If enough customers encounter situational factors that prompt them to purchase standard 2, despite its inferiority, then standard 2 obtains the critical market share needed to become the market leader. Recent empirical research in the context of operating systems find that IT managers devote more resources to technologically inferior standards if they believe that inferior standards have higher network-related benefits (Frels, Shervani, and Srivastava 2003), a finding that is consistent with our explanation.

Two possible marketing strategies that firms with a non-pioneering technology standard can use to obtain critical market shares are licensing the standard and exploiting the standard’s dominance in a related market. Licensing can increase the number of manufactured products that are compatible with the licensor’s standard, which will increase sales of products compatible with the licensor’s standard, which in turn will improve the chances of the follower reaching the market share tipping point. For
example, for many years, Apple did not license its operating system and preferred to focus instead on doing everything in house. In contrast, Microsoft licensed its operating system to other personal computer manufacturers. Although many factors led to Microsoft overtaking Apple in graphical user interface–based operating systems, licensing probably was a crucial factor.

In addition, by exploiting its dominance in a related but separate market, a later entrant can tie the market in which it is dominant with the market in which it is competing with the first mover. For example, when Microsoft introduced Internet Explorer, it tied the operating system market (in which it was dominant) with the browser market by bundling Internet Explorer with the Windows operating system. The consequence of this bundling for Microsoft was an installed base of customers overnight, which possibly was greater than the market share tipping point. This strategy of exploiting dominance in a related market has been suggested elsewhere as well (e.g., Frels, Shervani, and Srivastava 2003).

Our findings suggest that in markets with network externalities, a second mover can overtake the first mover successfully if it is sufficiently superior. In such a situation, the first mover either survives with a very low market share (in the real world, this condition would result in the standard exiting from the market) or survives with a reasonable but lower market share than the second mover. What determines survival is the extent of superiority of the second mover compared with the first mover. An extremely superior second mover will drive the first mover out of the market, whereas a second mover that is only slightly superior might allow the first mover to survive in the marketplace as a fringe player.
A sufficiently superior standard offers sufficiently high advantages to customers that it eventually becomes the market leader. Therefore, it may make sense for a firm to wait and improve its standard sufficiently instead of trying to enter the market early. Previous empirical research (Shankar, Carpenter, and Krishnamurthi 1998), conducted in the context of the pharmaceutical industry, suggests that late movers can overtake pioneers by introducing innovative products. Our theoretical findings in the context of network externalities are similar. In recent research, Shankar and Bayus (2003) demonstrate how Nintendo overtook Sega even though Sega, a first mover, had a higher installed base in the 16-bit video game market. They find that Nintendo was able to overtake Sega primarily because Nintendo had greater network strength, which they define “as the marginal impact of a unit increase in network size on demand, after controlling for the effects of other variables” (Shankar and Bayus 2003). Their findings suggest that followers can overtake pioneers by exploiting network-related advantages such as asymmetric network strengths. Our theoretical findings suggest that, in addition to these methods, followers can overtake pioneers in networked markets by introducing superior standards.

Our findings also suggest a trade-off between entry timing and the extent to which a firm can improve the quality of its standard. A higher quality standard will make overtaking the first mover easier, because the market share tipping point decreases as the later entrant improves its quality. However, the probability of reaching that tipping point decreases as entry is delayed, because the later entrant must overcome the higher installed base advantage of the first mover. Managing this trade-off represents an extremely important issue that we hope to address in further research.
From a theoretical perspective, we contribute to the existing literature on first-mover advantages by offering a contingent explanation for the market outcomes of pioneers. Our theoretical findings suggest that future empirical studies must account for the nature of market structures when they investigate the presence of first-mover advantages in networked markets. From a managerial perspective, our findings also offer insights into how later entrants can overtake pioneers in networked markets. In the absence of significant quality superiority over the first mover, later entrants can exploit the presence of tipping points to overtake the first mover with strategies such as licensing, exploiting dominance in a related market, and so forth.

2.6 Limitations and Conclusions

Although we needed to make several assumptions, we believe these assumptions must be relaxed in further studies to investigate the robustness of our results. We believe the functional form of the utility function is not crucial; most of our results remain even with a more general specification for the utility function. However, two assumptions that are crucial to our results—customer homogeneity and customers who do not exit the market once they enter—may affect the results if they are relaxed. It is also important to incorporate firm-level strategic actions into the model and analyze the resulting richer model for the presence of first-mover advantages. Finally, our research, as well as related research in marketing, suggests that the presence of first-mover advantages in networked markets is contingent on many factors, and further research therefore is needed to achieve a good understanding of all these contingent factors.
2.7 Appendices

2.7.1 List of Assumptions

*Substantive Assumptions*

1. The utility that a customer gets from standard $i$ is $U_i = V_i + \varepsilon_i$,

where

   $i = 1, 2$ represent the two standards,

   $V_i = \delta_i + k n_i$ is the deterministic component of utility,

   $\varepsilon_i$ = a random error associated with unobserved factors that affect the purchase decision of the customer and assumed to be a Type-I extreme value random variable,

   $\delta_i$ = the intrinsic preference of customers for standard $i$,

   $n_i$ = number of customers using products made with standard $i$, and

   $k$ = strength of network externality.

2. Eventually, all potential customers enter the market, and no customers leave the market.

*Technical Assumptions*

1. Inter-purchase times for a customer in the market are distributed as an exponential distribution with parameter $\lambda$.

2. Potential customers arrive in the market at an exponential arrival rate $\beta$.

3. Inter-purchase time is independent of the standard the customer chooses on the subsequent occasion.

4. In a small time interval $[t, t+\Delta t]$, one of the following events occurs: (a) a new customer enters the market or (b) an existing customer decides whether to switch.
2.7.2 Proof of Lemma 1

The market share process we outline in Section 2.4.1 is a finite state, continuous time stochastic process. We know from the theory of continuous time, discrete space stochastic processes (Kulkarni 1995, p. 285) that a state of continuous time, discrete space is transient (recurrent) if and only if the state of the underlying discrete time, discrete space Markov chain is transient (recurrent). The following rules hold for a discrete time, discrete space, finite state Markov chain (Kulkarni 1995, pp. 82–83):

(a) Not all states are transient,
(b) All states in a closed communicating class are recurrent, and
(c) All states that are not in a closed communicating class are transient.

Initially, the state of the system is \( n_2 = 0, n_0 = N \). As customers arrive in the market, \( n_0 \) starts to decrease, and eventually, all customers enter the market. Therefore, each state \( \{n_2, n_0\} \) in which \( n_0 > 0 \) is not a closed communicating class, because once the chain leaves the state, it can never return, and hence, each state \( \{n_2, n_0\} \) in which \( n_0 > 0 \) is a transient state. Similarly, the set of states \( \{n_2, n_0\} \) in which \( n_0 = 0 \) forms a closed communicating class, and hence, all these states are recurrent. Because in finite state, continuous time Markov chains, there cannot be any null recurrent states, it follows that all states of the market share process \( \{n_2, n_0\} \) in which \( n_0 > 0 \) are transient, and all states of the market share process \( \{n_2, n_0\} \) in which \( n_0 = 0 \) are positive and recurrent.

Therefore, to characterize the stationary distribution \( r^*(n_2, n_0) \), it is sufficient to characterize the stationary distribution for the set of states \( \{n_2, n_0\} \) in which \( n_0 = 0 \), because the stationary probability of a transient state is 0. The task of characterizing the stationary distribution \( r^*(n_2, n_0) \) therefore reduces to characterizing the stationary distribution \( r^*(n_2, n_0 = 0) = \lim_{t \to \infty} r(n_2, n_0 = 0; t) \).
2.7.3 Derivation of the Stochastic Differential Equation Governing \( r(n_2; t) \)

Consider a small time interval \([t, t + \Delta t]\), in which \( t \) is greater than the time when all customers enter the market. Because there are no potential customers who have yet to enter, the only event that can take place in the time interval \([t, t + \Delta t]\) is that, at most, one customer will consider switching brands. Therefore, \( r(n_2; t + \Delta t) \) is given by

\[
r(n_2; t + \Delta t) = r(n_2; t) \text{ Prob}(n_2 \rightarrow n_2) + r(n_2 - 1; t) \text{ Prob}(n_2 - 1 \rightarrow n_2) + r(n_2 + 1; t) \text{ Prob}(n_2 + 1 \rightarrow n_2)\]

that is,

\[
r(n_2; t + \Delta t) = r(n_2; t) [1 - \text{ Prob}(n_2 \rightarrow n_2 - 1) - \text{ Prob}(n_2 \rightarrow n_2 + 1)] + r(n_2 - 1; t) \text{ Prob}(n_2 - 1 \rightarrow n_2) + r(n_2 + 1; t) \text{ Prob}(n_2 + 1 \rightarrow n_2)
\]

Consider \( \text{ Prob}(n_2 \rightarrow n_2 - 1) \), or the probability of the system moving from state \( n_2 \) to state \( n_2 - 1 \). This movement can happen only if a customer switches from standard 2 to standard 1 during the time interval \( \Delta t \). The probability of this event is given by \( p_{21}(n_2) \lambda \Delta t \). (Note: Because \( p_{12}() \) and \( p_{21}() \) are functions of \( U_1 \) and \( U_2 \), they are functions of the state variable \( n_2 \).) Therefore, the probability of the system moving from state \( n_2 \) to state \( n_2 - 1 \) is given by \( n_2 p_{21}(n_2) \lambda \Delta t \), because there are \( n_2 \) customers who use standard 2 at time \( t \). Therefore,

\[
\text{ Prob}(n_2 \rightarrow n_2 - 1) = n_2 p_{21}(n_2) \lambda \Delta t.
\]

Similarly, we can show that

\[
\text{ Prob}(n_2 \rightarrow n_2 + 1) = (N - n_2) p_{12}(n_2) \lambda \Delta t,
\]

\[
\text{ Prob}(n_2 - 1 \rightarrow n_2) = (N - n_2 + 1) p_{12}(n_2 - 1) \lambda \Delta t , \text{ and}
\]

\[
\text{ Prob}(n_2 + 1 \rightarrow n_2) = (n_2 + 1) p_{21}(n_2 + 1) \lambda \Delta t.
\]

Substituting for C2.9, C2.10, C2.11, and C2.12 in C2.8, we obtain

\[
\Rightarrow r(n_2; t + \Delta t) = r(n_2; t) [1 - n_2 p_{21}(n_2) \lambda \Delta t - (N - n_2) p_{12}(n_2) \lambda \Delta t] + r(n_2 - 1; t) (N - n_2 + 1) p_{12}(n_2 - 1) \lambda \Delta t + r(n_2 + 1; t) (n_2 + 1) p_{21}(n_2 + 1) \lambda \Delta t.
\]

Rearranging terms, we get
\[
\frac{r(n_2; t + \Delta t) - r(n_2; t)}{\Delta t} = [k(n_2 + 1)p_{21}(n_2 + 1)\lambda r(n_2 + 1; t) - n_2 p_{21}(n_2)\lambda r(n_2; t)] + \\
[(N-n_2+1)p_{12}(n_2-1)\lambda r(n_2 - 1; t) - (N-n_2)p_{12}(n_2)\lambda r(n_2; t)]
\]  

\text{We define } w_{\uparrow}(n_2) \text{ and } w_{\downarrow}(n_2) \text{ as follows:}

\begin{align*}
  w_{\uparrow}(n_2) &= (N-n_2) p_{12}(n_2)\lambda \\
  w_{\downarrow}(n_2) &= n_2 p_{21}(n_2)\lambda
\end{align*}

Clearly, \( w_{\uparrow}(n_2) \) is the probability per unit of time for the entire system to move from \( n_2 \rightarrow n_2 + 1 \), and \( w_{\downarrow}(n_2) \) is the probability per unit of time for the entire system to move from \( n_2 \rightarrow n_2 - 1 \). Taking the limits as \( \Delta t \to 0 \) on both sides of C2.14 and using the definitions in C2.15, we obtain the required equation.
2.7.4 Transforming the Stochastic Differential Equation Governing \(r(n_2;t)\) to Continuous Space

Consider

\[
\frac{\partial r(n_2;t)}{\partial t} = [w_i(n_2 + 1) r(n_2 + 1; t) - w_i(n_2) r(n_2; t)] + [w_i(n_2 - 1) r(n_2 - 1; t) - w_i(n_2) r(n_2; t)].
\] (C2.16)

We introduce the following transformations: \(x_2 = \frac{n_2}{N}\), \(\Delta x_2 = \frac{1}{N}\) so \(0 \leq x_2 \leq 1\), and the probability density function on \(x_2\) as \(R(x_2;t) = N r(n_2;t)\). Note that \(R(x_2;t)\) is a proper density function, as follows:

\[
\int_0^1 R(x_2;t) \, dx_2 = \sum_{n_2=0}^{n_2=N} N r(n_2;t) \frac{1}{N} = 1.
\]

We also define the continuous versions of \(w_\uparrow(n_2)\) and \(w_\downarrow(n_2)\) as follows:

\[
\begin{align*}
W_\uparrow(x_2) &= (1 - x_2) p_{12}(x_2) \lambda \tag{C2.17}
\end{align*}
\]

\[
\begin{align*}
W_\downarrow(x_2) &= x_2 p_{21}(x_2) \lambda
\end{align*}
\]

The lhs of C2.16 can be written as

\[
\frac{\partial r(n_2;t)}{\partial t} = \frac{1}{N} \frac{\partial R(x_2;t)}{\partial t},
\] (C2.18)

and the rhs of C2.16 can be approximated as

\[
\begin{align*}
&\left[W_\uparrow(x_2 + \Delta x_2) R(x_2 + \Delta x_2; t) - W_\uparrow(x_2) R(x_2; t)\right] + \left[W_\downarrow(x_2 - \Delta x_2) R(x_2 - \Delta x_2; t) - W_\downarrow(x_2) R(x_2; t)\right].
\end{align*}
\] (C2.19)

Using Taylor series expansion, we get:

\[
\begin{align*}
W_\uparrow(x_2 + \Delta x_2) R(x_2 + \Delta x_2; t) &= W_\uparrow(x_2) R(x_2; t) + \Delta x_2 \frac{\partial}{\partial x_2} [W_\uparrow(x_2) R(x_2; t)] + \frac{(\Delta x_2)^2}{2} \frac{\partial^2}{\partial x_2^2} [W_\uparrow(x_2) R(x_2; t)] + o((\Delta x_2)^3) \\
W_\downarrow(x_2 - \Delta x_2) R(x_2 - \Delta x_2; t) &= W_\downarrow(x_2) R(x_2; t) - \Delta x_2 \frac{\partial}{\partial x_2} [W_\downarrow(x_2) R(x_2; t)] + \frac{(\Delta x_2)^2}{2} \frac{\partial^2}{\partial x_2^2} [W_\downarrow(x_2) R(x_2; t)] + o((\Delta x_2)^3)
\end{align*}
\]

, and

\[
\begin{align*}
W_\uparrow(x_2 - \Delta x_2) R(x_2 - \Delta x_2; t) &= W_\uparrow(x_2) R(x_2; t) - \Delta x_2 \frac{\partial}{\partial x_2} [W_\uparrow(x_2) R(x_2; t)] + \frac{(\Delta x_2)^2}{2} \frac{\partial^2}{\partial x_2^2} [W_\uparrow(x_2) R(x_2; t)] + o((\Delta x_2)^3) \\
W_\downarrow(x_2 + \Delta x_2) R(x_2 + \Delta x_2; t) &= W_\downarrow(x_2) R(x_2; t) + \Delta x_2 \frac{\partial}{\partial x_2} [W_\downarrow(x_2) R(x_2; t)] + \frac{(\Delta x_2)^2}{2} \frac{\partial^2}{\partial x_2^2} [W_\downarrow(x_2) R(x_2; t)] + o((\Delta x_2)^3)
\end{align*}
\]

. Substituting these expansions into C2.19 and simplifying, we get:

\[
\begin{align*}
\Delta x_2 \frac{\partial}{\partial x_2} [W_\uparrow(x_2) R(x_2; t)] + \frac{(\Delta x_2)^2}{2} \frac{\partial^2}{\partial x_2^2} [W_\uparrow(x_2) R(x_2; t)] + o((\Delta x_2)^3) \\
- \Delta x_2 \frac{\partial}{\partial x_2} [W_\downarrow(x_2) R(x_2; t)] + \frac{(\Delta x_2)^2}{2} \frac{\partial^2}{\partial x_2^2} [W_\downarrow(x_2) R(x_2; t)] + o((\Delta x_2)^3)
\end{align*}
\] (C2.20)

If we combine C2.20 and C2.18, we obtain:
We simplify by noting that \( \Delta x = \frac{1}{N} \), and by neglecting terms of the order of \((\Delta x)^2\), we get the required equation:

\[
\frac{1}{N} \frac{\partial R(x_2 ; t)}{\partial t} = \Delta x \frac{\partial}{\partial x} \left( W'_i(x_2) R(x_2 ; t) \right) + \frac{(\Delta x)^2}{2} \frac{\partial^2}{\partial x^2} \left( W'_i(x_2) R(x_2 ; t) \right) + o((\Delta x)^3)
\]

\[
-\Delta x \frac{\partial}{\partial x} \left( W'_i(x_2) R(x_2 ; t) \right) + \frac{(\Delta x)^2}{2} \frac{\partial^2}{\partial x^2} \left( W'_i(x_2) R(x_2 ; t) \right) + o((\Delta x)^3)
\]

\[
\frac{\partial R(x_2 ; t)}{\partial t} = \frac{\partial}{\partial x} \left[ (W'_i(x_2) - W'_i(x_2)) R(x_2 ; t) \right] + \frac{1}{2N} \frac{\partial^2}{\partial x^2} \left[ (W'_i(x_2) + W'_i(x_2)) R(x_2 ; t) \right].
\]
2.7.5 Expression for Stationary Distribution and Necessary and Sufficient Conditions for a Point to be a Maximum of $R'(x_2)$

Equation C2.5 can be rewritten as

$$\frac{\partial R(x_2; t)}{\partial t} = -\frac{\partial I(x_2; t)}{\partial x_2},$$

(C2.21)

where

$$I(x_2; t) = K(x_2) R(x_2; t) - \frac{1}{2N} \frac{\partial [Q(x_2) R(x_2; t)]}{\partial x_2}.$$  

The stationary distribution for $R(x_2, t)$ will satisfy the condition $\frac{\partial R'(x_2)}{\partial t} = 0$, which means that

$$I_{st}(x_2) = K(x_2) R'(x_2) - \frac{1}{2N} \frac{\partial [Q(x_2) R'(x_2)]}{\partial x_2} = 0.$$  

The above equation can be solved to get the stationary distribution:

$$R'(x_2) = s \frac{\exp[2N g(x_2)]}{Q(x_2)},$$

(C2.22)

where

$s$ is a normalization constant, and

$$g(x_2) = \int \frac{K(x_2^\prime)}{Q(x_2^\prime)} dx_2^\prime = \int \frac{W_1(x_2^\prime) - W_\downarrow(x_2^\prime)}{W_1(x_2^\prime) + W_\downarrow(x_2^\prime)} dx_2^\prime.$$  

Applying the conditions for a point to be an extreme,

$$\left. \frac{\partial R'(x_2)}{\partial x_2} \right|_{x_2 = x_m} = s \frac{\exp[2N g(x_m)] 2N g'(x_m) Q(x_m) - \exp[2N g(x_m)] Q'(x_m)}{[Q(x_m)]^2} = 0$$

$$\Rightarrow 2N g'(x_m) Q(x_m) - Q(x_m) = 0$$

$$\Rightarrow 2N \frac{K(x_m)}{Q(x_m)} Q(x_m) - Q'(x_m) = 0$$

$$\Rightarrow K(x_m) - \frac{Q'(x_m)}{4N} = 0$$

$$\Rightarrow K(x_m) = 0 \quad (\frac{Q'(x_m)}{2N} \sim 0 \therefore N \gg 1)$$
Taking the second derivative of $R^*(x_2)$ and using algebraic manipulations, we can show that the sufficient condition for $x_m$ to be a maximum/minimum, as determined by the preceding equation, is given by (as long as $N \gg 1$):

\[
\left. \frac{\partial K(x_2)}{\partial x_2} \right|_{x_2 = x_m} < 0 \text{ for a maximum, and } \left. \frac{\partial K(x_2)}{\partial x_2} \right|_{x_2 = x_m} > 0 \text{ for a minimum.}
\]
2.7.6 Proof of Proposition 1

To establish Proposition 1, we must find the roots of the equation \( K(x_2) = 0 \).

\[
K(x_2) = 0
\]

\[
\Rightarrow \quad K(x_2) = W_1(x_2) - W_1(x_2) = (1 - x_2)p_{12}(x_2)\lambda - x_2p_{21}(x_2)\lambda = 0
\]

\[
\Rightarrow \quad x_2 = \frac{p_{12}(x_2)}{p_{12}(x_2) + p_{21}(x_2)}.
\]

We denote \( F(x_2) = \frac{p_{12}(x_2)}{p_{12}(x_2) + p_{21}(x_2)} \). Therefore, the problem of solving for the roots of the equation \( K(x_2) = 0 \) is equivalent to finding the fixed points of the function \( F(x_2) \).

To find the fixed points of this function, we need to establish several lemmas.

**Lemma 2:** \( F(x_2) = \frac{p_{12}(x_2)}{p_{12}(x_2) + p_{21}(x_2)} \) is a continuous and increasing function in \( x_2 \), and \( F(x_2) \in (0,1) \).

**Proof:** Continuity of \( F(x_2) \) is trivial because the functional forms \( p_{ij}(.) \) are continuous. The derivative of the function \( F(x_2) \) with respect to \( x_2 \) is

\[
\frac{2e^\gamma k(e^\gamma + \cosh[2k - 4kx_2 + \delta_1 - \delta_2])}{(1 + e^\gamma \cosh[2k - 4kx_2 + \delta_1 - \delta_2] )^2},
\]

which is greater than 0. Therefore, the function \( F(x_2) \) increases in \( x_2 \). Because \( \delta_1, \delta_2, \gamma \) are finite, it follows that \( \lim_{x_2 \to 0} p_{12}(x_2) > 0 \), and \( \lim_{x_2 \to 0} p_{12}(x_2) < 1 \). Therefore, it follows that \( F(x_2) \in (0,1) \).

**Lemma 3:** The function \( F(x_2) \) has an odd number of fixed points. With points labeled as 1, 2, 3, and so on, odd numbered points are the maxima of the function \( K(x_2) \), and even points are the minima of \( K(x_2) \).

**Proof:** By definition, \( x_2 \in [0,1] \), and we have shown that \( F(x_2) \in (0,1) \) is a continuous, increasing function. Therefore, according to a simple geometrical argument, \( F(x_2) \) intersects the line \( y = x \) alternately from above and below, which results in an odd number of fixed points.
To prove that odd numbered points are maxima and even numbered points are minima, we must show that \( \frac{dK(x_2)}{dx} < 0 \) at the odd numbered points and \( \frac{dK(x_2)}{dx} > 0 \) at even numbered points. Consider that

\[
K(x_2) = [(1-x_2)p_{12}(x_2) - x_2p_{21}(x_2)]\lambda
\]

\[
= [p_{12}(x_2) - x_2(p_{12}(x_2) + p_{21}(x_2))]\lambda(p_{12}(x_2) + p_{21}(x_2))
\]

\[
= [F(x_2) - x_2]\lambda(p_{12}(x_2) + p_{21}(x_2)).
\]

Then,

\[
\frac{dK(x_2)}{dx_2} = \left[ \frac{dF(x_2)}{dx_2} - 1 \right] \lambda(p_{12}(x_2) + p_{21}(x_2)) + \left[ F(x_2) - x_2 \right] \lambda \left( \frac{dp_{12}(x_2)}{dx_2} + \frac{dp_{21}(x_2)}{dx_2} \right).
\]

Because \( F(x_2) \big|_{x_2=x_m} = x_m \), we get

\[
\frac{dK(x_2)}{dx_2} \bigg|_{x_2=x_m} = \left[ \frac{dF(x_2)}{dx_2} \bigg|_{x_2=x_m} - 1 \right] \lambda(p_{12}(x_m) + p_{21}(x_m)).
\]

At odd numbered points, the function \( F(x_2) \) intersects the line \( y=x \) from above, and therefore, \( \frac{dF(x_2)}{dx_2} \bigg|_{x_2=x_m} - 1 < 0 \), and \( p_0(.) > 0 \) by definition. At odd numbered points, \( \frac{dK(x_2)}{dx_2} < 0 \). Similarly, at even numbered points, the function \( F(x_2) \) intersects the line \( y=x \) from below, and therefore, \( \frac{dF(x_2)}{dx_2} \bigg|_{x_2=x_m} - 1 > 0 \). At odd numbered points,

\[
\frac{dK(x_2)}{dx_2} > 0.
\]

**Proof of Proposition 1**

From Lemmas 2 and 3, we know that the stationary distribution has two maxima if and only if the function \( F(x_2) \) has three fixed points. Given that it is a continuous and increasing function in \( x_2 \) and that its range and domain are the same (i.e., \([0,1] \)), a simple geometrical argument indicates that the function \( F(x_2) \) will have three fixed points if
and only if the slope of the function \( F(x_2) \) attains the value of 1 at two different values of \( x_2 \). We can show that the equation \( \frac{dF(x_2)}{dx_2} = 1 \) has four roots:

\[
x_2 = \begin{cases} 
\frac{2KN - \text{ArcCosh}\left[-e^{-\gamma} + e^{-\gamma}k - e^{-\gamma}\sqrt{k}\sqrt{-2 + 2e^{2\gamma} + k}\right] + \delta_1 - \delta_2}{2k} \\
\frac{2KN + \text{ArcCosh}\left[-e^{-\gamma} + e^{-\gamma}k - e^{-\gamma}\sqrt{k}\sqrt{-2 + 2e^{2\gamma} + k}\right] + \delta_1 - \delta_2}{2k} \\
\frac{2KN - \text{ArcCosh}\left[-e^{-\gamma} + e^{-\gamma}k + e^{-\gamma}\sqrt{k}\sqrt{-2 + 2e^{2\gamma} + k}\right] + \delta_1 - \delta_2}{2k} \\
\frac{2KN + \text{ArcCosh}\left[-e^{-\gamma} + e^{-\gamma}k + e^{-\gamma}\sqrt{k}\sqrt{-2 + 2e^{2\gamma} + k}\right] + \delta_1 - \delta_2}{2k}
\end{cases}
\]

However, the first two roots are not real. The argument of \( \text{ArcCosh}[.] \) in the first two roots is \( -e^{-\gamma} + e^{-\gamma}k - e^{-\gamma}\sqrt{k}\sqrt{-2 + 2e^{2\gamma} + k} \). Now, \( -2 + 2e^{2\gamma} + k = 2(-1 + e^{2\gamma}) + k \), but \( 2(-1 + e^{2\gamma}) + k > 2(-1 + 1) + k = k \) \( (\because \gamma \geq 0) \)

\[
\Rightarrow -e^{-\gamma} + e^{-\gamma}k - e^{-\gamma}\sqrt{k}\sqrt{-2 + 2e^{2\gamma} + k} < -e^{-\gamma} + e^{-\gamma}k - e^{-\gamma}\sqrt{k}\sqrt{k}
\]

\[
\Rightarrow -e^{-\gamma} + e^{-\gamma}k - e^{-\gamma}\sqrt{k}\sqrt{-2 + 2e^{2\gamma} + k} < -e^{-\gamma} < 0.
\]

However, the minimum value of \( \text{Cosh}[.] \) is 1, and thus, the first two roots are complex.

The argument for the next two roots is \( -e^{-\gamma} + e^{-\gamma}k + e^{-\gamma}\sqrt{k}\sqrt{-2 + 2e^{2\gamma} + k} \).

Substituting \( k = \frac{1 + \exp(-\gamma)}{2} \) into this expression and simplifying, we get 1, and \( \text{ArcCosh}[1] = 0 \). Therefore, when \( k = \frac{1 + \exp(-\gamma)}{2} \), the third and fourth roots are identical and equal \( \frac{1}{2} + \frac{\delta_1 - \delta_2}{2k} \). Now the argument for the third and fourth roots is an increasing function of \( k \), and as \( k \) goes beyond the critical value \( \frac{1 + \exp(-\gamma)}{2} \), the value of the argument goes higher than 1, which implies that the third and fourth roots are nonidentical, real roots, which means we may have two maxima for the stationary
distribution. However, we cannot have more than two, because we have only two roots, which proves part (a) of Proposition 1.

To prove part (b), note that when \( \delta_1 = \delta_2 \), the third and fourth roots are symmetrical around the point \( x_2 = 0.5 \). It is easy to show that \( F(0.5 - x_2) = 1 - F(-0.5 + x_2) \). Therefore, the presence of two nonidentical roots, where the slope of the function is 1, indicates that the three fixed points are unique, and the symmetry of the roots and the fact that \( F(0.5 - x_2) = 1 - F(-0.5 + x_2) \) indicates that the second fixed point occurs at \( x_2 = 0.5 \). Therefore, the first fixed point represents where standard 1 dominates because the value of \( x_2 \) at that point is less than 0.5, and the third fixed point is where standard 2 dominates because the value of \( x_2 \) at that point is greater than 0.5. Note that these odd numbered points represent points of maxima for the stationary distribution.
2.7.7 Proof of Proposition 2

Suppose that instead of the ensemble approach, we model the market share stochastic process as a deterministic process but add an error term that accounts for the stochastic nature of the process. Specifically, let

\[
\frac{dx}{dt} = k_L(x_2) + q_L(x_2)\xi(t),
\]

where \(\xi(t)\) is assumed to be Gaussian-correlated random errors, such that \(E(\xi(t)) = 0\) and \(E(\xi(t + \delta), \xi(t)) = \delta(t)\).

Note that the subscript \(L\) is used to reflect that this formulation is consistent with the Langevin approach to modeling a stochastic process. We can show that the formulation is equivalent to the ensemble approach if we set

\[
k_L(x_2) = K(x_2) \quad \text{and} \quad Q_L(x_2) = \left(\frac{Q(x_2)}{N}\right)^{1/2} \quad \text{(for details, see Weidlich and Haag 1973)}.
\]

Because \(N \gg 1\), we can approximate C2.23 as follows:

\[
\frac{dx_2}{dt} = K(x_2).
\]

In turn, the stationary point of the stochastic process is equivalent to the point at which

\[
\frac{dx_2}{dt} = 0,
\]

that is, when \(K(x_2) = 0\), which is the necessary condition that we derived for a point to be the maximum for the stationary distribution. Note that if \(K(x_2) < 0\), then \(\frac{dx_2}{dt}\) is less than 0, which indicates the variable \(x_2\) decreases with time. In contrast, if \(K(x_2) > 0\), then \(\frac{dx_2}{dt}\) is greater than 0, which indicates the variable \(x_2\) increases with time. We previously showed that \(K(x_2) = [F(x_2) - x_2]l(p_{12}(x_2) + p_{21}(x_2))\). Therefore, \(F(x_2) < x_2\) implies that \(K(x_2) < 0\), and \(F(x_2) > x_2\) implies that \(K(x_2) > 0\).

Consider the schematic sketch of the function \(F(x_2)\) shown next.
To the left of the first fixed point denoted by 1, $F(x_2) > x_2$ implies that $K(x_2) > 0$, which indicates that $x_2$ increases with time. To the right of the fixed point 1, $F(x_2) < x_2$ implies that $K(x_2) < 0$, which indicates that $x_2$ increases with time. We can draw similar conclusions about the behavior of $x_2$ to the right and left of fixed point 3. Therefore, fixed point 2 is the critical point, to the left of which the process converges to the dominance of standard 1 and to the right of which the process converges to the dominance of standard 2. In this case, fixed point 2 happens to be 0, and hence, $x_{2c} = 0$. 
2.7.8 Proof of Proposition 3

It is easy to show that the function \( \frac{dF(x_2)}{d\delta_1} < 0 \). Therefore, the function \( F(x_2) \) starts moving downward as \( \delta_1 \) increases. It is also easy to show that \( \lim_{\delta_1 \to \infty} F(x_2) = 0. \) Therefore, the only fixed point of the function \( F(x_2) \) as \( \delta_1 \to \infty \) occurs at \( x_2 = 0. \) Because if \( \delta_1 = \delta_2 \), the function \( F(x_2) \) has three fixed points, \( \frac{dF(x_2)}{d\delta_1} < 0 \), and there exists only fixed point of the function \( F(x_2) \) as \( \delta_1 \to \infty \), there must be a critical value of \( \delta_1 \) beyond which the function \( F(x_2) \) has only one fixed point. We show by construction that such a point exists.

There exists a value of \( \delta_1 \) such that the function \( F(x_2) \) qualitatively looks like that shown in the graph. The circled point is not only a fixed point of the function \( F(x_2) \) but also a point at which the slope of the function \( F(x_2) \) is 1. Beyond this point, the slope of the function \( F(x_2) \) is always less than 1; therefore, this point must be the greatest root of the equation \( \frac{dF(x_2)}{dx_2} = 1 \). That is, it must be

\[
x_2 = \frac{2k + \text{ArcCosh} \left( e^{-\gamma} + e^{-\gamma} k + e^{-\gamma} k \sqrt{-2 + 2 e^{2 \gamma} + k} \right)}{4k} + \delta_1 - \delta_2.
\]

Substituting this value into the equation \( F(x_2) = x_2 \) and solving for \( \delta_1 \), we get the critical value of \( \delta_1 \):
\[
\frac{1}{2} + \exp(\gamma + \text{ArcCosh}\left[\frac{\exp(-\gamma)\left(-1 + kN + \sqrt{kN} \sqrt{-2 + 2e^{\gamma} + kN}\right)}{2kN + 2\sqrt{kN} \sqrt{-2 + 2e^{\gamma} + kN}} + \frac{\delta_2}{4kN}\right]) \]

Note that as \( \delta_i \) increases, the function \( F(x_2) \) decreases further, which implies that the curve comes down and leaves only the left-most fixed point, the fixed point at which firm 1 dominates. In contrast, if \( \delta_i \) decreases, the function \( F(x_2) \) increases further, which implies that the curve comes up and leaves three fixed points and thus two maxima.
2.7.9 Proof of Proposition 4

The proof of Proposition 4(a) is similar to that of Proposition 2. The only thing we must prove is that the critical value of $x_2$ is greater than 0.5. Consider the case when $\delta_1 = \delta_2$, for which the critical point is 0.5. As $\delta_1$ increases, the function $F(x_2)$ decreases, and a simple geometrical argument suggests that the second fixed point, which is the critical value, increases above 0.5. As $\delta_2$ increases, the function $F(x_2)$ increases, and thus, the second fixed point moves to the right. Therefore, $x_{zc}$ decreases as $\delta_2$ increases.

To prove Proposition 4(b), we consider the evolution of the market share stochastic process from time $t_2$ onwards, or the time after standard 2 has entered the market. We define an event in the market share process as either (a) a new customer entering the market or (b) an existing customer deciding whether to switch. Without loss of generality, let $E$ be the number of events that occur between $t_2$ and the time when all customers enter the market. Let $e_j$, where $j = 1, 2, \ldots, E$, be the times when the events occur. Consider the time $e_j$ and let the value of $n_2$ at this time be denoted by $n_2(e_{j-1})$. Therefore,

$$
\text{Prob}(n_2(e_j) = l) = \text{Prob}(n_2(e_j) = l \mid n_2(e_{j-1}) = l - 1) \text{Prob}(n_2(e_{j-1}) = l - 1) + \\
\text{Prob}(n_2(e_j) = l \mid n_2(e_{j-1}) = l + 1) \text{Prob}(n_2(e_{j-1}) = l + 1)
$$

Then,

$$
\text{Prob}(n_2(e_j) = l \mid n_2(e_{j-1}) = l - 1) = \text{Prob}(\text{a customer switches from } 1 \text{ to } 2) e^{-\lambda(e_j-e_{j-1})} + \\
\text{Prob}(\text{a new customer arrives and chooses standard } 2) e^{-\beta(e_j-e_{j-1})}
$$

and

$$
\text{Prob}(n_2(e_j) = l \mid n_2(e_{j-1}) = l + 1) = \text{Prob}(\text{a customer switches from } 2 \text{ to } 1) e^{-\lambda(e_j-e_{j-1})} + \\
\text{Prob}(\text{a new consumer arrives and chooses standard } 1) e^{-\beta(e_j-e_{j-1})}
$$

Now the following hold:

(a) $\text{Prob}(\text{a customer switches from } 1 \text{ to } 2)$ decreases in $n_1$.

(b) $\text{Prob}(\text{a new customer arrives and chooses standard } 2)$ decreases in $n_1$.

(c) $\text{Prob}(\text{a customer switches from } 2 \text{ to } 1)$ increases in $n_1$. 
(d) Prob(a new customer arrives and chooses standard 1) increases in $n_1$.

These claims are true at any time $e_j$, and as standard 2 delays entry, the value of $n_1$ increases. Therefore, the density function of $n_2$ at any point in time is skewed toward lower values of $n_2$ as standard 2 delays entry, which suggests that the probability of firm 2 attaining any particular market share decreases as it delays entry.
2.7.10 Proof of Proposition 7

When the strength of network externality decreases below the critical value of \( \frac{1 + \exp(-\gamma)}{2} \), the value of the argument of \( \text{Arccosh} \) in the third and fourth roots of the equation \( \frac{dF(x_2)}{dx_2} = 1 \) goes below 1, which implies that the third and fourth roots are complex. Therefore, the equation \( \frac{dF(x_2)}{dx_2} = 1 \) has no real roots and the function \( F(x_2) \) can only have one fixed point, which implies that there exists only one maximum when \( k \leq \frac{1 + \exp(-\gamma)}{2} \).

Now consider the function \( F(x_2) = \frac{p_{12}(x_2)}{p_{12}(x_2) + p_{21}(x_2)} \). If \( \delta_1 = \delta_2 \), the function is such that \( F(0.5 - x_2) = 1 - F(-0.5 + x_2) \), and the fixed point must be 0.5. Therefore, both firms get 50% market share. The function \( F(x_2) \) decreases in \( \delta_1 \) and increases in \( \delta_2 \). Therefore, as \( \delta_1 \) increases, the function \( F(x_2) \) comes down, and hence, the fixed point also goes down, which implies that \( x_2 < 0 \), as well as a higher market share for firm 1. In turn, the more \( \delta_1 \) increases, the more the function \( F(x_2) \) comes down, and therefore, firm 1 attains more market share. The converse argument holds for firm 2.
2.8 References


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3. Incentive-Compatible Compensation Schemes for Reverse Auction Market Makers

3.1 Introduction

The use of online reverse auctions in material procurement continues to grow despite concerns about the negative impact of these auctions on buyer–supplier relationships. For example, in a survey of 294 manufacturers, 27.2% reported using an online auction in the fourth quarter of 2002, as opposed to 18.8% in the third quarter of the same year (Chin 2003). In a more recent survey of distributors, 32% of survey respondents participated in reverse auctions in 2004, up significantly from the 18% in 2003 (Griffiths 2004). In a reverse auction, the buyer or a third-party firm (hereafter, “market maker”) acts as the auctioneer, and potential suppliers are bidders. In contrast to a standard auction, in a reverse auction, the price declines as suppliers drop their prices to win the business of the buyer. The attractiveness of online reverse auctions for buyers lies in the large savings they achieve; estimated savings range from 10% to 20% below historical prices (Beall et al. 2003).

Despite the increasing popularity and tremendous savings potential of reverse auctions, there have been instances in which buyers have realized unfavorable auction outcomes. For example, consider the following:

…the company awarded a contract to an aerospace parts supplier based on its bid in a reverse auction. After a year of trying to get parts from the supplier, Pratt & Whitney had to terminate the contract because the supplier could not produce the product lot it had quoted…. Then Pratt & Whitney had to go back to its original supplier and pay a hefty premium to get the parts (Anonymous 2002).

Often the retailers must source in China to get the prices they want, and when the fixtures arrive they are not the quality the retailer wanted (Scheraga 2002).
We have seen many instances where the business was sourced again within a year, to a new supplier after a reverse auction, due to the incompetence of the winning bidder, the substitution of inferior materials, or the simple inability to deliver (Larson 2004).

These quotes suggest that the outcomes of reverse auctions are not always beneficial. Although these incidents involve undesirable auction outcomes associated with the supply of a poor quality product, the buyer also might forego cost savings because it failed to include a low-cost supplier in the auction. The generally recommended solution for purchasing professionals to avoid undesirable outcomes is to prequalify suppliers.

The most effective method to ensure that appropriate suppliers are invited to review and participate in reverse auctions is to pre-qualify the suppliers prior to their participation in the auction event (E-Auction PlayBook 2002).

However, qualifying suppliers on a global basis is problematic, because qualifying overseas suppliers is both costly and time consuming. Market makers add value in these situations by collecting information about suppliers and their capabilities on a global basis, as the following quotes illustrate:

They [the buyer] may not have a team of 20 people to go wandering around finding machine component suppliers in Taiwan, but working with an e-sourcing provider, they can touch far more potential suppliers than they would otherwise (Armes et al. 2002).

Sourcing consultants and specialized and online market makers such as FreeMarkets can do substantial legwork as well. These entities have global resources for identifying supplier candidates, conducting site visits, and validating their existence and competence (E-Auction PlayBook 2002).

When a buyer employs a market maker to qualify suppliers, the qualifying effort exerted is unobservable by the buyer. Therefore, the buyer must appropriately incentivize the market maker to ensure appropriate supplier qualification. A compensation scheme that does not offer correct incentives to the market maker likely will lead to the same
undesirable auction outcomes for the buyer that the market maker was supposed to solve (e.g., inadequate cost savings). Therefore, from a managerial point of view, designing incentive-compatible compensation schemes—namely, compensation schemes that align the interests of the market maker with the interests of the buyer—is extremely important.

Despite the practical importance of this topic, we could not locate any theoretical research on the issue of designing incentive-compatible compensation schemes for reverse auction market makers. Most existing auction theory assumes that the seller (a buyer in the context of a reverse auction) is the auctioneer and focuses on issues such as determining equilibrium bidding behavior or establishing the revenue properties of different auction mechanisms (e.g., Klemperer 1999; Krishna 2002). The only article we found that explicitly models the auctioneer is a paper by Greenleaf and Sinha (1996), who address how a combination of buy-in penalties and commissions charged by an auctioneer can increase the expected auction revenue by motivating sellers to lower their reserve prices. Although their research discusses incentive issues related to attracting a sufficient number of bidders to participate in the auction, it does not address inviting the “right” set of bidders, a primary focus of this chapter. Herein, we consider a class of compensation schemes used in practice—namely, compensation schemes based on winning bids—and analyze how such compensation schemes can be structured such that the buyer invites the “right” set of bidders to participate in the auction. We also analyze how the structure of these compensation schemes varies systematically across different procurement situations. Our main contribution to existing auction literature is the resolution of a substantive problem in reverse auctions: designing incentives such that the market maker qualifies suppliers rigorously.
Our model consists of three sets of players: a buyer planning to hold a reverse auction, a market maker that qualifies suppliers on behalf of the buyer, and new suppliers that participate in the auction. We model the interaction between the buyer and the market maker using the principal–agent framework. The buyer (i.e., the principal) appoints the market maker (i.e., the agent) to qualify suppliers and, in return, offers it a compensation scheme. The buyer must choose its compensation scheme from three classes of schemes: (1) fixed fee; (2) increasing scheme, in which the market maker’s compensation increases as a function of the winning bid (e.g., a percentage of the contract value); and (3) decreasing scheme, in which the market maker’s compensation is a decreasing function of the winning bid (e.g., a percentage of the savings). After the qualification process, the buyer invites a fixed number of new suppliers, as well as the incumbent supplier, to bid in the auction. At the end of the auction, the buyer awards its business to the lowest bidder. The buyer’s objective is to choose the optimal compensation scheme, such that the expected procurement cost is the lowest.

Intuition suggests that the buyer should offer the market maker a decreasing scheme (e.g., percentage of the savings), because the primary purpose of the reverse auction is to lower the buyer’s procurement costs. However, counter intuitively, we find that the buyer sometimes should offer the market maker an increasing scheme (e.g., percentage of the contract value), which offers an incentive to the market maker to reduce rather than increase savings. This counterintuitive finding emerges because when the costs associated with poor quality are very high, the buyer wants to avoid awarding the business to a low quality supplier. When the market maker is offered an increasing scheme, its profits increase if the winning bid is high. Winning bids are likely to be high
if the buyer screens out low quality suppliers, a task that is made easier if the market maker qualifies suppliers rigorously. Therefore, when offered an increasing scheme, the market maker qualifies suppliers rigorously. In contrast, if the buyer offers a decreasing scheme, the market maker has no incentive to expend enough effort in qualifying suppliers, which leads to low quality suppliers being identified as high quality suppliers. If these low quality suppliers are invited, they will win the auction, which increases profits for the market maker but which represents an undesirable outcome for the buyer.

The rest of this chapter is organized as follows: In Section 3.2, we describe the supplier qualification process to enable our subsequent model development. In Section 3.3, we discuss the modeling implications that follow from our description of the qualification process. Then in Section 3.4, we state and justify the assumptions of our theoretical model. Subsequently, we discuss issues related to model analysis. We present our analysis, which shows how the optimal incentive-compatible compensation scheme depends on the nature of the procurement situation in Section 3.5, and discuss the impact of relaxing some of our key assumptions in Section 3.6. Because deriving incentive-compatible compensation schemes analytically is intractable, we illustrate how optimal incentive-compatible compensation contracts can be computed numerically in Section 3.7. Finally, in Section 3.8, we discuss the implications of our research for theory and business practice and conclude.

### 3.2 Supplier Qualification Process

To understand the issues involved in the supplier qualification process, consider how a buyer in the electronics industry qualifies printed circuit board suppliers (for an overview, see *CACLE News* 2003). The first step involves sending all suppliers a
standard survey form developed by IPC, an electronics industry association (www.ipc.org). The purpose of the survey is to obtain information from suppliers in the following areas: basic company description (e.g., manufacturing locations, management details, contact information), supplier capabilities (e.g., testing equipment, processes used to fabricate printed board products), equipment available from the supplier, the technology the supplier uses, and so forth. After all the suppliers respond to the survey, the buyer determines which supplier has the potential to satisfy the buyer’s requirements and designates a certain number of suppliers for further audit. The supplier audit examines various aspects of the supplier, such as the rigor of its quality-control processes (e.g., by examining quality-control documents like control charts), maintenance documents, knowledge and training of workers in the plant, and so forth. At the end of the qualification process, the audit team scores the supplier on various criteria and generates an overall score that represents the best assessment of the supplier’s ability to supply parts to the buyer. In addition, the buyer can supplement this process by requesting references from the supplier.

The steps that a market maker follows when it qualifies suppliers resemble the above process. For example, consider the supplier qualification process of Threecore (www.threecore.com), a reverse auction market maker. Threecore divides its supplier qualification process into three components: (1) capabilities audit, (2) process audit, and (3) reference checks. A capabilities audit ascertains whether the supplier has the financial and operational ability to supply the product lot if the buyer awards it the contract. It consists of collecting background information about the supplier, such as its number of years in business, annual reports, number of employees, industries served, and so forth
A process audit verifies the supplier’s ability to produce and deliver the quality that the buyer desires and consists of an audit of the supplier’s quality-control procedures, inspection and testing procedures, and policies for training employees, among other things (see Figure 3.2). Finally, during reference checks, the market maker collects information from the customers of the supplier about the extent to which they receive quality products, how price competitive the supplier is, and so forth (see Figure 3.3).

**Figure 3.1: Capabilities Audit**

![Capabilities Audit](http://example.com/capabilities.png)

**Figure 3.2: Process Audit**

![Process Audit](http://example.com/process.png)
Figure 3.3: Reference Checks

Figure 3.4 outlines the sequence of actions that take place during the supplier qualification process. First, the buyer specifies the items that the market maker should measure to qualify suppliers. These items are designed to assess various aspects of a supplier, such as product quality, cost competitiveness, and financial stability. The buyer also specifies how a supplier’s ratings for these items should be combined to form the overall rating (e.g., overall rating equals the weighted average of the item ratings). Second, the market maker evaluates all the suppliers according to the items specified by the buyer by collecting the information from its databases, surveys sent to suppliers, and site visits. Third, at the end of the qualification process, the market maker calculates an overall score using the buyer’s prespecified rule (e.g., weighted average), and those suppliers with the highest overall scores are invited to participate in the auction.
3.3 Modeling Issues

A few important points follow from our description of the qualification process. First, the qualification process involves measuring the indicators of the supplier’s quality and cost competitiveness. For example, the quality of a supplier’s product often is measured by inspecting a random sample of parts. Therefore, the success of the supplier qualification effort depends on the extent to which the underlying variables can be measured accurately by the corresponding indicators. High measurement error in these indicators is problematic because a poor quality supplier might be judged as a good quality supplier, which would lead to undesirable outcomes for the buyer. One way to decrease measurement error is to measure the variable of interest in multiple ways. For example, quality can be assessed by inspecting samples, examining the rigor of the
quality-control processes at the plant, and examining the supplier’s quality improvement efforts. However, measuring the same variable using multiple indicators increases the qualification effort. In our model development in Section 3.4, we account for these issues by explicitly modeling the measurement error that arises in the qualification process, as well as the impact of the qualification effort on measurement error.

Second, the buyer is unable to observe the market maker’s qualification effort. Although the buyer can observe the output of the qualifying effort, such as the questionnaires filled out by the suppliers, the site visit reports of the market maker, and so forth, it cannot observe the actual qualification process. The inability of the buyer to observe the market maker’s effort and the uncertain nature of the qualifying process results in a problem known in principal–agent literature as the moral hazard problem. In the moral hazard problem, the interests of the market maker and the buyer may be misaligned. Misaligned interests can be costly to the buyer, because the market maker may not qualify suppliers according to the buyer’s requirements. One solution to the moral hazard problem is to design an incentive-compatible compensation scheme for the market maker that will align the interests of the market maker and the buyer and thereby motivate the market maker to qualify suppliers according to the buyer’s requirements.

Note that even when the market maker’s compensation is incentive incompatible, it still may qualify suppliers rigorously because of its long-term need to build a good reputation in the market. However, an incentive-incompatible compensation scheme actually conflicts with this long-term need. To understand how the conflict arises, consider a consumer who requests that a travel agent find the cheapest deal possible for his or her travel needs. Until recently, most travel agents received a fixed percentage of
the purchase price of the ticket as compensation from airlines. Therefore, it has been in the travel agent’s short-term interest to offer the highest possible fare to the consumer. However, offering a high fare is not in the long-term interest of the agent as the agent may obtain a poor reputation among consumers. The presence of these conflicting short- and long-term incentives may become problematic, because the agent must trade off certain short-term gains against uncertain long-term gains. Similarly, when the market maker’s compensation is incentive incompatible, it will need to trade off the certain short-term gains that can be obtained through improper qualification of suppliers against the uncertain long-term gains of obtaining a good reputation in the market.

We seek to align the market maker’s short- and long-term interests by structuring an incentive-compatible compensation scheme using the principal–agent paradigm (Holmstrom 1979; Laffont and Martimort 2002). The principal–agent paradigm has been used previously to understand other business situations in which the moral hazard problem exists, such as salesperson compensation structures (Basu et al. 1985; Joseph and Thevaranjan 1998; Lal and Srinivasan 1993; Lal and Staelin 1986; Mrinal and John 2000; Raju and Srinivasan 1996), executive compensation (e.g., Murphy 2003), and the use of incentive pay for investment bankers (e.g., Nash 2003).

Before we turn to our model development, note that compensation schemes based on winning bids are both contemporaneous with the auction and informative about the outcome of the qualifying effort exerted by the market maker. Clearly, the winning bid is contemporaneous with the auction, but to understand why the winning bid is informative about the outcome of the qualifying effort, consider a situation in which the buyer wants to identify low quality suppliers to exclude them from the auction. When qualification
effort is low the measurement error in the items used to assess quality will be very high and therefore a low quality supplier may be mistakenly identified as a high quality supplier. In such a situation, the winning bid is likely to be low, because the low quality suppliers will bid low and therefore likely will win the auction. In contrast, if the qualifying effort is high, measurement error will be minimal. Consequently, a low quality supplier will not be mistakenly identified as a high quality supplier and will be excluded from the auction. In such a situation, the winning bid is likely to be high; the low quality suppliers are not present in the auction and cannot submit low bids. This intuition thus suggests that the winning bid can be informative about the quality of the winning supplier.

We now turn to our model setup.

3.4 The Model

In our model development, we make several assumptions to capture the key issues involved in the supplier qualification process. After we complete the analysis of our model, we will discuss in Section 3.6 how relaxing some of the key assumptions will affect our results. Our model uses the principal–agent framework to model the interaction between the buyer and the market maker and familiar models from auction literature for the behavior of suppliers. We first describe the buyer’s objective function and model the qualification process. Subsequently, we develop the principal–agent model and describe the auction model. For a list of the symbols we use and their meanings, see Table 3.1 (p. 98).
3.4.1 Buyer’s Objective Function and Supplier Qualification

Consider a buyer, the principal in our model, that intends to procure a single product lot using a first-price, sealed bid auction format, in which the supplier with the lowest bid wins the auction and receives payment from the buyer equal to its bid. The buyer’s objective is to minimize its expected procurement cost, that is, minimize $E[b_w + I(q_w) + s(b_w)]$, where $b_w$ is the winning bid, $q_w$ is the quality of the winning supplier (measured as a percentage of acceptable parts), $I(q_w)$ are the quality costs associated with procuring the product from the winning supplier, and $s(b_w)$ is the compensation paid to the market maker. Note that with “quality costs,” we refer to all costs the buyer incurs because of poor quality, including those associated with inspection, manufacturing problems, warranty repair, or lost customer goodwill in the event of a product recall. Because quality costs are likely to decline as quality improves, and consistent with previous literature (Tagaras and Lee 1996), we assume that

Assumption 1: $I(q_w) = \theta (1 - q_w)$, where $\theta$ is the quality cost per defective part.

As we mentioned in Section 3.2, the supplier qualification process is designed to assess a supplier on many dimensions, such as cost competitiveness, quality, service support, financial stability, and so forth. For the sake of simplicity, we assume that

Assumption 2: The buyer cares primarily about a supplier’s product quality and cost competitiveness.

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4 Considering multiple product lots adds complexity to the modeling framework without providing additional insight.
5 Approximately about 50% of reverse auctions are sealed bid auctions. (Jap 2003).
6 In any case, we can re-interpret “quality” as an overall measure of how well good a supplier performance is on all non-price dimensions, including such as quality of the product, service support, and financial stability etc.
Let the $i$th new supplier’s cost of fulfilling the contract be $c_i$, where $i = 1, 2, \ldots, N$. Let $r_{q,i} = q_i + \varepsilon$ and $r_{c,i} = -c_i + \varepsilon$ be the ratings that the $i$th supplier obtains on the quality and cost competitiveness dimensions, respectively, where $\varepsilon \sim N(0, \sigma^2)$ is the measurement error in the ratings. Let $w_q$ ($0 \leq w_q \leq 1$) and $1 - w_q$ be the weights that the buyer assigns to the quality and cost dimensions, respectively, and let $r_i = w_q r_{q,i} + (1 - w_q) r_{c,i}$ be the overall rating for a supplier. Then,

Assumption 3: The buyer invites the suppliers with the $n$ highest overall ratings to participate in the auction.

After the qualification of suppliers is complete, the buyer must decide how many suppliers to invite to participate in the auction. As the following quote suggests, buyers face a trade-off between how competitive they want the auction to be and the ease of managing the auction: “More qualified suppliers participating are better than fewer, but keep the total number of suppliers manageable” (Davies 2002).

A higher number of qualified suppliers in the auction is likely to lead to greater savings to the buyer but also can cause problems in managing the auction. As illustrated by the following, it is often recommended that buyers invite a fixed number of suppliers to participate in the auction:

The average auction for production materials can be executed with 4 to 8 qualified suppliers. This is typically enough to ensure a competitive market (www.threecore.com).

As a general guideline, it is recommended to have four to six participating suppliers to increase the likelihood of an active and successful bid event (Grasso et al. 2002).
Assumption 3 reflects these recommendations about how many suppliers a buyer should invite to an auction.

Let \( c_{(1)} > c_{(2)} \ldots > c_{(N-1)} > c_{(N)} \) be the order statistics of the costs of the \( N \) suppliers that potentially can be invited to participate in the auction. As we show in our analysis (see Lemma 1), the winning supplier is the supplier with the lowest cost among the \( n \) suppliers in the auction. Therefore, the cost of the winning supplier will be one of the following order statistics: \( c_{(n)}, c_{(n-1)}, \ldots, c_{(N-1)}, c_{(N)} \). Let \( p_k \) \((k = n, \ldots, N)\) be the probability that the \( k \)th highest cost supplier wins the auction. Then, we assume that

\[ \text{Assumption 4: As } \sigma^2 \to \infty, \ p_k \to \frac{C_{n-1}^{k-1}}{C_1^{N}}, \text{ where } C_{n-1}^{k-1} = \text{the number of ways in which we can select } n-1 \text{ suppliers from } k-1 \text{ suppliers, and } C_1^{N} = \text{the number of ways in which we can select } n \text{ suppliers from } N \text{ suppliers.} \]

As the variance in the measurement error increases, the true quality and cost ratings of a supplier will become dominated by error. Therefore, as the variance in the measurement error increases to high levels, choosing \( n \) suppliers to invite to participate in the auction is equivalent to choosing \( n \) suppliers at random from among the \( N \) potential suppliers. Consequently, the supplier with the \( k \)th highest cost (where \( k = n, \ldots, N \)) wins the auction if and only if the remaining \( n-1 \) suppliers have higher costs. The number of suppliers that have higher costs than supplier \( k \) is \( k-1 \). Therefore, when \( n \) suppliers are invited to participate, supplier \( k \) wins in \( C_{n-1}^{k-1} \) situations. However, there are \( C_n^{N} \) ways of choosing \( n \) suppliers from among \( N \) suppliers. Therefore, when error variance is high, the probability that the \( k \)th highest cost supplier will win the auction is
given by \( \frac{C_0^{k-1}}{C_0^N} \). In light of this reasoning, Assumption 4 simply states that as error variance increases, the probability that that the \( k \)th highest cost supplier will win the auction approaches \( \frac{C_0^{k-1}}{C_0^N} \).

### 3.4.2 The Principal–Agent Model

The buyer appoints the market maker, the agent in our model, to qualify suppliers. The market maker exerts effort, such as collecting information from suppliers, deputing personnel to audit a supplier’s facilities, and maintaining a database of information about various suppliers, to qualify new suppliers. Let the amount of money \( e \) expended by the market maker to qualify suppliers according to the buyer’s requirements be the measure of the qualifying effort. The market maker’s choice of qualifying effort remains its private knowledge. The objective of the market maker is to maximize the compensation it earns, net the cost of the qualifying effort, or to maximize \( s(b_u) - e \). As we discussed in Section 3.3, as the qualification effort increases, the accuracy of the quality and cost competitiveness ratings should increase. Therefore, we assume the following relationship between error variance in ratings (\( \sigma^2 \)) and qualifying effort \( e \):

**Assumption 5:** \( \frac{d\sigma^2}{de} < 0, \lim_{e \to \infty} \sigma^2 \to 0, \lim_{c \to \infty} \sigma^2 \to 0, \) and \( \lim_{c \to 0} \sigma^2 \to \infty. \)

In addition to specifying that measurement error decreases as qualifying effort increases, Assumption 5 also states that as qualifying effort continues to increase, measurement error eventually should fall to negligible levels (i.e., \( \lim_{c \to \infty} \sigma^2 \to 0 \)). Finally, as the qualifying effort decreases to 0, the error variance should rise to very high levels.
In return for the qualification services provided by the market maker, the buyer offers a take-it-or-leave-it compensation scheme, $s(b_w)$. The buyer’s objective is to structure a compensation scheme such that the market maker exerts the level of qualification effort that minimizes the buyer’s procurement cost. In other words, the buyer’s problem is to choose $w_q, s(.)$, such that the market maker exerts the optimal qualifying effort $e^*$ according to the following optimization program:

\[
\min_{w_q, s(.)} E[b_w + I(q_w) + s(b_w)],
\]

subject to

IC: $e^* \in \arg\max E[s(b_w) - e]$ and

PC: $E[s(b_w) - e^*] \geq 0$,

where

$E[.]$ is the expectation operator,

$b_w$ is the winning bid,

$q_w$ is the quality of the winning supplier,

$w_q$ is the weight parameter used to calculate the overall ratings,

$s(b_w)$ is the compensation paid to the market maker, and

$e^*$ is the optimal effort expended by the market maker.

Thus, the buyer’s optimization problem is to find the optimal compensation scheme and optimal level of effort, subject to two constraints. The first constraint, known as the incentive-compatible constraint (Equation C3.2), requires that the market maker find it optimal to exert optimal effort, given the optimal scheme. The second constraint,
known as the participation constraint (Equation C3.3), requires that the market maker’s compensation is greater than some minimum value (which we specify as 0 for simplicity).

3.4.3 The Auction

We make two assumptions regarding the suppliers in the auction.

Assumption 6a: A supplier’s private cost of fulfilling a contract \( c_i \) is distributed i.i.d on \([c, \bar{c}]\) according to an increasing cumulative distribution function \( F(.) \) with corresponding density \( f(.) \).

Assumption 6b: A supplier’s quality is given by \( q_i = q(c_i) \), such that \( \frac{dq(c_i)}{dc_i} > 0 \).

Assumption 6a is quite general and reflects that information about costs is proprietary. Assumption 6b states that a supplier’s quality increases with the cost of fulfilling the contract, an assumption that is likely to hold in a wide range of procurement situations. Previous researchers of procurement auctions have made somewhat similar assumptions (Branco 1997; Che 1993). However, unlike previous research, Assumption 6b does not allow for the presence of suppliers with high costs who can deliver only a low quality product. We discuss the effect of including such suppliers on our results in Section 3.6.

3.5 Analysis

Before we present our analysis, we note that the buyer’s optimization problem, as presented in Equations C3.1–C3.3, is analytically intractable. Therefore, we first qualitatively characterize how the nature of the market maker’s compensation scheme depends on the buyer’s procurement situation. Specifically, we analyze in what
procurement situations the buyer needs to offer (1) a fixed fee (i.e., \( \frac{ds(b_w)}{db_w} = 0 \)), (2) an increasing scheme (i.e., \( \frac{ds(b_w)}{db_w} > 0 \)), or (3) a decreasing scheme (i.e., \( \frac{ds(b_w)}{db_w} < 0 \)).

Subsequently, we illustrate how the buyer’s optimization problem (Equations C3.1–C3.3) can be solved numerically to compute optimal compensation schemes.

We analyze the model in four steps. In the next section, we characterize the equilibrium bids of suppliers in the auction. Building on our analysis of the auction, we analyze the impact of the measurement error on auction outcomes. In Section 3.5.3, we analyze how the market maker’s choice of qualifying effort depends on the compensation scheme offered by the buyer. Finally, we analyze how the compensation scheme offered by the buyer depends on the procurement situation.

### 3.5.1 Equilibrium Bids

First, note that the first-price, sealed bid reverse auction of our model is analogous to a first-price, sealed bid standard auction in which there is a single seller and multiple buyers. Second, because qualification reports are confidential, a supplier does not know the ratings obtained by competing suppliers. Therefore, a supplier invited to participate in an auction cannot update the cost distribution of competing suppliers and hence bids with the assumption that the cost distribution is \( F(\cdot) \). We know from auction theory that in a first-price, sealed bid standard auction, the buyer with the highest valuation wins the auction (Krishna 2002, pp. 16–18). In Appendix 3.9.1, we offer proof that the lowest cost supplier wins the auction and that its bid increases with the cost of fulfilling the contract. The proof follows the same techniques as in the case of a standard auction. We formally state Lemma 1 as follows:
Lemma 1: The supplier that wins the auction is the lowest cost supplier from among the \( n \) suppliers in the auction, and its bid increases with the cost of fulfilling the contract.

Proof: See Appendix 3.9.1.

3.5.2 Measurement Error and Auction Outcomes

Let \( c_w \) be the cost of the winning supplier, and let \( G(.) \) be its distribution. Then we can show that

Lemma 2: Depending on \( w_q \) and \( \sigma^2 \), it must be the case that either \( \frac{dG(.)}{d\sigma^2} < 0 \) or \( \frac{dG(.)}{d\sigma^2} > 0 \).

Proof: See Appendix 3.9.2.

To understand the intuition behind Lemma 2, consider the case of the lowest cost supplier. Depending on the levels of \( w_q \) and \( \sigma^2 \), the probability that the lowest cost supplier wins the auction is either greater or less than the probability that it will win the auction when suppliers are invited at random to participate in the auction. As the measurement error increases, the probability that the lowest cost supplier will win the auction tends toward a fixed value (per Assumption 4). If the initial probability is greater than this fixed value, then as measurement error increases, the suppliers whose costs are higher than that of the lowest cost supplier will tend to have a better chance of winning the auction, and hence, \( \frac{dG(.)}{d\sigma^2} < 0 \). If, in contrast, the initial probability is less than this fixed value, then as measurement error increases, the suppliers that have higher costs than the lowest cost supplier will tend to have a lesser chance of winning the auction, and hence, \( \frac{dG(.)}{d\sigma^2} > 0 \).
3.5.3 Market Maker’s Choice of Qualification Effort

In this section, we analyze how the market maker’s choice of qualifying effort depends on the way the measurement error affects the outcome of the auction and the compensation scheme offered by the buyer. Our result is formally stated as Proposition 1:

**Proposition 1:** The following table summarizes the market maker’s choice of qualifying effort in various situations:

<table>
<thead>
<tr>
<th>If $w_q$ is chosen, such that $dG(.)/d\sigma^2 &lt; 0$</th>
<th>And the compensation scheme offered is $ds(b_w)/db_w = 0$</th>
<th>Then the market maker’s choice of qualifying effort will be $e^* = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decreasing, $ds(b_w)/db_w &lt; 0$</td>
<td>$e^* &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>Increasing, $ds(b_w)/db_w &gt; 0$</td>
<td>$e^* = 0$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>If $dG(.)/d\sigma^2 &gt; 0$</th>
<th>$ds(b_w)/db_w = 0$</th>
<th>$e^* = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decreasing, $ds(b_w)/db_w &lt; 0$</td>
<td>$e^* = 0$</td>
<td></td>
</tr>
<tr>
<td>Increasing, $ds(b_w)/db_w &gt; 0$</td>
<td>$e^* &gt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

Proof: See Appendix 3.9.3.

The intuition behind Proposition 1 is as follows: The market maker knows that the lowest cost supplier among those in the auction will win and that a supplier’s bid increases with the supplier’s costs (Lemma 1). Therefore, when offered an increasing scheme, the market maker prefers that the winning bid be as high as possible. When $dG(.)/d\sigma^2 < 0$, the market maker knows that, as measurement error decreases, there is a greater likelihood that the winning supplier will be a low cost supplier. Consequently, as measurement error decreases (or equivalently, as qualification effort increases), there is a
greater chance that the winning bid will be low, which is not a desired outcome by the market maker. Therefore, when offered an increasing scheme with \( \frac{dG(.)}{d\sigma^2} < 0 \), the market maker does not exert any qualifying effort.

In contrast, when offered a decreasing scheme, the market maker has an incentive to exert a non-zero level of qualifying effort. With a decreasing scheme, the market maker benefits if the winning bid is low, and its exertion of a non-zero level of qualifying effort increases the chances that the winning bid will be low. Therefore, it is in the best interest of the market maker to exert a non-zero level of qualifying effort. Finally, offering the market maker a fixed fee does not provide any incentives to the market maker to exert effort, and therefore, the optimal level of qualifying effort is zero. A similar argument holds when \( \frac{dG(.)}{d\sigma^2} > 0 \) because low quality suppliers are likely to be low cost suppliers as well.

We now turn to our analysis of the benchmark case in which we assume quality is verifiable ex-post. Proposition 2 summarizes what the incentive-compatible compensation scheme should be in this situation.

**Proposition 2:** When quality is verifiable ex-post, the buyer should offer a decreasing scheme to the market maker.

**Proof:** See Appendix 3.9.4.

The intuition behind Proposition 2 is as follows: When quality is verifiable, the buyer does not incur quality costs. Therefore, the buyer’s goal is to reduce the purchase price as much as possible, and because the purchase price (i.e., the winning bid) increases with the costs of the winning supplier, the buyer wants the market maker to identify cost-
competitive suppliers. We know from Proposition 1 that if the market maker is offered an increasing scheme or a fixed fee, it will not exert any qualifying effort, which results in a suboptimal outcome for the buyer. Instead, when the market maker’s compensation is a decreasing scheme, the market maker chooses a positive level of qualifying effort. Therefore, offering a decreasing scheme is the optimal strategy for the buyer.

When quality is verifiable, the buyer focuses on reducing the purchase price as much as possible. However, when quality is non-verifiable, the quality costs may be very high. Therefore, the optimal strategy for the buyer will depend on the structure of its procurement costs, which are \( b_w + I[q(c_w)] \), where \( b_w \) is the winning supplier’s bid, and \( c_w \) is its cost to fulfill the contract. Proposition 3 establishes the conditions in which the buyer should offer a decreasing scheme and those in which the buyer should offer a increasing scheme to the market maker when quality is non-verifiable.

**Proposition 3:**

(a) If \( \frac{db}{dc} + \frac{dI[q(c)]}{dc} > 0 \forall c \in [\underline{c}, \bar{c}] \), the buyer should offer a decreasing scheme to the market maker.

(b) If \( \frac{db}{dc} + \frac{dI[q(c)]}{dc} < 0 \forall c \in [\underline{c}, \bar{c}] \), the buyer should offer an increasing scheme to the market maker.

(c) If \( \frac{db}{dc} + \frac{dI[q(c)]}{dc} = 0 \) for some \( c \in [\underline{c}, \bar{c}] \), the buyer should offer an increasing scheme to the market maker provided \( \theta > \theta_0 \), a decreasing scheme provided \( \theta < \theta_0 \), and a fixed fee provided \( \theta = \theta_0 \), where \( \theta_0 \) is a critical value determined by the following equation:
\[
\theta_0 = \frac{-\frac{\int (db(c) + ds[b(c)]) dG(c)}{de}}{\int \frac{d[1-q(c)]}{dc} dG(c) dc} \text{ at } e = 0.
\]

Proof: See Appendix 3.9.5.

To understand Proposition 3, note that when \( \frac{db}{dc} + \frac{dI[q(c)]}{dc} > 0 \forall c \in [\underline{c}, \overline{c}] \), the buyer obtains the lowest procurement cost if the cost of the winning supplier is as close to \( \underline{c} \) as possible. Therefore, the buyer prefers that low cost suppliers win the auction. From Proposition 2, we know that offering a decreasing scheme to the market maker ensures that low cost suppliers are more likely to win the auction. The converse logic holds when \( \frac{db}{dc} + \frac{dI[q(c)]}{dc} < 0 \forall c \in [\underline{c}, \overline{c}] \).

However, when \( \frac{db}{dc} + \frac{dI[q(c)]}{dc} = 0 \) for some \( c \in [\underline{c}, \overline{c}] \), the buyer’s choice of the compensation scheme must depend on how fast quality costs increase with poor quality, or \( \theta \). When \( \theta < \theta_0 \), quality costs do not increase very fast with poor quality, and therefore, the decrease in the purchase price that stems from awarding business to a low cost supplier more than compensates for the increase in quality assessment costs due to the lower quality. The buyer thus prefers to award business to a low cost supplier and offers a decreasing scheme to the market maker to increase the likelihood that a low cost supplier will win the auction. In contrast, when \( \theta > \theta_0 \), quality costs increase very fast with poor quality, so awarding business to a high quality supplier ensures that quality costs are very low, which compensates for the increase in the purchase price. In contrast to the benchmark case, when quality is non-verifiable, the buyer may need to offer an
increasing scheme to the market maker. When the buyer incurs substantial quality costs associated with poor quality, the failure to properly qualify suppliers exposes the buyer to a high risk of substantial quality costs. Therefore, the buyer will accept a higher purchase price upfront in return for lower quality costs in the future and therefore offers an increasing scheme to the market maker.

We now turn to two examples that provide additional insight into how the nature of the procurement situation drives the choice of the compensation scheme.

### 3.5.4 Illustrative Examples

In the first example, we consider a procurement situation in which quality increases linearly with cost; in the second example, we consider a situation in which quality increases at a decreasing rate with quality. Both examples assume that the cost of a supplier is uniformly distributed on \([c, \overline{c}]\).

#### Example 1: Quality increases linearly with cost

Consider the case in which a supplier’s quality increases linearly with cost; for example, suppose that \(q(c) = \alpha c\), where \(\alpha\) is the marginal increase in quality with respect to cost. We show in Appendix 3.9.6 that conditions (a) and (b) of Proposition 3 are sufficient to characterize the buyer’s choice of compensation scheme. Specifically, if

\[
\text{Var}(q) < \frac{1}{\theta^2} \left( \frac{n}{n+1} \right)^2 \frac{(\overline{c} - c)^2}{12},
\]

then the buyer should offer a decreasing scheme to the market maker, where \(\text{Var}(q)\) is the quality variation among suppliers. In contrast, if

\[
\text{Var}(q) > \frac{1}{\theta^2} \left( \frac{n}{n+1} \right)^2 \frac{(c - \underline{c})^2}{12},
\]

then the buyer needs to offer an increasing scheme to the market marker (for details, see Appendix 3.9.6).
To understand the intuition behind these results, consider Figure 3.5 which is a schematic sketch depicting how the type of compensation scheme depends on quality variation and the rate of increase in quality costs. As the vertical arrow suggests, for a given rate of increase in quality costs and as quality variation increases from point 1 to point 2, the optimal compensation scheme changes from a decreasing to an increasing scheme. At point 1, quality variation is quite low, so the buyer is unlikely to award business to a low quality supplier, which implies that the buyer is unlikely to incur high quality costs. Consequently, the buyer is interested in lowering the purchase price as much as possible and therefore offers the market maker a decreasing compensation scheme as motivation to identify cost-competitive suppliers. In contrast, at point 2, the quality variation is sufficiently high that there is a high risk the buyer will award business to a low quality supplier. In these circumstances, the buyer wants to hedge against this risk and therefore offers the market maker an increasing scheme to motivate it to qualify suppliers properly so that the buyer can exclude them from the auction.
Also consider points 3 and 4 in Figure 3.5. As the horizontal arrow suggests, for a given quality variation, as the rate of quality costs increases from point 3 to point 4, the optimal compensation scheme changes from a decreasing to an increasing scheme. At point 3, despite substantial quality variation, the rate of increase in quality costs with poor quality is not very high, which might occur if quality were relatively standardized and verifiable. In such a situation, even if the buyer awards the business to a low quality supplier, it will not incur high quality costs. Thus, the buyer is interested in lowering the purchase price as much as possible and offers a decreasing scheme to the market maker. In contrast, at point 4, the rate of increase in quality costs is sufficiently high, which might occur when quality is not easily verifiable and hence the buyer is likely to incur substantial quality costs if a low quality supplier wins the auction. Therefore, the buyer offers an increasing scheme to the market maker.

In the above example, we assume that quality increases linearly with cost, a somewhat restrictive assumption. However, in our next example, we find that the results do not change if the supplier’s quality increases at a decreasing rate with cost.

**Example 2: Quality increases at a decreasing rate with cost**

In contrast to example 1, consider the case in which a supplier’s quality increases at a decreasing rate with cost. Specifically, suppose that \( q(c) = \alpha \sqrt{c} \). Just as in Example 1 and using conditions (a) and (b) of Proposition 3, we show in Appendix 3.9.7 that if \( \text{Var}(q) < \frac{1}{\theta^2} \left( \frac{2n}{n+1} \right)^2 c \text{Var}(\sqrt{c}) \), then the buyer should offer a decreasing scheme to the market maker. In contrast, if \( \text{Var}(q) > \frac{1}{\theta^2} \left( \frac{2n}{n+1} \right)^2 c \text{Var}(\sqrt{c}) \), then the buyer needs to offer an increasing scheme to the market maker.
However, unlike the previous example, conditions (a) and (b) of Proposition 3 are not sufficient here to characterize completely the type of compensation scheme that the buyer should offer the market maker. When \( q(c) = \alpha \sqrt{c} \), the buyer’s procurement cost is given by \( \frac{nc}{n+1} + \theta(1 - \alpha \sqrt{c}) \). It is easy to show that when \( c < \frac{\theta^2 \alpha^2 (n + 1)^2}{4n^2} < c \), the buyer’s procurement cost is minimized at a point \( c \in (\underline{c}, \bar{c}) \), the situation addressed by condition (c) of Proposition 3. With suitable algebraic manipulations, we can show that if \( \text{Var}(q) > \tau \left( \frac{1}{\theta_o} \right) \), the buyer should offer an increasing scheme, and if \( \text{Var}(q) < \tau \left( \frac{1}{\theta_o} \right) \), the buyer should offer a decreasing scheme (for details, see Appendix 3.9.7.). The preceding analysis suggests that the insights generated from the first example hold even for the second example.

To summarize, in both examples, we demonstrate how quality variations among suppliers and quality costs interact to determine the nature of the compensation scheme the buyer should offer the market maker. When the buyer perceives that there is significant quality variation among suppliers and when quality costs are very high, the buyer must hedge against the possibility of awarding business to a low quality supplier, so it offers an increasing scheme to the market maker. In contrast, when quality costs are not very high, the buyer prefers that a low cost supplier win the auction, so it offers a decreasing scheme to the market maker. In the next section, we discuss how our results are likely to change if we relax some of our key assumptions.

### 3.6 Discussion of Key Assumptions

We now discuss how our results might change if we were to relax several of the key assumptions we made during our model development.
3.6.1 Incumbent Supplier and Reserve Price

We implicitly assumed that there was no incumbent supplier in our model, which may not hold in many procurement situations. The presence of an incumbent supplier has two implications from a modeling standpoint. First, unlike the case of a new supplier, the buyer knows the quality of the incumbent’s product from previous experience, which makes qualification of the incumbent supplier unnecessary. Second, when there is an incumbent supplier, the buyer often specifies that a new supplier will be awarded business only if the winning bid of that supplier is below a certain reserve price. The reserve price is set by the buyer to account for any additional costs involved in transferring business to a new supplier.

When a reserve price is present, bidders that cannot meet the reserve do not bid in the auction; in our context, some new suppliers that have high costs do not participate. However, simply because some new suppliers do not participate does not remove the need for the proper qualification of the remaining suppliers, because improper qualification still can lead to negative outcomes for the buyer. For example, consider a situation in which there are ten new suppliers, out of which the buyer wants to invite five to bid in the auction along with the incumbent. Suppose that one of the new suppliers has the lowest cost among all suppliers, including the incumbent. In such a situation, if the buyer prefers that the low cost supplier win the auction, then improper qualification of suppliers might cause the lowest cost supplier to be left out of the auction because it might be classified as a high cost supplier. Therefore, proper qualification is again essential.
However, offering the market maker an increasing scheme is not optimal in this scenario. If offered an increasing scheme, the market maker will not qualify suppliers rigorously, because improper qualification will increase the chances that a low cost supplier will be mistaken as a high cost supplier, which would exclude the low cost supplier from the auction. In contrast, offering a decreasing scheme actually motivates the market maker to qualify suppliers rigorously, so that it may ensure that low cost suppliers participate in the auction. A similar logic holds when the buyer does not want low quality suppliers to participate in the auction.

3.6.2 Auction Format

We also have assumed that the auction format is a first-price, sealed bid auction. However, our results are independent of the specific details of the auction format, provided the actual auction format satisfies two features. First, the winner in the auction must be the lowest bidder. Second, a supplier’s bid must increase with its cost. As long as these two conditions are met, our proofs hold, and consequently, our results generalize to other auction settings, such as first-price, ascending auctions.

However, not all auction formats used by buyers satisfy these features. In certain procurement auctions, the buyer awards the business to a supplier on the basis of a combination of the supplier’s bid and its quality rating. Essentially, the buyer uses the supplier’s bid and quality rating to calculate an estimate of the procurement cost for that supplier. The buyer then awards the business to the supplier whose estimated procurement cost is the lowest. We modify our model to accommodate such an auction format as follows: Let the estimate of the procurement cost be represented by $b + I(r_q)$, where $b$ is the bid of a supplier and $r_q$ is the quality rating of that supplier. Note that
when the buyer selects the winner on the basis of the total procurement cost, the process of ensuring that only qualified suppliers enter the auction is moot. Therefore, we drop Assumption 3, that the buyer invites \( n \) suppliers to participate in the auction, from our model. Instead, we assume that the buyer invites all suppliers to participate and bid in the auction.

We speculate that our results will not substantively change even when the buyer awards business on the basis of the total procurement cost. Consider a procurement situation when quality costs rise rapidly with poor quality. In such a situation, offering a decreasing scheme to the market maker does not offer it an incentive to qualify suppliers rigorously. This lack of rigor in supplier qualification might lead to a low quality supplier obtaining a high quality rating, which would enable a low quality supplier to win the auction, an outcome that is not favorable to the buyer. In contrast, offering an increasing scheme motivates the market maker to qualify suppliers rigorously, because rigorous qualification ensures that a high quality supplier wins the auction, which results in higher compensation for the market maker. A similar argument holds when quality costs do not rise rapidly with poor quality. We leave a formal analysis of the above issues for future research.

### 3.6.3 Stochastic Cost–Quality Relationship

In addition, we have assumed a deterministic relationship between a supplier’s cost and the quality of the product it supplies to the buyer, but this assumption does not account for suppliers that have high costs but relatively poor quality. One way to accommodate such suppliers in our model is to assume that the cost–quality relationship is stochastic rather than deterministic. To make the cost–quality relationship stochastic,
we might assume that \( q = q(c) + \tau \) (instead of \( q = q(c) \)), where the error term \( \tau \) is such that \( E(\tau) = 0 \) and \( E(\tau^2) > 0 \) (i.e., \( \tau \) has 0 mean and finite variance). If the variance of \( \tau \) is sufficiently high, a high cost supplier may be a low quality supplier. We speculate that our results are likely to hold as long as the error variance \( E(\tau^2) \) is not very high, but they are unlikely to hold when error variance is very high. When error variance is high, the winning bid is not correlated with the quality of the supplier because a low quality, high cost supplier will bid high, which prevents the buyer from inferring that a supplier that bids high is a high quality supplier. In the absence of such an inference, it may not be possible for the buyer to use winning bids to ensure that market makers qualify suppliers rigorously.

However, if the market maker qualifies suppliers rigorously, the presence of a high cost, low quality supplier is not an issue. For example, consider a procurement situation in which the buyer has high quality costs and therefore prefers not to invite low quality suppliers to participate in the auction. If the market maker qualifies suppliers rigorously, low quality suppliers will be identified correctly and the buyer will not invite them to the auction. But, it is not necessary for the market maker to report a supplier’s ratings correctly. When the market maker’s compensation is an increasing function of the winning bid, the market maker prefers that a high cost supplier win the auction. Therefore, the market maker may misrepresent quality and cost competitiveness ratings, such that a high cost, low quality supplier could be considered a supplier with low cost and high quality. If the high cost, low quality supplier wins the auction, the buyer will realize the true quality of the supplier long after the auction is over. Therefore, compensation schemes based on variables that are contemporaneous with the auction
(e.g., winning bids) are unlikely to prevent the market maker from such a misrepresentation.

In light of the above, the buyer must either rely on the market maker’s need to build a long-term reputation or design compensation schemes that are based on outcomes (e.g., actual quality of the supplier’s products) to ensure that the market maker does not misrepresent high cost, low quality suppliers as suppliers of high quality products. Because constructing such compensation schemes is outside the scope of our research, we leave a formal analysis of these issues for future research.

3.6.4 Alternative Solutions to Solve Problem of Supplier Qualification

Auction theory suggests at least two other ways to solve the problem of assessing a supplier’s ability. One solution relies on designing a truth-revealing auction, such that suppliers are motivated to reveal their true cost and quality levels and thereby eliminate the need to appoint a market maker. However, it not optimal for a supplier to reveal its true cost and quality levels in the present auction if there is a possibility that it will be asked to compete for the buyer’s business in the future. Revealing its true cost and quality levels in the present auction would enable the buyer to offer a take-it-or-leave it contract in the future, in which the buyer sets the price at a few cents above the supplier’s cost, an outcome that is not in the best interest of the supplier. If, however, the supplier does not reveal its true cost and quality levels in the present auction, the buyer cannot structure a take-it-or-leave-it contract in the future, which may enable the supplier to obtain a higher profit than that offered through a take-it-or-leave-it contract.

The second solution involves “selling the firm” to the market maker. In this solution, the buyer offers to sell the business of procuring materials to the market maker
in return for a fixed fee. If the market maker accepts the buyer’s offer, the market maker’s objective function becomes identical to the buyer’s procurement cost function. Therefore, the market maker is motivated to qualify suppliers rigorously so that it can optimize procurement costs. In this scenario, the buyer’s problem becomes choosing the minimum fixed fee acceptable to the market maker, which successfully solves the problem of assessing the supplier’s ability to meet the buyer’s requirements. Although this solution seems to solve the buyer’s problem, it is not clear whether it can be implemented in a practical setting. We leave for additional research issues such as whether truth-revealing auctions or “selling the firm” help the buyer solve the supplier qualification problem.

We have not yet shown how to compute optimal incentive-compatible compensation schemes, a topic we turn to in the next section.

3.7 Deriving Optimal Incentive-Compatible Compensation Schemes

The analytical derivation of the structure of optimal compensation schemes based on winning bids (hereafter, optimal schemes) is intractable in our context. Therefore, we illustrate how to derive optimal schemes computationally. We restrict our illustration to designing compensation schemes in which the buyer wants to offer an increasing scheme, because the same methodology can be followed for a decreasing scheme. The buyer’s optimization problem is:

$$\min_{w, s(e), e} \int \limits_{\mathcal{C}} [b(c) + I(q(c)) + s(b(c))] \ g(c \mid e, w_q) \ dc,$$  \hspace{1cm} (C3.4)

subject to

$$\text{IC: } e^* \in \arg\max_{w, s(e), e} \int \limits_{\mathcal{C}} [s(b_w) - e] \ g(c \mid e, w_q) \ dc \text{ and }$$  \hspace{1cm} (C3.5)
One approach to this problem, known as the first-order approach, involves substituting the market maker’s maximization problem (Equation C3.5) by the corresponding first-order condition (Equation C3.5') and solving the corresponding nonlinear constrained optimization problem,

\[
\begin{align*}
PC: \int_{c}^{\cdot} \left[ s(b_w) - e^* \right] g(c \mid e, w_q) \, dc & \geq 0. \\
\end{align*}
\]  

(C3.6)

However, C3.5' is a necessary but not sufficient condition for the market maker’s optimization problem. The first-order approach remains valid only with extremely stringent assumptions about the impact of the agent’s effort on the outcome, or \( g(.1e) \) in Equation C3.5' (Rogerson 1985). Our model does not meet these assumptions, and they are unlikely to be met in practice either. Therefore, we turn to another approach that attempts to transform the nonlinear program into a linear program and then solve the linear program to derive the optimal scheme (Prescott 1998). We first transform the optimization problem from continuous space into discrete space and then consider mixed instead of pure strategies. We present our methodology for a fixed value of \( w_q \). Finding the optimal \( w_q \) requires running the numerical procedure mentioned subsequently for many values of \( w_q \) and then selecting the optimal \( w_q \) from among those tested.

For simplicity, let the market maker’s compensation be a fraction of the winning bid, \( s(b_w) = d b_w \), where \( 0 \leq d \leq 1 \). We transform the optimization problem into discrete space by assuming that (1) the fraction of the winning bid the buyer plans to offer to the
market maker $d$, (2) a supplier’s cost $c$, and (3) the market maker’s effort $e$ all belong to a finite set. That is, let $d_i \in \{d_1, d_2, \ldots, d_n\}$, $c_j \in \{c_1, c_2, \ldots, c_m\}$, and $e_k \in \{e_1, e_2, \ldots, e_p\}$.

Then, let

$$p(c_j \mid e_k) = \text{the probability that the winning supplier’s cost is } c_j \text{ when the market maker’s effort is } e_k,$$

and

$$\pi(d_i, c_j, e_k) = \text{the probability that the buyer offers a compensation rate of } d_i \text{ when the winning supplier’s cost is } c_j \text{ and the market maker’s effort is } e_k.$$

In turn, the buyer’s optimization problem can be written as:§

$$\min \sum_{d_i} \sum_{c_j} \sum_{e_k} \pi(d_i, c_j, e_k) \left[ \beta(c_w) + IC(q(c_w)) + d \beta(c_w) \right],$$

subject to

**PC:** \(\sum_{d_i} \sum_{c_j} \sum_{e_k} \pi(d_i, c_j, e_k) \left[ d \beta(c_w) - V(e) \right] \geq 0,

**IC:**

\[
\sum_{d_i} \sum_{c_j} \sum_{e_k} \pi(d_i, c_j, e_k) \left[ d \beta(c_w) - V(e) \right] \geq \sum_{d_i} \sum_{c_j} \sum_{e_k} \pi(d_i, c_j, e_k) \frac{p(c_w \mid e')} {p(c_w \mid e)} \left[ d \beta(c_w) - V(e') \right],
\]

**BT:** \(\sum_{d_i} \pi(d_i, c_w, e') = p(c_w \mid e') \sum_{d_i} \sum_{c_j} \pi(d_i, c_j, e')\),

**P1:** \(\sum_{d_i} \sum_{c_j} \sum_{e_k} \pi(d_i, c_w, e) = 1\), and

**P2:** \(\pi(d_i, c_w, e) \geq 0\).

---

§ Note that the joint probability of $d, c_w, e$ is given by $\pi(d_i, c_j, e_k) = \pi(d \mid c_w, e) \cdot p(c_w \mid e) \cdot \pi(e)$. 
The participation constraint PC ensures that the market maker earns a minimum level of compensation, and the incentive-compatible constraint IC ensures that the market maker does not profit from deviating from the optimal effort level. Constraint BT follows from the Bayes theorem, and constraints P1 and P2 follow from basic probability theory. The market maker chooses \( \pi(d, c_w, e) \), subject to the five constraints (PC, IC, BT, P1, and P2). Because the program is linear in \( \pi(d, c_w, e) \), it can be solved using well-developed algorithms. In general, \( \pi^*(d, c_w, e) \) need not be a degenerate probability distribution, which implies that the market maker should randomize among the compensation schemes suggested by the solution. However, such a recommendation does not have practical appeal. Therefore, we suggest that the buyer choose the compensation scheme that has the highest probability, a recommendation that may be suboptimal but is practical.

Before illustrating how to solve the linear program, we provide details about how a buyer can operationalize the preceding model. The variables can be operationalized as follows:

\[
\begin{align*}
    c_j & = \text{supplier’s cost of fulfilling the contract in $ per unit of product supplied}, \\
    e_k & = \text{expense incurred by the market maker in qualifying suppliers, and} \\
    d_i & = \text{fraction of the winning bid paid to the market maker as compensation.}
\end{align*}
\]

The range of \( c_j \) and \( e_k \) can be assessed by the buyer on the basis of its previous experience with suppliers and previous qualification efforts, respectively. The range of \( d_i \) depends on the buyer’s discretion; it is a variable controlled by the buyer. The buyer also must assess the distribution of the winning supplier’s cost, conditional on the market
maker’s effort, or \( p(c_j | e_k) \) in our model. This assessment can be problematic because the buyer must assess \( nc \times ne \) probabilities. Therefore, we propose the following procedure to assess \( p(c_j | e_k) \).

**Step 1:** Select a few effort levels, say, \( e_1, e_{nc} \), and an intermediate effort level. At each effort level, obtain the buyer’s assessment of \( p(c_j | e) \) for a certain number of cost levels.

**Step 2:** For each cost level, estimate \( p(c_j | e) \) for the effort levels (not considered in step 1) and ensure that the resulting estimates are bound between 0 and 1.

**Step 3:** For each effort level, estimate \( p(c_j | e) \) for all the cost levels not considered in step 2 and ensure that the resulting estimates are greater than 0.

**Step 4:** For each effort level, normalize \( p(c_j | e) \) by \( \sum_{j=1}^{ne} p(c_j | e) \) to make the probabilities sum to 1.

**Step 5:** Obtain the buyer’s feedback on the resulting estimates and repeat steps 1–4 if needed to fine tune the estimates of \( p(c_j | e) \).

The buyer also must assess the functional form of its procurement cost, which may be based on previous experience with existing suppliers and industry trends in terms of defect rates. In the following example, we simplify by assuming the following:

(a) \( e \in \{50000, 10000, 15000\} \), \( ne = 2 \).

(b) \( c_v \in \{20, 25, 30\} \), \( nc = 3 \).

(c) \( d \in \{0.01, 0.025, 0.04\} \), \( nd = 3 \).
(d) The buyer intends to procure 1 million parts and the procurement cost per part is given by 
\[ \frac{n c_w + 30}{n+1} + \theta(1 - \alpha \sqrt{c_w}), \] 
where \( n = 5, \alpha = 0.223, \) and \( \theta = 41.071. \)

(e) \( p(c_j \mid e) \) is set as follows, which satisfies \( \sum_{j=1}^{nc} p(c_j \mid e) = 1, \) as well as the requirement that as effort increases, the distribution shifts to the right:

\[
\begin{align*}
c_1 &= 20 & c_2 &= 25 & c_3 &= 30 \\
e_1 &= 5 & 0.6 & 0.2 & 0.2 \\
e_2 &= 10 & 0.5 & 0.2 & 0.3 \\
e_3 &= 15 & 0.2 & 0.1 & 0.7
\end{align*}
\]

The optimal solution to this linear program is given by the following table:

<table>
<thead>
<tr>
<th>( \pi(d, c_w, e) )</th>
<th>( d )</th>
<th>( c )</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.01</td>
<td>$20</td>
<td>15000</td>
</tr>
<tr>
<td>0.1</td>
<td>0.01</td>
<td>$25</td>
<td>15000</td>
</tr>
<tr>
<td>0.7</td>
<td>0.01</td>
<td>$30</td>
<td>15000</td>
</tr>
</tbody>
</table>

To interpret the solution, note that \( \pi(e) = \sum_{d, c_w} \sum_{e} \pi(d, c_w, e) = 1. \) Similarly, \( \pi(d \mid c_w, e) = \frac{\pi(d, c_w, e)}{\pi(c_w, e)} = 1. \) Therefore, the buyer need not randomize among compensation schemes; its optimal solution is to offer to the market maker 1% of the winning bid per part as compensation. In return, the market maker spends $15,000 in qualifying suppliers, thereby allocating an effort level greater than the minimum level of effort that the market maker might have exerted.

3.8 Discussion and Conclusion

Our findings suggest that the nature of an incentive-compatible compensation scheme should depend on the extent of quality variation among suppliers and the
structure of the quality costs. Specifically, when quality costs rise very rapidly with poor
good quality and when quality varies significantly among suppliers, the buyer must hedge
against the risk of awarding its business to a low quality supplier. In return for
“insurance” against awarding business to a low quality supplier, the buyer pays a higher
upfront purchase price. In contrast, when quality costs do not rise very fast with poor
quality and when quality does not vary much, the buyer need not buy such insurance and
can request that the market maker try to locate low cost suppliers. Our findings caution
buyers not to reject seemingly counterintuitive compensation schemes that increase with
the winning bid. Depending on the procurement situation, a scheme based on the contract
value may be better than a scheme based on savings.

Our research contributes to both theory and practice. We extend existing auction
theory by explicitly modeling the role of an auctioneer (i.e., the market maker) and
providing insight into the structure of the incentive-compatible compensation scheme that
a buyer should offer to a market maker. From the point of view of e-business practice,
when market makers offer online reverse auction services, they essentially create an e-
marketplace that allows market transactions to take place on a global basis. For an e-
marketplace to prosper, it is essential that only qualified buyers and sellers participate in
the market, and different marketplaces adopt various solutions to ensure that participants
are properly qualified (e.g., eBay’s reputation ratings). In the context of online reverse
auctions, rigorous supplier qualification is a means to solve the credibility problem.
Misaligned compensation schemes for a market maker create problems because they
encourage market makers to slack off during the supplier qualification process.
Our research also contributes to practice by identifying how the nature of compensation schemes changes systematically according to different procurement situations, a finding that buyers can use to structure appropriate compensation schemes for market makers. We also have proposed a computationally feasible procedure to compute incentive-compatible compensation schemes that has real potential to be applied in practice.

However, our research also has some limitations that suggest future research opportunities. Because we assume a deterministic relationship between quality and cost, future research might explore the impact of relaxing this assumption on the nature of the incentive-compatible compensation scheme. Another important issue pertains to the market maker. When offered compensation on the basis of the contract value, the market maker may try to influence the number of new suppliers to be invited to the auction, because fewer auction participants would result in a higher bid price.\(^8\) Finally, computing incentive-compatible compensation schemes in practice to guide buyers in their attempts to structure appropriate compensation for market makers should be explored further. We leave these interesting extensions for future research.

\(^8\) We thank an anonymous reviewer of the eBRC Doctoral Award competition for pointing out this extension to us.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Index for supplier</td>
</tr>
<tr>
<td>$b_w$</td>
<td>Winning bid in the auction</td>
</tr>
<tr>
<td>$q_w$</td>
<td>Quality of the winning supplier</td>
</tr>
<tr>
<td>$I(q_w)$</td>
<td>Quality costs associated with the winning supplier’s quality $q_w$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Quality cost per defective part</td>
</tr>
<tr>
<td>$s(b_w)$</td>
<td>Compensation paid to the market maker</td>
</tr>
<tr>
<td>$c \in [c, \bar{c}]$</td>
<td>Cost of a supplier</td>
</tr>
<tr>
<td>$q(c)$</td>
<td>Quality of a new supplier whose cost is $c$</td>
</tr>
<tr>
<td>$r_{q,i}$</td>
<td>Overall rating that supplier $i$ obtains on the quality dimension</td>
</tr>
<tr>
<td>$r_{c,i}$</td>
<td>Overall rating that supplier $i$ obtains on the cost dimension</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Overall rating that supplier $i$ needs to obtain to be qualified</td>
</tr>
<tr>
<td>$w_q, 1-w_q$</td>
<td>Weights the buyer assigns to the quality and cost dimensions, respectively</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Measurement error associated with quality and cost ratings</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>Variance of measurement error associated with quality and cost ratings</td>
</tr>
<tr>
<td>$e$</td>
<td>Amount in $ spent by market maker in qualifying suppliers</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of new suppliers that a buyer considers to invite to the auction</td>
</tr>
<tr>
<td>$n$</td>
<td>Actual number of new suppliers to invited to participate in the auction</td>
</tr>
<tr>
<td>$F(c), f(c)$</td>
<td>Distribution and density, respectively, of the cost of a supplier</td>
</tr>
<tr>
<td>$G(.), g(.)$</td>
<td>Distribution and density, respectively, of the winning supplier’s cost</td>
</tr>
<tr>
<td>$d$</td>
<td>Fraction of the winning bid that the buyer offers to market maker as compensation</td>
</tr>
<tr>
<td>$nd, nc, ne$</td>
<td>Number of compensation rates, cost levels, and effort levels, respectively</td>
</tr>
<tr>
<td>$\pi(d_i, c_j, e_k)$</td>
<td>Probability that the buyer offers a compensation rate $d_i$ when the winning supplier’s cost is $c_j$ and the market maker’s effort is $e_k$</td>
</tr>
<tr>
<td>$p(c_j \mid e_k)$</td>
<td>Probability that the winning supplier’s cost is $c_j$ when the market maker’s effort is $e_k$</td>
</tr>
</tbody>
</table>
3.9 Appendices

3.9.1 Proof of Lemma 1

Assume that all suppliers except supplier $i$ follow the same strategy of bidding $b(c_j)$, where $j \neq i$. We assume that the bidding function is an increasing function of the supplier’s cost, such that $\frac{db(c_j)}{dc_j} > 0$. The expected profits that supplier $i$ obtains by bidding $b_i$ are given by

$$\pi(b_i; c_i) = R[b^{-1}(b_i)](b_i - c_i),$$  \hspace{1cm} (C3.7)

where $R[b^{-1}(b_i)]$ is the probability that supplier $i$’s bid $b_i$ is the lowest among all bids.

Differentiating C3.7 w.r.t $b_i$, we obtain the first-order condition for a maximum:

$$\frac{r[b^{-1}(b_i)]}{b'(b^{-1}(b_i))}(b_i - c_i) + R[b^{-1}(b_i)] = 0.$$  \hspace{1cm} (C3.8)

At a symmetric equilibrium, $b_i = b(c_i)$, and therefore, by simplifying C3.8, we get:

$$r(c_i) b_i + b'(c_i) R[c_i] = r(c_i) c_i.$$  \hspace{1cm} (C3.9)

Rewriting Equation C3.9, we obtain:

$$b'(c_i) = \frac{r(c_i)(c_i - b_i)}{R(c_i)}.$$  \hspace{1cm} (C3.10)

Because $(c_i - b_i) < 0$ and $\frac{r(c_i)}{R(c_i)} < 0$, it follows that $b'(c_i) > 0$. 
3.9.2 Proof of Lemma 2

Let \( c_{(1)} > c_{(2)} > \ldots c_{(n)} > c_{(n+1)} > \ldots c_{(N)} \) be the order statistics of the costs of the \( N \) suppliers that potentially can be invited to participate in the auction. Because the buyer invites \( n \) suppliers to participate in the auction, the cost of the winning supplier must be one of the following order statistics: \( c_{(n)}, c_{(n+1)}, \ldots c_{(N)} \). That is, the suppliers with the \( n-1 \) highest costs can never win the auction. Let \( p_k \) \((k = n, n+1,\ldots N)\) be the probability that the supplier with the \( k \)th highest cost wins the auction. Therefore, it follows that

\[
G(c) = \sum_{k=n}^{N} \Pr[c_k > c] \cdot p_k .
\]

Note that the following relationships hold:

\[
\Pr[c_{(n)} > c_w] < \Pr[c_{(n+1)} > c_w] < \ldots < \Pr[c_{(N-1)} > c_w] < \Pr[c_{(N)} > c_w] , \text{ and} \tag{C3.11}
\]

\[
\sum_{k=n}^{N} p_k = 1 . \tag{C3.12}
\]

To prove the lemma, note that

a. \( \Pr[c_k > c] \) is independent of both \( w_q \) and \( e \), because it is the \( k \)th highest cost among the \( N \) suppliers and therefore depends only on \( f(.) \) and \( N \).

b. Therefore, the measurement error has an impact on \( p_k \) alone.

c. Suppose that \( w_q \) and \( \sigma^2 \) are such that \( p_N > \frac{C_{o_{n-1}^N}}{C_{o_n^N}} \).

d. From Assumption 4, we know that \( p_N \to \frac{C_{o_{n-1}^N}}{C_{o_n^N}} \) as \( \sigma^2 \to \infty \).
e. Because $p_n > \frac{Co_n^{N-1}}{Co_n^N}$ and $\sum_{k=n}^{N} p_k = 1$ as $p_n \to \frac{Co_n^{N-1}}{Co_n^N}$, it must be the case that some $p_k$ ($k \neq i$) are increasing.

f. In light of C3.11 and $G(c) = \sum_{k=m}^{N} \Pr[c_k > c] p_k$, it must be that $G(.)$ decreases as $\sigma^2 \to \infty$.

Using similar logic, we can show that $\frac{dG(.)}{d\sigma^2} > 0$ when $p_n < \frac{Co_n^{N-1}}{Co_n^N}$. 

3.9.3 **Proof of Proposition 1**

The market maker’s problem is to choose the optimal rigor of qualifying effort, given the compensation scheme offered by the buyer and the specified value of $w_q$.

**Case A:** $\frac{dG(.)}{d\sigma^2} < 0$ and $\frac{ds(b_w)}{db_w} < 0$.

Note that $\frac{dG(c)}{de} = \frac{dG(c)}{d\sigma^2} \frac{d\sigma^2}{de}$. Given that $\frac{dG(c)}{d\sigma^2} < 0$ and, by assumption, $\frac{d\sigma^2}{de} < 0$, it follows that $\frac{dG(c)}{de} > 0$. The following optimization program gives the market maker’s choice for the optimal level of its qualifying effort:

$$\max_e \quad \pi(e) = E[s(b_w)] - e,$$

s.t.

$$\pi(e) \geq 0, \ e \geq 0.$$  

Now,

$$E[s(b_w)] = \int_{\zeta} s(b(c)) \ g(c) \ dc.$$  \hfill (C3.12)

Using integration, C3.12 can be rewritten as

$$\int_{\zeta} s(b(c)) \ g(c) \ dc = s(b(c)) \ G(c) \left[ \zeta \right] - \zeta \left[ ds(b(c)) \ G(c) \right] dc;$$

that is,

$$\int_{\zeta} s(b(c)) \ g(c) \ dc = s(b(C)) - \zeta \left[ ds(b(c)) \ G(c) \right] dc.$$

Therefore,
Differentiating C3.13 w.r.t \( e \), we get:

\[
\frac{dE[s(b_w)]}{de} = -\int_{\xi}^{\bar{\xi}} \frac{ds(b(c))}{dc} \frac{dG(c)}{de} dc.
\]

(C3.14)

Now consider the first term under the integral in C3.14, which can be rewritten as

\[
\frac{ds(b(c))}{dc} = \frac{ds(b_w)}{db_w} \frac{db_w}{dc}.
\]

(C3.15)

It follows from C3.15 that \( \frac{ds(b(c))}{dc} > 0 \), because \( \frac{db_w}{dc} = \frac{db(c)}{dc} > 0 \) (see Lemma 1).

Now consider the second term under the integral in C3.14. When \( w_q = 0 \), as we already have shown, \( \frac{dG(c)}{de} > 0 \). Because \( \frac{ds(b_w)}{dc} > 0 \) and \( \frac{dG(c)}{de} > 0 \), it follows from C3.13 that \( \frac{dE[s(b_w)]}{de} < 0 \). Differentiating the market maker's objective function w.r.t \( e \), we get:

\[
\frac{d\pi(e)}{de} = \frac{dE[s(b_w)]}{de} - 1.
\]

Because \( \frac{dE[s(b_w)]}{de} < 0 \) and \( \frac{de}{de} > 0 \), it follows that \( \frac{d\pi(e)}{de} < 0 \). Therefore, the optimal level of the market maker's qualifying effort is \( 0 \) (i.e., \( e^* = 0 \)), and the market maker's profit is

\[
\pi_A^*(0) = E[s(b_w) | e^* = 0],
\]

(C3.16)

where \( \pi_A^*(.) \) refers to the optimal profit in Case A.
Case B: $\frac{dG(c_w)}{d\sigma^2} < 0$ and $\frac{ds(b_w)}{db_w} < 0$.

In contrast with the previous case, because $\frac{ds(b(c_w))}{dc_w} < 0$ and $\frac{dG(c_w)}{de} > 0$, it follows from C3.14 that $\frac{dE[s(b_w)]}{de} > 0$. Given an appropriately structured compensation scheme, the market maker’s choice of optimal effort is greater than 0 (i.e., $e^* > 0$), and its profit is

$$\pi_B^*(e^*) = E[s(b_w) \mid e^*] - e^*,$$

where $\pi_B^*(\cdot)$ is the optimal profit in Case B, and $e^* > 0$ is the optimal level of the qualifying effort, which is a solution to the equation

$$\frac{dE[s(b_w) \mid e]}{de} - 1 = 0.$$

Now we show that $\pi_B^*(\cdot) > \pi_A^*(\cdot)$. Note that when the market maker’s qualifying effort is not rigorous, the distribution of the lowest cost supplier is the same in both Cases A and B, and hence, $E[s(b_w) \mid e]$ is the same for both alternatives. Therefore, it follows that $\pi_B(e = 0) = \pi_A(e = 0)$. However, we also know that $\pi_B^*(e^*)$ is the optimal profit for Case B, so it follows that $\pi_B^*(\cdot) > \pi_A^*(\cdot)$.

Case C: $\frac{dG(c_w)}{d\sigma^2} < 0$ and $\frac{ds(b_w)}{db_w} = 0$.

When $\frac{ds(b(c_w))}{dc_w} = 0$, it follows from C3.8 that $\frac{dE[s(b_w)]}{de} = 0$, and therefore, the market maker does not exert any qualifying effort (i.e., $e^* = 0$). Therefore,
\( \pi_c^*(e^*) = \pi_A^*(e^*) < \pi_B^*(e^*) \). By a similar argument, we can show that when \( \frac{dG(c_w)}{d\sigma^2} > 0 \), the market maker exerts a non-zero level of qualifying effort only when it is offered a decreasing scheme.
3.9.4 Proof of Proposition 2
The proof for Proposition 2 is essentially the same as that for Proposition 3, so we omit it for the most part. The only difference between the two proofs is that, because quality is verifiable and relatively standardized, the buyer incurs minimal quality costs. Therefore, we can assume that $I(q) = 0$ in Proposition 2.
3.9.5 Proof of Proposition 3

The buyer’s problem is to minimize the total cost of procurement (TCP). The buyer’s expected cost of procurement is

$$\tilde{c} \left[ (b(c) + I[q(c)] + s[b(c)]) g(c) \right] dc. \quad \text{(C3.17)}$$

Integrating C3.17 by parts, we obtain the buyer’s objective:

$$\left[ (b(c) + I[q(c)] + s[b(c)]) \right] - \tilde{c} \left[ \frac{db(c)}{dc} + \frac{dI(q(c))}{dc} + \frac{ds[b(c)]}{dc} \right] G(c) dc. \quad \text{(C3.18)}$$

We know from Proposition 1 that if the buyer offers the market maker an increasing scheme, the market maker chooses a non-zero level of qualifying effort, such that low quality suppliers do not participate in the auction. In contrast, if the buyer offers the market maker a decreasing scheme, the market maker will chose a non-zero level of qualifying effort, such that low cost suppliers participate in the auction. Therefore, the buyer will offer an increasing scheme to the market maker only if the buyer gains when the market maker exerts a non-zero level of qualifying effort to exclude low quality suppliers from the auction. Conversely, the buyer will offer a decreasing scheme only if the buyer gains when the market maker exerts a non-zero level of qualifying effort to include low cost suppliers.

Differentiating C3.18 w.r.t $e$, we get:

$$- \tilde{c} \left[ \frac{db(c)}{dc} + \frac{dI(q(c))}{dc} + \frac{ds[b(c)]}{dc} \right] G(c) dc.$$

We know that if the market maker exerts effort to include low cost suppliers in the auction, then \( \frac{dG(c)}{dc} > 0 \). Therefore, it follows that the buyer decreases its expected TCP if \( \frac{db(c)}{dc} + \frac{dI[q(c)]}{dc} + \frac{ds[b(c)]}{dc} > 0 \forall c \in [c, \bar{c}] \). A similar argument shows that when \( \frac{db(c)}{dc} + \frac{dI[q(c)]}{dc} + \frac{ds[b(c)]}{dc} < 0 \forall c \in [c, \bar{c}], \) the buyer’s procurement cost falls as the market maker exerts effort to exclude low quality suppliers from the auction.

We now claim that \( \frac{db(c)}{dc} + \frac{dI[q(c)]}{dc} > 0 \forall c \in [c, \bar{c}] \) is a necessary condition for the buyer to encourage the market maker to locate and include low cost suppliers in the auction. As an explanation, consider \( \frac{db(c)}{dc} + \frac{dI[q(c)]}{dc} + \frac{ds[b(c)]}{dc} > 0 \). Proposition 1 indicates that if the buyer wants the market maker to locate and include low cost suppliers in the auction, the market maker’s compensation must be set such that \( \frac{ds[b(c)]}{db(c)} < 0 \). Therefore,

\[
\frac{db(c)}{dc} + \frac{dI[q(c)]}{dc} + \frac{ds[b(c)]}{dc} > 0
\]

\[
\Rightarrow \quad \frac{db(c)}{dc} + \frac{dI[q(c)]}{dc} > -\frac{ds[b(c)]}{dc} = -\frac{ds[b(c)]}{db(c)} \cdot \frac{db(c)}{dc} > 0.
\]

In turn, \( \frac{db(c)}{dc} + \frac{dI[q(c)]}{dc} > 0 \forall c \in [c, \bar{c}] \) is a necessary condition for the buyer to encourage the market maker to locate and include low cost suppliers in the auction. Similarly, we can show that \( \frac{db(c)}{dc} + \frac{dI[q(c)]}{dc} < 0 \forall c \in [c, \bar{c}] \) is a necessary condition for
the buyer to encourage the market maker to exclude low quality suppliers from the auction.

Now consider the situation in which \( \frac{db}{dc} + \frac{dI(q(c))}{dc} = 0 \) for some \( c \in [c, \bar{c}] \). Our proof in this case proceeds in three steps:

1. If there exists a \( \hat{\theta} \) such that the buyer’s optimal strategy is to offer an increasing scheme, it is optimal to offer an increasing scheme for all \( \theta > \hat{\theta} \).

2. If there exists a \( \hat{\theta} \) such that the buyer’s optimal strategy is to offer a decreasing scheme, it is optimal to offer an increasing scheme for all \( \theta < \hat{\theta} \).

3. There is a unique \( \theta \) at which the buyer will be indifferent between offering a decreasing or an increasing scheme.

From steps 1, 2 and 3, the required proof of (c) is evident.

Suppose that there exists a \( \hat{\theta} \) such that the buyer’s optimal strategy is to offer an increasing scheme. Consider \( \hat{\theta} + \Delta \hat{\theta} \), where \( \Delta \hat{\theta} > 0 \). Because at \( \hat{\theta} \) it is optimal for the buyer to offer an increasing scheme, it must be the case that the buyer’s objective function satisfies the first-order condition. That is,

\[
-\int_{\hat{c}}^{\bar{c}} \left( \frac{db(c)}{dc} + \hat{\theta} \frac{d[1 - q(c)]}{dc} + \frac{ds(b(c))}{dc} \right) \frac{dG(c)}{de} dc < 0 \text{ at } e = 0.
\]

We also know that for the buyer to offer an increasing scheme, it must be necessary that \( \frac{dG(c)}{de} < 0 \). Now consider,

\[
-\int_{\hat{c}}^{\bar{c}} \left( \frac{db(c)}{dc} + (\hat{\theta} + \Delta \hat{\theta}) \frac{d[1 - q(c)]}{dc} + \frac{ds(b(c))}{dc} \right) \frac{dG(c)}{de} dc \text{ at } e = 0.
\]
Simplifying this equation, we get:

\[- \int_\xi \left( \frac{db(c)}{dc} + (\hat{\theta} + \Delta \hat{\theta}) \frac{d[1 - q(c)]}{dc} + \frac{ds[b(c)]}{dc} \right) \frac{dG(c)}{de} dc \]

\[= - \int_\xi \left( \frac{db(c)}{dc} + \hat{\theta} \frac{d[1 - q(c)]}{dc} + \frac{ds[b(c)]}{dc} \right) \frac{dG(c)}{de} dc - \int_\xi \left( \Delta \hat{\theta} \frac{d[1 - q(c)]}{dc} \right) \frac{dG(c)}{de} dc \]

\[= - \int_\xi \left( \frac{db(c)}{dc} + \hat{\theta} \frac{d[1 - q(c)]}{dc} + \frac{ds[b(c)]}{dc} \right) \frac{dG(c)}{de} dc + \int_\xi \left( \Delta \hat{\theta} \frac{dq(c)}{dc} \right) \frac{dG(c)}{de} dc < 0. \]

Therefore, it follows that the buyer decreases its procurement cost with an increase in the qualifying effort. The converse argument holds for step 2.

For step 3, the buyer will be indifferent between offering an increasing scheme and a decreasing scheme if and only if its objective function is such that

\[- \int_\xi \left( \frac{db(c)}{dc} + \hat{\theta} \frac{d[1 - q(c)]}{dc} + \frac{ds[b(c)]}{dc} \right) \frac{dG(c)}{de} dc = 0 \text{ at } e = 0. \]

Alternatively,

\[\hat{\theta} = - \frac{\int_\xi \left( \frac{db(c)}{dc} + \frac{ds[b(c)]}{dc} \right) \frac{dG(c)}{de} dc}{\int_\xi \left( \frac{d[1 - q(c)]}{dc} \right) \frac{dG(c)}{de} dc} = \theta_0. \]
3.9.6 Proof of Example 1 Results

Note that \( q(c) = \alpha c \) implies that quality variation equals \( \alpha^2 \frac{(\tilde{c} - q)^2}{12} \)

(\( \therefore \) \( \text{Var}(q) = \text{Var}(\alpha c) = \alpha^2 \text{Var}(c) \) because \( \text{Var}(c) = \frac{(c - q)^2}{12} \)). Now, \( \frac{db_w}{dc_w} = \frac{n}{n+1} \) and

\[
\frac{dI[\varphi(c_w)]}{dc_w} = -\alpha \theta. \quad \text{Therefore,} \quad \frac{db_w}{dc_w} + \frac{dI[\varphi(c_w)]}{dc_w} > 0 \quad \text{reduces to} \quad \frac{n}{n+1} - \alpha \theta > 0, \quad \text{and}
\]

\[
\frac{db_w}{dc_w} + \frac{dI[\varphi(c_w)]}{dc_w} < 0 \quad \text{reduces to} \quad \frac{n}{n+1} - \alpha \theta < 0. \quad \text{Using elementary algebraic manipulations and the fact that} \quad \text{Var}(q) = \alpha^2 \frac{(\tilde{c} - q)^2}{12}, \quad \text{we obtain:}
\]

\[
\frac{n}{n+1} - \alpha \theta > 0 \quad \Rightarrow \quad \text{Var}(q) < \frac{1}{\theta^2 \left( \frac{n}{n+1} \right)^2} \frac{(\tilde{c} - q)^2}{12}, \quad \text{and}
\]

\[
\frac{n}{n+1} - \alpha \theta < 0 \quad \Rightarrow \quad \text{Var}(q) > \frac{1}{\theta^2 \left( \frac{n}{n+1} \right)^2} \frac{(\tilde{c} - q)^2}{12}.
\]

Therefore, when \( \text{Var}(q) < \frac{1}{\theta^2 \left( \frac{n}{n+1} \right)^2} \frac{(\tilde{c} - q)^2}{12} \), the buyer must offer a decreasing scheme to the market maker, and when \( \text{Var}(q) > \frac{1}{\theta^2 \left( \frac{n}{n+1} \right)^2} \frac{(\tilde{c} - q)^2}{12} \), the buyer must offer an increasing scheme to the market maker.
3.9.7 Proof of Example 2 Results

Note that \( q(c) = \alpha \sqrt{c} \) implies that the quality variation equals \( \alpha^2 \text{Var}(\sqrt{c}) \). Now,

\[
\frac{db_w}{dc_w} = \frac{n}{n+1}, \quad \text{and} \quad \frac{dII(q(c_w))}{dc_w} = -\frac{\alpha \theta}{2\sqrt{c}}. \quad \text{Therefore,}
\]

\[
\frac{db_w}{dc_w} + \frac{dII[q(c_w)]}{dc_w} > 0 \quad \Rightarrow \quad \frac{n}{n+1} - \frac{\alpha \theta}{2\sqrt{c}} > 0 \quad \forall \ c \in [\xi, \bar{c}], \quad \text{and}
\]

\[
\frac{db_w}{dc_w} + \frac{dII[q(c_w)]}{dc_w} < 0 \quad \Rightarrow \quad \frac{n}{n+1} - \frac{\alpha \theta}{2\sqrt{c}} < 0 \quad \forall \ c \in [\xi, \bar{c}].
\]

In other terms,

\[
\frac{n}{n+1} - \frac{\alpha \theta}{2\sqrt{c}} > 0 \quad \forall \ c \in [\xi, \bar{c}] \quad \Rightarrow \quad \frac{\alpha \theta}{2\sqrt{c}} < \frac{n}{n+1} \quad \forall \ c \in [\xi, \bar{c}], \quad \text{and}
\]

\[
\frac{n}{n+1} - \frac{\alpha \theta}{2\sqrt{c}} < 0 \quad \forall \ c \in [\xi, \bar{c}] \quad \Rightarrow \quad \frac{\alpha \theta}{2\sqrt{c}} > \frac{n}{n+1} \quad \forall \ c \in [\xi, \bar{c}].
\]

Because \( \bar{c} > \xi \), it follows that

\[
\frac{\alpha \theta}{2\sqrt{\xi}} < \frac{n}{n+1} \quad \Rightarrow \quad \frac{\alpha \theta}{2\sqrt{\xi}} < \frac{n}{n+1} \quad \forall \ c \in [\xi, \bar{c}], \quad \text{and}
\]

\[
\frac{\alpha \theta}{2\sqrt{\bar{c}}} > \frac{n}{n+1} \quad \Rightarrow \quad \frac{\alpha \theta}{2\sqrt{\bar{c}}} > \frac{n}{n+1} \quad \forall \ c \in [\xi, \bar{c}].
\]

Using elementary algebraic manipulations and the fact that \( \text{Var}(q) = \alpha^2 \text{Var}(\sqrt{c}) \), we recognize that if \( \text{Var}(q) < \frac{1}{\theta^2} \left( \frac{n}{n+1} \right)^2 4 \xi \text{Var}(\sqrt{c}) \), then the buyer should offer a decreasing scheme to the market maker. In contrast, if \( \text{Var}(q) > \frac{1}{\theta^2} \left( \frac{n}{n+1} \right)^2 4 \bar{c} \text{Var}(\sqrt{c}) \), the buyer should offer an increasing scheme to the market maker.

Now consider the condition involved in part (C) of Proposition 2,
Substituting \( q(c) = \alpha \sqrt{c} \) and using the fact that \( \text{Var}(q) = \alpha^2 \frac{(c - \bar{c})^2}{12} \), we get the following:

\[
\theta_0 = -\frac{\int \left( \frac{db(c_w)}{dc_w} + \frac{ds[b(c_w)]}{dc_w} \right) dG(c_w) \frac{dc_w}{de}}{\int \left( \frac{d[1 - q(c_w)]}{dc_w} \right) dG(c_w) \frac{dc_w}{de}}.
\]

or

\[
\theta_0 = -\frac{\int \left( \frac{db(c_w)}{dc_w} + \frac{ds[b(c_w)]}{dc_w} \right) dG(c_w) \frac{dc_w}{de}}{\int \left( \frac{d[1 - \alpha \sqrt{c_w}]}{dc_w} \right) dG(c_w) \frac{dc_w}{de}}.
\]

or

\[
\alpha = \frac{\int \left( \frac{db(c_w)}{dc_w} + \frac{ds[b(c_w)]}{dc_w} \right) dG(c_w) \frac{dc_w}{de}}{\theta_0 \int \left( \frac{d\sqrt{c_w}}{dc_w} \right) dG(c_w) \frac{dc_w}{de}}.
\]

In turn, \( \theta > \theta_0 \) implies that \( \text{Var}(q) > t\left( \frac{1}{\theta_0} \right) \), and \( \theta < \theta_0 \) implies that \( \text{Var}(q) < t\left( \frac{1}{\theta_0} \right) \).

Therefore, the required conclusion follows.
3.10 References


Murphy, Kevin (2003), “Stock Based Pay in New Economy Firms,” Journal of Accounting and Economics, 34 (1-3), 129


4. Conclusion

This dissertation has provided plausible answers to the two business issues posed in the first chapter. With Chapter 2, we have demonstrated how customer preferences, network externalities, switching costs, and entry timing interact to determine the outcome of market leadership in the competition between standards. As is indicated by our analysis, being the first mover in a technology market does not necessarily lead to market leadership. In addition, arriving later in a market with an inferior standard does not necessarily mean that the inferior standard has no chance to become the market leader. The network nature of technology markets is such that even an inferior standard that enters later can go on to become the market leader. Some anecdotal evidence, albeit controversial, suggests that inferior standards sometimes go on to become market leaders (e.g., VHS, QWERTY keyboards). Further empirical studies in the context of standards competition should be able to shed even more light on the issues we consider in Chapter 2.

The analysis included in Chapter 3 shows that buyers sometimes need to offer compensation schemes that, at first glance, seem counter to their best interests (e.g., percentage of the contract value). However, as our analysis suggests, the buyer must carefully analyze the trade-off between short-term gains (i.e., lower prices) and long-term gains (i.e., lower quality costs). Depending on the procurement situation, the buyer may need to sacrifice short-term gains in return for long-term gains, or vice versa. We require further research to test the robustness of our findings and to design optimal compensation schemes in general.
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