A COMBINED EMPIRICAL AND COMPUTATIONAL APPROACH TO CREEP IN REPLICA S OF HISTORIC MORTAR

A Thesis in Architectural Engineering by Sally Jean Gimbert

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ABSTRACT

Experimental, analytical and computational methods of analysis are applied to the nonlinear problem of creep in historic masonry. Twelve (12) historically replicated mortars are studied in two (2) sets of six (6) stacked masonry prisms. In addition to the use of a ‘crushed brick’ pozzolan, three (3) popular Middle Ages additives, eggs, beeswax and beer, are also studied across the masonry sets.

The mortars are placed under a compressive configuration at a constant stress of 309psi (2.13MPa) for 120 days. Elastic and creep strains are monitored; shrinkage is also noted as a competing variable to the magnitude of creep. Average strains generate a homogenized performance trend; a viscoelastic analytical model is matched to this trend as a predictive mathematical description. The experimental data is found to closely match a modified Burger’s body. Further, a set of material properties are furnished from the derivation of the model including Young’s modulus of Elasticity, $E_b$; the Bulk modulus, $K_b$; the Elastic Shear modulus, $G_2$; and other suitable variables to describe the viscosity; $G_1$, $\eta_1$, and $\eta_2$.

The analytical results are implemented as a constitutive law in the commercial program ANSYS by using implicit creep numerical integration schemes. Following, the nonlinear material formulation of the mortar is employed in five (5) preliminary time-dependent models of various solid-unit volume fractions. The numerical results are translated into an overall effective Modulus of Elasticity. These stiffness values are to be implemented as viable numerical modeling references in holistic deformability models of historical masonry structures.
# TABLE OF CONTENTS

LIST OF FIGURES .......................................................................................................................... vii

LIST OF TABLES ............................................................................................................................. ix

ACKNOWLEDGMENTS ...................................................................................................................... x

Chapter 1 A Study of Creep in Replica Mortars of Historical Masonry .............................................. 1
  1.1 Background and Scope .................................................................................................................. 1
  1.2 Problem Scope and Definition .................................................................................................... 3
  1.3 Objectives .................................................................................................................................. 5
  1.4 Research Methodology ................................................................................................................ 7
  1.5 Outline of Study .......................................................................................................................... 8

Chapter 2 The Behavior and Analysis of Historic Mortars ................................................................. 10
  2.1 The Definition of Historic Mortars ........................................................................................... 10
    2.1.1 The Interactive Role of Mortar in Masonry ......................................................................... 10
    2.1.2 The Composition of Historic Mortars ................................................................................. 11
      2.1.2.1 A Brief History of ‘Lime-Based’ Cementitious Materials ........................................... 11
      2.1.2.2 The Use of ‘Organic’ Admixtures ............................................................................... 12
    2.2 Analysis of ‘Lime-Based’ Historic Mortars .............................................................................. 14

Chapter 3 The Phenomenon of Creep .............................................................................................. 18
  3.1 The Engineering Concept of Creep ............................................................................................ 18
    3.1.1 Phases of Creep Behavior .................................................................................................... 19
    3.1.2 Basic Mathematical Representations of Creep ................................................................. 20
    3.1.3 Pertinent Studies on Basic Creep in Masonry and Cementitious Materials ..................... 22
  3.2 Creep in Historical Masonry Structures .................................................................................... 24
    3.2.1 Early and Delayed Deformations ......................................................................................... 24
    3.2.2 Long-Term Behavior, Creep before Collapse .................................................................... 25
    3.2.3 Observed Damage on Historical Masonry Structures ....................................................... 26
  3.3 Analysis of Creep, Time-Dependency in Historical Masonry ................................................... 29
    3.3.1 In-Situ Experimental Studies .............................................................................................. 29
    3.3.2 Analytical Studies, Parametric Theory Applications .......................................................... 30

Chapter 4 Experimental Analysis ..................................................................................................... 36
  4.1 Significance and Use .................................................................................................................... 36
  4.2 Experimental Apparatus ............................................................................................................. 37
  4.3 Test Prisms ................................................................................................................................ 39
    4.3.1 Prism Size and Organization ............................................................................................... 39
    4.3.2 Mortar ................................................................................................................................. 41
      4.3.2.1 Lime Selection .............................................................................................................. 42
      4.3.2.2 Sand Conformity .......................................................................................................... 43
      4.3.2.3 Pozzolan, ‘crushed brick’ ............................................................................................ 43
      4.3.2.4 Additives ....................................................................................................................... 44
    4.3.3 Mortar Matrix ...................................................................................................................... 45
    4.3.4 The Fabrication Process ....................................................................................................... 45
LIST OF FIGURES

Figure 1.3-I: Objective B, analyze the experimental data for behavioral trends.........................7
Figure 1.3-II: Objective A, experimentally determine and confirm creep characteristics. ............7
Figure 3.1-I: Regions of creep behavior; (a) primary, (b) secondary, (c) tertiary.......................19
Figure 3.1-II: Linear Viscoelastic Models showing graphical response to creep tests. ..............21
Figure 4.2-I: Test setup, a schematic of the creep apparatus without masonry prisms...............38
Figure 4.2-II: Test setup: [a] the threaded rod in place and, [b] the load cell in place.............39
Figure 4.3-I: Standard bricks: [a] Three-dimensional view; [b] Brick measurements (mm). .......40
Figure 4.3-II: Prism 01.004 with mortar joint b highlighted.............................................41
Figure 4.3-III: An Illustration of the Mortar Matrix ...........................................................46
Figure 4.4-I: Test apparatus showing prisms 001 thru 006..................................................48
Figure 4.4-II: Application of initial load onto the second series, February 24, 2008.............50
Figure 4.4-III: Twelve control mortar specimens...............................................................50
Figure 4.4-IV: Layout of Demec System using 50mm (2") and 100mm (4") gauge lengths.....51
Figure 4.5-I: Graphic Results of the Pozzolan Lime Sand Mortars, Prism Series_01 ..........53
Figure 4.5-II: Graphic Results of the Lime Sand Mortars, Prism Series_02 .........................54
Figure 4.5-III: The Experimental Creep Behavior; An Averaged Performance Trend ..........57
Figure 4.5-IV: Examples of shrinkage rates over the primary phase.................................60
Figure 4.6-I: The Experimental Creep Behavior; An Averaged Performance Trend ..........69
Figure 5.2-I: The Initial Strain Phase, showing $\varepsilon_i$ ......................................................72
Figure 5.2-II: The Primary Phase, showing $\varepsilon_t$ and a theoretical $\varepsilon_t$ .........................72
Figure 5.2-III: The Secondary Phase, showing $\varepsilon_{ss}$ .....................................................73
Figure 5.3-I: Creep in uniaxial compression under deviatoric stress; Burger’s body..........76
Figure 5.3-II: Plot of Log $q(t)$ .........................................................................................80
Figure 5.3-III: The Homogenized Behavior, A Modified Burger’s Body .........................83
Figure 5.4-I: Experimental and Analytical Results ..............................................................85
Figure 7.1-I: Geometry and Meshed Elements of 8%-92% Masonry ANSYS model.........................100
Figure 7.1-II: Curve Fitting results used to define the mortar material model. ..........................104
Figure 7.2-I: Numerical v Analytical Results of 8%-92% Masonry ANSYS model.....................105
Figure 7.2-II: Average inelastic strain in the mortar along the z axis ........................................106
Figure 7.2-III: Average creep strains in the mortar along the z axis @ 120 days ..........................106
Figure 7.3-I: Effective Modulus of Elasticity Chart, $E_{ref}$ v Time ........................................109
Figure 7.3-II: $E_{ref}$ Chart over the first 5 years ........................................................................109
Figure 7.3-III: Mortar Volume Fraction v. Effective Modulus of Elasticity .................................110
Figure A-I: The nave of the Beverley Minster, Beverley, England ............................................126
LIST OF TABLES

Table 2.1-A: List of ‘Organic’ Materials and their date of implementation...........................................13
Table 3.2-A: Observations on the State of Historical Masonry Structures .............................................28
Table 3.3-A: Common Theories and Models applied to Masonry as Constitutive Laws......................31
Table 4.3-A: A Summary of the Mortar Constituents..............................................................................42
Table 4.3-B: Sand Conformity by ASTM C144.........................................................................................43
Table 4.3-C: The Mortar Matrix, a list of the Mortar Mixes.................................................................45
Table 4.3-D: Mortar Batch Factors and Weight of Materials.................................................................46
Table 4.4-A: Observed Range of Compressive Stresses in Historical Masonry Structures ..............48
Table 4.5-A: Results and Comparisons of Lime-based Mortar Strength Properties .....................55
Table 4.5-B: Summary of the Percentage of Shrinkage to Overall Strain...........................................58
Table 4.5-C: Summary of Absolute Differential in Creep and Phase Creep Rates..............................62
Table 5.4-A: Summary of Homogenized Material Properties [MP] of the Mortars .......................83
Table 5.4-B: Numerical Comparison of Experimental and Analytical Results..................................88
Table 7.1-A: Linear Elastic Material Properties for Solid / Brick Units...........................................102
Table 7.1-B: Linear Elastic Material Properties for Mortar....................................................................103
Table 7.1-C: Coefficients for Implicit Creep Equation in ANSYS......................................................105
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Chapter 1

A Study of Creep in Replica Mortars of Historical Masonry

1.1 Background and Scope

A majority of technical literature states that masonry, the oldest of construction materials, is the least understood in terms of its strength and deformation properties. Indeed, the definition of masonry in terms of its structural and non-structural performance is often obscured and leads to confusion in practice and design. In the study of historical masonry, it is also common to find many ambiguities due in part to the extensive and/or the irretrievable state of historical records. Today, research into the art of masonry continues to present new theories and techniques that delight and intrigue both modern engineers and historians alike.

Over the past twenty years, there have occurred an increased number of failures in medieval structures, particularly of towers and massive masonry buildings under heavy persistent loads. While collapse is infrequent, the series of events have prompted new studies to re-evaluate the manner in which ancient masonry responds to its large self-weight. It is well known that the construction techniques employed by ancient builders were relatively effective and qualified intelligent methods. The discovery of alarming stress states at historic sites, and in some cases, serious signs of imminent collapse, are forcing analysts to look beyond the traditional stress-strain laws conventionally applied to masonry structures. The objective is to further understand long-term response that is not fully explained by theories of elasticity, plasticity and limit-state analysis. From inspection and experimentation on masonry assemblies, collective data trends show evidence that masonry does have the ability to creep over time; its long-term performance more closely replicated by viscoelastic models (Binda et al. 1991).

While simple elastic and analytical methods in masonry have been used for decades to understand the non-homogenous material, new performance-based methods and advanced techniques in computational analysis have also allowed masonry engineering practice to continue evolving at an equivalent rate to that of other traditional materials such as steel, timber and reinforced concrete. Using Finite Element Analysis (FEA), researchers develop time-dependent models to
accurately reproduce the complex structural behavior of masonry. These computational models can simulate long-term structural effects imposed by the construction process, building alterations and/or changing states of physical damage. Simultaneously, the modeling allows analysts to understand plausible scenarios of structural response over periods of time extending through centuries. However, while numerical models are a valid and widespread means for assessment of historical masonry, the results can be called into question because the models, and in turn the analysts, lack accurate quantitative and qualitative inputs needed to produce solutions that are later validated through comparison to in situ structural conditions. Often, holistic structures of historical masonry in FEA are modeled as a single smeared material with induced linear properties. This is often done due to limitations on file size and the element discretization. Analysts can employ constitutive laws to increase the sophistication and induce a nonlinear response. This, however, can be time consuming and futile. Improvements to qualitative inputs can be made to better represent the time dependency of masonry materials in Finite Element models.

In a modern analysis of historical masonry, it is not always easy to accurately characterize the strengths and deformability properties of its materials. Those materials used to construct historical masonry seriously lack measured and defined standards. In regards to computational analysis, the selection and typology of the materials can dictate the accuracy of the results and the overall understanding of the structures response. To better define and predict deformability and life-cycle solutions, it is apparent that further knowledge about lime-based historic mortars, in particular their creep behavior, is needed to more truthfully depict the response of masonry as a time-dependent material in finite element analysis.

The mortar material particularly has a large impact on the overall behavior and performance of a masonry structure. It is critical to note that over 60 to 80 percent of the overall deformation in masonry wall occurs in the mortar joint (Sickels 1988). Most studies on mortars in general employ Portland cement as the main binding ingredient. Studies on historical masonry have addressed long-term response from a holistic standpoint, testing on sample masonry prisms. Historic mortars, however, are often not properly considered in the analysis of masonry because of the limited knowledge regarding its basic properties and performance.

By taking a close look at the time-dependent performance of creep in historically replicated mortars, this research develops structured computational references based on simple
mathematical laws to improve the methods by which analysts represent material properties as performance-based finite element analysis inputs in models of historical structures. An analytical model and its applicability to a numerical ‘homogenizing’ technique shown in this study are projected to enhance the ability of the analyst to address creep phenomenon in historic masonry.

1.2 Problem Scope and Definition

The scope of this thesis is specifically derived from a series of questions regarding the structural degradation of two prominent structures built in the Gothic architectural styles: Santa Maria Novella in Italy and Beverley Minster in the United Kingdom.

At Beverley Minster, construction began in 1288. And while it is as a parish church, it clearly has the stature of a cathedral. The 11-bay division in the nave accentuates its 66ft high quadripartite vaulting. It is visually obvious, however, that the vault structure is losing curvature. The nave walls have undergone serious displacement, causing the vaults in the main arcade to flatten at the crown in the main arcade and separate from the upper clerestory walls. The Basilica of Santa Maria Novella is described as the first example of the Florentine gothic style, but with clear Romanesque and Cistercian origins. Its construction began in 1279 with various pauses along the way. Again, peering down the main nave, it is possible to discern that the upper clerestory walls are splaying outward.

Through the completion of this study, two variables can be more accurately compared at each aforementioned location, the rate at which the foundation settles and, by contrast, the rate of long-term creep behavior in the overall stonework, more importantly, in the component with significant impact on deformation – the mortar. The magnitude of deformation over time based on the rate comparisons can simultaneously be achieved.

Further, the importance of this study is also highlighted in the field of sustainability, where the paradigm of modern science has successfully integrated stiffer and stronger mortars into the building industry, claiming their fast setting abilities and reduction in construction time. The return to vernacular techniques is fast becoming a viable solution to our energy and environmental concerns. In fact, the low compressive strength of softer, historical mortars can enhance the overall performance of a masonry structure. Van Balen et al. (1995) provided
evidence stating that hardened mortars may reduce the durability and long-term performance of masonry. Henceforth, this thesis can also be seen as an informative study about lime-based products and can be understood as further evidence toward a successful integration of lime mortars as a ‘green’ building method in today’s construction field.

This thesis addresses two sets of problems in the field of structural engineering and architectural preservation.

**Architectural History.** The construction techniques used by ancient builders were relatively effective and qualified as coherent and intelligent mechanical methods. However, the need to understand masonry as a time-dependent material is emphasized by the recent series of sudden failures over the past twenty years. Indeed, this study marks the significance of deformation in the mortar with respect to its creep properties and consequently extends our understanding of the role in which mortar plays on the overall deformable state of historical masonry. This will be accomplished by addressing the difficulties of modeling the nonlinear nature of the masonry, particularly with respect to the material properties of historic lime-based mortars and their subsequent viscoelastic behavior and attempting to answer the following questions: with the current state of masonry, how can we further understand the performance of masonry as a time-dependent material? knowing that the mortar joint exhibits large deformation, does creep in the mortar have a significant impact on the overall behavior of the structure?

**Structural Analysis.** Structural mechanisms in historical masonry are most successfully analyzed when a diagnostic evaluation closely associated with unique properties of the structure have been formulated through computational and quantitative assessment. Unknown material properties and continuous modifications to historical structures make the task of prediction difficult for modelers and engineers alike. When neither the sequence of damage nor the effectiveness of previous repairs are known, the results of intervention are often futile; continued modifications to the structure add complex layers to the behavior of the existing structure, making the task of prediction difficult for modelers and engineers alike. The need to address nonlinear masonry simulations for predictive purposes is highlighted by recent failures. Making solid observations and assumptions about the nature of materials used during the medieval / middle ages is an important step for the success of these numerical model since it reduces subjective analysis and improves the accuracy of results. In turn, current predictions regarding the life span of historical structures can be better analyzed. Research regarding the performance of 500-year-old mortars is
rare. Literature provides a summary of historic admixtures to mortars including common pozzolans, air entrainers and plasticizers used over the ages. Using this as a beginning point, the study initiates a database regarding the performance trends and the mechanisms of these historic lime-based mortars. The study, therefore, attempts to answer; how can we improve the ability of the analyst to effectively and accurately simulate holistic masonry building with apparent nonlinear behaviors? can there be effective, non-futile numerical methods?

1.3 Objectives

To address the previously defined problems, this thesis will accomplish objectives that contribute information to three areas of research:

1. Knowledge about the effects of creep within the framework of historical masonry built in the medieval time period.
2. Methods to improve the performance of computational historical masonry models; to assess the performance of creep in mortar within the parameters of Finite Element Analysis.
3. Knowledge about the mechanics and behavior of lime-based (historic) mortars.

The first phase of research is accomplished through Objective A.

1.A To experimentally determine and confirm the creep characteristics of lime-based historic mortars.
   1.A.1 To offer the structural engineering field a standardized approach for the creation of lime-based historic mortars.
   1.A.2 To design a testing frame to successfully evaluate creep in lime-based historic mortars.
   1.A.3 To record the primary and secondary phases of creep strain in historic mortars over time.
   1.A.4 To understand the parameters of studying real time creep.

The second phase of research is accomplished through Objective B and Objective C as illustrated in [Figure 1.3-I] and [Figure 1.3-II], respectively.

2.B To assess a homogenized representation of the overall creep performance in historic lime-based mortars.
2.B.1 To select a mathematical model based on viscoelastic principles to describe the primary and secondary phases of creep in lime-based historic mortars.

2.B.2 To write the analytical body to describe the strain behavior over time.

2.B.3 To generate a set of creep strain data over time from the analytical model.

2.C To improve the simulations of historical masonry in Finite Element Analysis.

2.C.1 To create simplistic geometrically accurate three dimensional finite element models of unreinforced masonry.

2.C.2 To calibrate the FEA models with the experimental data of Objective B as a means of validation.

2.C.3 To determine an appropriate and accurate method that implements the analytical model of Objective B into FEA models.

2.C.4 To provide an advanced method of a refined numerical simulation technique for the study of historical masonry structures.

The Objective D provides a summary of those points throughout this study that contribute to the third phase of research. In general, the entity of this thesis adds to the knowledge about the behavior of lime-based historic mortars.

3.D To present a collection of findings regarding the mechanics, behavior and properties of lime-based historic mortars.


3.D.2 To suggest to the structural engineering field a standardized approach for the creation of lime-based historic mortars.

3.D.3 To establish a set of material parameters on lime-based historic mortars, for reference within the field of masonry and finite element analysis.

3.D.4 To summarize substantial findings in regards to the use of historic additives in lime-based mortars so that these constituents may be considered as a sustainable practice in today’s industry.
Figure 1.3-I: Objective B, analyze the experimental data for behavioral trends.

Figure 1.3-II: Objective A, experimentally determine and confirm creep characteristics.

1.4 Research Methodology

A moderate experimental program was devised in which particular portions and combinations of sand, lime and pozzolan were established as reasonable replicas of historic mortar. Selective admixtures were also used in conjunction to the aforementioned materials. Twelve (12) mixes
were researched in a final matrix. The various combinations were representative of what was deemed commonly admissible mortar during the Middle Ages. Six (6) masonry prisms were created with the basic formula of lime and sand in conjunction with the select additives. The remaining six (6) prisms were created with pozzolan as the main binder in addition to sand, lime and the additives. After understanding the parameters to study real time creep, a testing frame was designed and built in the laboratory. The configuration of the prisms and the testing frame follow examples available in the literature and ASTM standards. Over a period of approximately 120 days, axial strains across the mortar joint were monitored. The readings were adjusted for shrinkage; the remaining magnitude of strain was recorded as the creep strain behavior and was illustrated in graphical form for further analysis.

An averaged representation of the overall creep performance was determined by analyzing the experimental data over time for distinct material properties and statistical behavioral trends of the twelve mortar prisms. Following, the averaged ‘homogenized’ strains were related to a basic viscoelastic rheological model. This is considered an appropriate mathematical body to describe creep behavior within the parameters of the material mechanics of this study. The devised equation is envisioned to be representative of a constitutive law applicable to computational simulations.

After a review of literature for specifications on finite element modeling parameters and other technical procedures regarding the simulation of creep in ANSYS, the homogenized viscoelastic creep behavior was simulated in simple three-dimensional FE models. The models were calibrated noting linear deformability and correlation to the experimental data. Inserting the effects of creep in the mortar joint, an effective modulus of elasticity is determined for the given time period of 120 days. This is deemed representative of a valid numerical simulation technique for analysts; this effective variable can later be applied in holistic models of historical masonry as a reliable representation of its overall deformability under self-weight, and other structural actions.

1.5 Outline of Study

To complete the scoped objectives, the study is accomplished through three general phases of research: [1] an experimental program, [2] an analytical approach, and [3] numerical simulations.
The empirical results are used to understand further the performance of lime-based historic mortars, and, later, to derive a homogenized mathematical model. Following, this model is applied as a nonlinear computational tool in FEA to further sensitize the complex results obtained for the analysis of historical masonry structures.

To summarize the importance of this study, Chapter 2 provides a detailed literature review on the historic definition and the behavior of lime-based mortars of the Middle Ages. The chapter summarizes the role of the mortar joint and provides a briefing on the use of additives in historic mortars. Section 2.2 introduces precedence on the research exploring the role of lime in cementitious materials. Further, Chapter 3 expands the literature review by defining the phenomenon of creep and its pertinence in the field of historic masonry. A number of authors who have begun analytical studies on the time-dependent nature of masonry are listed in Section 3.4.

The experimental phase of this study is detailed in Chapter 4. Sections 4.1 through Section 4.4 lay out the prism preparation, the testing setup and the experimental procedure. Section 4.5 completes this chapter with an extensive look at the results of the experiment over the 120-day time period, including a discussion on the roles of the additives on the strain behavior. Chapter 5 is a detailed analysis on the selection of an appropriate viscoelastic body to match the experimental creep strains. In addition, a section is dedicated to the comparison of the empirical and the derived results.

Before launching into the computational analysis, Chapter 6 completes the literature review by summarizing appropriate methods of Finite Element Analysis and the role that it plays as a prominent computational tool for understanding the numerous complexities of historical masonry. Chapter 7 is the final iteration. It describes the simulation of the derived mathematical model in preliminary FE models that are based on simple geometric and quantitative forms.

Finally, Chapter 8 provides conclusions to the study and brief summary for future research and the transmissibility of the findings. A section is dedicated to the suitability of a future case study of Beverley Minster in the United Kingdom.
Chapter 2

The Behavior and Analysis of Historic Mortars

2.1 The Definition of Historic Mortars

Masonry is an assembly-based construction. In the Middle Ages and medieval times, it was often an unreinforced structural material; a solid unit and a mortar fill act as a composite material to resist self-weight, lateral pressure and service loads. Units are usually a stone or brick material and are stacked together with mortar that acts as a cushioning unit. The presence of a joint makes masonry an inhomogeneous system. A masonry assembly may be considered to act as a rigid body where its compartmentalized components, with individual material properties and mechanisms, control the overall behavior of the structure. The movement of individual units determines the movement of the structure as a whole.

The challenge in assessing the structural integrity of masonry is in quantifying the deformation differentials between units and the mortar joints. While each single masonry unit may exhibit familiar behavior, the combination of irregular materials in various configurations induces a rather complex structural state; each component strains in a different manner due to the various stress distributions in their physical properties (Binda et al. 1991).

2.1.1 The Interactive Role of Mortar in Masonry

Mortar is employed as a bonding agent between the solid units of the masonry; it aligns uneven surfaces, provides protection from moisture penetration and aids in resisting external and internal lateral forces. One of its primary functions is to distribute loads evenly throughout the structure. With good composite properties, this is accomplished by conforming to irregularities of the blocks and by undergoing plastic deformation during the life of the building.
2.1.2 The Composition of Historic Mortars

A majority of the buildings under investigation and found to be susceptible to long-term damage and/or collapse are of the medieval time period. The composition of sand and lime was often employed during the Middle Ages when Romanesque and Gothic Architecture reigned as the supreme style. Henceforth, an historic mortar in the scope of this study is replicated as a cementitious paste with its basic ingredients being sand and non-hydraulic lime putty. Regardless of this scope, an historical mortar is often defined as a ‘lime-based’ composite with sand and pozzolan as additives, its formulae specifically inherited from ancient roman times. (Sickels 1998; Velosa et al. 2001; Taylor 2001).

2.1.2.1 A Brief History of ‘Lime-Based’ Cementitious Materials

Physically, mortar is defined as a ‘cement,’ capable of binding small mineral fragments into a compact composite that exhibiting both adhesive and cohesive properties. Cement has been in use for thousands of years. Evidence of its existence in Israel dates back to over 12,000 BC where, through a process of spontaneous combustion, deposits of limestone and oil shale are said to have formed a cementitious compound. It was known that when lime was used as the binding agent, a strengthened cement developed through the process of carbonation. To follow, the Assyrians and Babylonians used the naturally occurring binder in brick and gypsum construction; and the Egyptians further improved the lime-gypsum technology (Sickels 1988). The combination of lime and sand as a mortar was revolutionary.

The main reactive bitumen, lime, was also well known to the Greeks. However, it was the Romans who discovered and noted its hydraulic property. Roman cements and concrete were primarily formed by the combination of slaked lime and pozzolan, a siliceous tuff found in volcanic zones, most notably from Pozzuoli near Naples (Lane 2000). The methods and mixtures used by architects and artisans of the roman era were substantiated by the writings of Vitruvius. His handbook, De Architectura, documents the techniques and proportions of the ancient sand-lime formulae, noting the significance of the important additive, pozzolan. When pozzolan was added to the basic sand-lime ingredients, strength developed through a chemical reaction between fine siliceous and/or aluminous materials and hence did not require the presence of carbon.
dioxide from the air. A lime-pozzolan-sand mix is therefore said to be hydraulic because of its ability to set under water.

For a period of time, the art of ancient mortar was forgotten. By the Renaissance, its usefulness had been re-discovered and masons of the era created some of the world’s most magnificent masonry structures. For centuries, it was accepted that the strongest lime was pure white and came from the hardest of limestone. By the late eighteenth century, a number of debates sparked a century long discussion about the validity of this fact. Scientists became concerned with three main issues: [1] the chemistry of lime and what composition determined hydraulicity; [2] the quality of mortar acquired by the addition of additives, either those prepared by man, such as iron fillings, or those of natural origins, such as pozzolan; and [3] the care of mortar materials and its effect on the ultimate use (Sickels 1988, p.3). In 1756, John Smeaton, aiming to produce a mortar that would withstand salt water washing for the construction of a lighthouse, became the first to disprove the theories previously set forth. Louis J. Vicat followed suit with experiments in 1818 to pinpoint the source of hydraulicity in clay as Smeaton had discovered previously. Together with other such as Horace-Benedict de Saussure and Tobern O. Bergmann, results linked hydraulicity to clay or silica and/or alumina found in certain limestones. In 1824, Joseph Aspdin invented what today is known as the modern Portland cement. The concrete technology has since developed over the centuries with the addition of new admixtures and mineral based substances. (Sickels 1988).

2.1.2.2 The Use of ‘Organic’ Admixtures

Lime cements dominated the industry for centuries. Due to the prohibitive costs and scarcity of the material over the ages, mortar was often created with supplemental ‘organic’ compounds to balance out the degradation or lack of the employed lime. These additives are no longer in use today, but their inception has, within the last half a century, introduced ‘synthetic mortars’ as a derivative of the ‘organics,’ both chemically and physically. While the use of ‘synthetics’ is relatively new, ‘organics’ have been globally employed as successful admixtures for over 2000 years [see Table 2.1-A]. While these additives are considered to be no longer of consequence in today’s market, their re-invention could establish resurgence in the vernacular, a leading concept for future sustainable practices.
### Table 2.1-A: List of ‘Organic’ Materials and their date of implementation

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<tr>
<th>Constituent</th>
<th>Egyptian BC</th>
<th>Vetusius BC</th>
<th>Pliny AD</th>
<th>Middle Ages</th>
<th>Beverley</th>
<th>Basileia</th>
<th>Santa Maria</th>
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<th>Neve, Moxon 1653</th>
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* Specific materials used in this study

The properties each ‘organic’ was to impart has been documented over the ages. One commonly mentioned property was that of the setting quality and the ability of an ‘organic’ to “…. ‘regulate’ the set or ‘retard’ the set of mortar” (Sickels 1988 p. 46). Blood and egg whites were employed as popular means to both retard and increase workability of the set. Some retarders also carried the property of increased hardness or durability; beer as an air entrainer, sugar as a plasticizer / mollifier, and beeswax as a non-shrinkage agent. Sugar was also known to accelerate the setting, along with others items such as rye dough, barley water, elm bark, starch, hogs’ lard, and curdled milk.

As can be seen in Table 2.1-A, some of the most popular agents employed over the ages were blood, eggs, and the egg white. During the Middle Ages, in addition to these three most common items, the set of mortar may also have been altered by other commonalities; beeswax, beer, sugar, malt, rice, glutens, fruit juices and urine.

2.2 Analysis of ‘Lime-Based’ Historic Mortars

The topic of preservation continues to motivate scientists to study lime-based mortars. Some of these studies make efforts to compare sand-lime mixes to those mixes with pozzolanic constituents. Studies continue to make significant efforts to augment further understanding of the mechanical properties of mortars, often investigating the hardening characteristics of industrial and lime-based mortars for appropriate repair mixtures and procedures.

Overall, most studies available in literature today do not fully address or detail the long-term performance of sand-lime-pozzolan mortars of the medieval ages. This formula, or some combination of these materials, is found to be consistently used in historical masonry buildings exhibiting nonlinear viscoelastic nature. It is proving difficult to account for the nonlinearity of masonry materials, particularly from a computational standpoint. Linear or plastic-rigid theories assume a holistic, simplified behavior that ignores the influence of time; these assumptions may not fully address the behavior of masonry as recently seen over the past few decades.

Individual studies dedicated to the topic of mortar with lime alone as the primary binding ingredient are scarce. Velosa and Veiga (2001) and Bosiljkov (2001) compared the mechanical, physical and chemical properties of hardened sand-lime mortars with pozzolanic additives. Their
studies were performed with the intent to improve the restoration process of old masonry buildings. Amid the tests to monitor the hardened mortars for environmental and climate effects, carbonation development, material strength and curing conditions, the behavioral performance of the mortars, particularly with respect to its plastic properties and creep over time was neglected.

Velosa and Veiga (2001) aimed to refine rehabilitation procedures by taking a closer look at lime-based mortars. Two selections of pozzolan from Portugal and from Cape Verde were studied. The research reports variations in the compressive and flexural strengths, obtained dynamic moduli of elasticity at 28 days and 90 days, the pozzolanic reactivity, Blaine’s specific surface, the water absorption coefficient, the drying capacity, carbonation tests and durability assessments. The results show that the Azores pozzolans did not contribute towards the mechanical properties of the lime-based mortars, the Cape Verde pozzolans, however, did significantly increase the strength of the lime-based mortars, showing values from 1.3 MPa (189 psi) to 2.0 MPa (290 psi).

Bosljikov (2001) studied closely plain lime-based mortars created using a variety of traditional, hydrated and industrial lime putties. The experiment was conducted over approximately 300 days and took note of various material properties including the air content, density, shrinkage, water retentivity, flow value, flexural strength, and compressive strength. Bosiljkov discovered a variable range of compressive strengths, from 0.95 MPa (138 psi) to 2.09 MPa (303 psi), and concludes that workmanship in the creation of the batches strongly influenced these results. He noted that the addition of small amounts of microsilica and metakaolin pozzolanic cement binders always increases the compressive strength, while their addition can increase or decrease its flexural strength, all dependent on the uniformity or particles and the amount of the pozzolanic material used. Portland cement had a retarding influence on the final strength properties of the lime-based mortar. The research made a substantial effort to note the shrinkage performance of lime-based mortars and by designing their own equipment, concluded that the deformation are 5 and 6% for plain lime mortars and around 9.5% for lime-based mortars using additives or fibers.

A majority of the studies that seek to make conclusions on the performance of mortars do so by listing portland cement as the main binder. Most of these research programs examine both lime and Portland cement as two primary ingredients, but eventually ask for further scientific study to the confirm results. The authors Neville (1970) and Leczner (1986) made significant conclusions regarding the performance of creep in mortars, masonry and concrete. Their work is discussed further in Chapter 3. However, it was Sickels (1988) who substantiated their findings
and validated the importance of their work.

Sickels (1988) devised an experimental program with seven individual mortars. The study was performed by monitoring creep, shrinkage and carbonation in mortars of a 1:1/4: 3 and a 1:1:6 volumetric ratio of portland cement : hydrated lime : sand and following the experimental precedent set by Neville and Lenczner. Sickels (1988) does however reach interesting conclusions about the effect of lime on the overall performance of mortar as previously mentioned.

With clearer intentions to evaluate the behavior of lime-based mortar, Boothby et al. (1999) monitored the performance of masonry arches, paying particular attention to the elasto-plastic hardening and shakedown behavior of the joint. The study shows that sand-lime mortars are more ductile allowing the joint to release beyond the yield state but before the collapse load.

Navedo (1995) presented results detailing sand-lime mortar joints of unreinforced masonry assemblies also, comparing analytical results assessed using basic rigid-plastic theories to numerical FEA models. Navedo reasonably correlates the FE models of voussoir arches and piers – in terms of mechanism, deformations and stability – clarifying various analytical and numerical techniques to assess lime-based masonry in light of plastic theories. The finite element models take into account linear responsive properties of sand-lime mortars and generate response in good agreement to in situ data. Both studies by Navedo and Boothby, however, do not reach a conclusion about the nonlinear long-term nature of masonry.

Conclusive information about mortar is difficult to attain because it is often erroneously assumed to behave similarly to concrete. The historical formula of mortar takes on a similar chemical combination to that of modern concrete bonding gels such that a close relationship can be observed between the calcium hydroxide molecules of portland cement and that of ancient wet lime, and between the amorphous silica of the pozzolanic fly ash and the silica of the volcanic pozzolan. While this may seemingly indicate similar behavioral qualities, studies have identified significant deterioration in concrete structures just over 100 years old due to high reactive hydration rates and over dosages of fine aggregates and cements, and / or additives. All these actions can damage bonds, cause cracking and develop spalling in the concrete, reducing its life span. In comparison, the life span of historical masonry, around 500 years, is clearly greater than that of modern concrete, around 100 years. Concrete is primarily designed to resist compressive
strength. While resisting force is a primary function of mortar, the success of its behavior relies heavily on its ability to transfer and distribute stress through the masonry composition. This is an indication that the material requires individual specifications for analyzing its long-term performance (Kirca 2005).
Chapter 3
The Phenomenon of Creep

Following the transfer of movement in masonry, the imposed stresses translate into physical alterations, notably strain. Strain is deformation measured per-unit length of change and presents itself in two forms, elastic and plastic. Elastic deformation is recoverable, meaning the material can return to its original state. If the initial state of a material is unrecoverable, then the material has undergone plastic deformation. Creep behavior is an important kind of plastic deformation in masonry structures. Through release and compensation of heightened states of stress and strain, creep allows a building to tolerate small shear, tensile and compressive movement without significant damage. It has been found however that under self-weight, massive masonry structures exhibit compressive strain that can severely accumulate. Often, this accumulation can induce excessive physical distortions in the dimension of the material.

3.1 The Engineering Concept of Creep

Most solids exhibit both immediate and delayed states of deformation. In soft, porous materials such as fresh cements, concrete, soft rocks, and stiff clays, if a constant load is applied and an increasing deformation occurs, this differential is identified as viscoelastic creep behavior. Viscosity is the phenomena of “...[a]n apparent solid material that distorts slowly and continuously in response to shearing stresses” (Goodman 1989, p.202). In solid entities, two types of mechanisms are used to describe creep action. The first, ‘mass flow,’ happens at relatively low deviatoric stress values; the body exhibits creep through internal gliding, compaction and movement of the inter- and intra-crystalline dislocations and vacancies of the material (Goodman 1989, p.202). Second and more commonly associated with harder solids like limestone, creep appears at levels of higher deviatoric stresses where crack growth is imminent. Here, the process is nonlinear since incremental steps in the stress cause new physical changes.
3.1.1 Phases of Creep Behavior

Over time, creep can be commonly subdivided into three stages: the so-called primary phase, secondary phase and tertiary phase. Accordingly, “…the appearance of one or more of these phases and the strain rate of the secondary creep phrase strongly depends on the stress level” (Binda et al. 2002, p. 3). The three phases are described in a strain versus time curve as seen in Figure 3.1-I and further elaborated on below:

[1] The initial strain of the graph is simply the elastic response of a material to the applied load (stress) and is usually noted as the engineering strain.

[2] The primary region is characterized by transient creep, indicated by a decreasing rate due to resistance of material and an increasing magnitude by virtue of accumulating deformation.

[3] The secondary region is identified by a constant creep strain rate, also known as the steady state creep where competing mechanisms of strain hardening and recovery may be present.

[4] The tertiary region is characterized by an increasing strain rate, where a constant load or constant stress causes excessive fracture prior to failure of the specimen.

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**Figure 3.1-I:** Regions of creep behavior; (a) primary, (b) secondary, (c) tertiary.
3.1.2 Basic Mathematical Representations of Creep

The three phases of creep behavior can be fitted to match mathematical exponential or power functions. The most common rheological models used to describe general viscoelastic creep in one-dimensional stressed and strained materials are the classical Maxwell model, Kelvin model, and the Burger’s model (England and Jordaan 1975; Jordaan et al. 1977; Shrive et al. 1997 (as cited in Shrive et al. 1997)). In general, these models define total strain at any time, $t$, under sustained stress, $\sigma$, by the following basic expressions (Shrive et al. 1997):

**The Maxwell model:**

$$\varepsilon(t) = \frac{\sigma}{E} + \frac{\sigma}{\lambda} t$$

**The Kelvin model:**

$$\varepsilon(t) = \frac{\sigma}{E} \left( 1 - e^{-\frac{t}{\lambda}} \right)$$

**The Burger’s model:**

$$\varepsilon(t) = \frac{\sigma}{E} + \frac{\sigma}{\lambda} t + \frac{\sigma}{E} \left( 1 - e^{-\frac{t}{\lambda}} \right)$$

The above expressions to describe long-term creep are technically the superposition of various strain-time curves. A close fit to describe the three phases of creep is obtained by (Jenkins 2002):

$$\varepsilon = \varepsilon_i + \varepsilon_i \left( 1 - e^{r t} \right) + t \varepsilon_{ss}$$

where, $r$ is a constant,

$\varepsilon_i$ is the strain at the transition from primary to secondary creep,

$\varepsilon_{ss}$ is the steady-state strain rate.

Although there are no generally accepted forms to describe the nonlinear nature of strain-time curves, the following is one such relation (Jenkins 2002):

$$\varepsilon = \varepsilon_i + B \sigma^m t + D \sigma^a \left( 1 - e^{b \sigma} \right)$$

where, $B, m, D, a$ and $b$ are mathematical constants related to the material.

Creep models can be represented in more detail using constants and/or elements, the two most
common being a linear spring and a linear dashpot dampers. The five simple linear viscoelastic models seen in Figure 3.1-II denote various ways in which the dashpot and the spring can be combined to describe one-dimensional viscoelastic behaviors.

Figure 3.1-II: Linear Viscoelastic Models showing graphical response to creep tests.

[a] Two-constant liquid (Maxwell body); “…flows, shear stress is applied and held constant” (Goodman 1989).

[b] Two-constant solid (Kelvin or Voight body); “…a suddenly applied constant shear stress causes shear strain at an exponentially decreasing rate, approaching zero as t grows without bound…” (Goodman 1989).

[c] Three-constant liquid (generalized Maxwell body); “…initially had an exponential rate of shear strain, decaying to become asymptotic to a constant rate of shear strain” (Goodman 1989).

[d] Three-constant solid; “…initial “instantaneous” strain followed by shear strain at an exponentially decreasing rate, eventually tapering off…” (Goodman 1989).

[e] Four-constant liquid (Burger’s body); composed of a Maxwell and a Kelvin body in series, responds to a suddenly applied sustained shear stress – “instantaneous” … strain followed by shear strain at an exponentially decreasing rate, becoming asymptotic to a line representing a constant rate of … strain (Goodman 1989).
The Burger’s body is considered the most appropriate representation of long-term creep in rock mechanics. Its behavior is based on two linear elastic constants, the bulk modulus of elasticity $K$ associated with dilation and the shear modulus of elasticity $G$ associated with distortion. The axial strain, $\varepsilon_1(t)$, with time, $t$, of a Burger’s body under deviatoric stress, $\sigma_i$, and acting as an elastic body under constant axial compression is more accurately represented as (Goodman 1989):

\[
\varepsilon_1(t) = \frac{2\sigma_1}{9K} + \frac{\sigma_1}{3G_2} + \frac{\sigma_1}{3G_1} - \frac{\sigma_1}{3G_1} e^{-(G_1/\eta_1)} + \frac{\sigma_1}{3\eta_1} t
\]

where, $K$, $E/[3(1-2\nu)]$; the bulk modulus $G_1, G_2, h_1, h_2$ are constants describing properties of the rock.

3.1.3 Pertinent Studies on Basic Creep in Masonry and Cementitious Materials

The behavior of creep in concrete and mortar has been studied for many decades (Lenczner 1965; Lenczner 1969; Shrive and England 1981; Shrive et al. 1997 (as cited in Binda et al. 2001)). Observations have been made about the influence of the strength, the additives, and the condition on the creep performance of the various cementitious mixes. In particular, the important relationship between creep and shrinkage has also been extensively noted. A majority of these results pertain to mortars with portland cement as the main binding ingredient. Studies have also demonstrated how closely related the selection of mortar materials and the magnitude of creep can be.

Neville (1970) (as cited in Sickels 1988) while studying creep in concrete made with portland cement, suggested that strength and fineness of the mix affected the creep, and specified that creep was inversely proportional to the strength of the concrete at the time of load application. Further, Neville (1957, 1959) (as cited in Sickels 1988) linked creep to shrinkage, observing that for low stresses creep was equal to, or no greater than shrinkage of an unloaded specimen. Ameny et al. (1990) (as cited in Shrive et al. 1997) provides data for creep in a lightweight concrete and by assessing the material over two stress states (0.2 and 0.4 of the ultimate strength) concludes that the creep behaved as a linear function of stress.

Over a period of eight years, Lenczner (1986) (as cited in Sayed-Ahmed et al. 1998) measured
creep strains rising to about three times the initial ‘elastic’ strains in a load-bearing wall of a tower block in the United Kingdom. On the whole, long-term creep strains were found or estimated to range from about 0.3 to about 5.0 times the elastic initial strain. The wide range is attributed to differences in strength of the masonry components and the variation in shape of the assemblages.

Lenczner (1965, 1969) (as cited in Sickels 1988) made numerous observations through theoretical and experimental accounts about the state of creep in historical masonry specimens. He noted that creep was more prevalent in walls than in columns and even greater in single leaf walls than in cavity walls. Interestingly, he also observed that block-work piers exhibit more creep than brickwork piers. Lenczner (1976) further studied the influence of brick type and mortar type on the creep of brickwork piers. Prisms with two different bricks (strengths of 58.0 MPa (8410 psi) and 99.0 MPa (14400 psi)) and two mortars with portland cement (strengths 7.80 MPa (1130 psi) and 16.1 MPa (2340 psi)) were created. A constant load was applied at 28 days and monitored for up to 32 weeks. Lenczner discovered that pier with the lower brick exhibited approximately 55% more strain than the pier made with the stronger brick. The lower mortar exhibited approximately 2 to 3 times more strain than the stronger mortar. Later, in a 1976 study, Lenczner and Salahuddin also concluded that the load eccentricity (provided that it is small) does not significantly affect the creep behavior of masonry (Sayed-Ahmed et al. 1998).

Sickels (1988) observed the effects of lime on the performance of mortars with portland cement, concluding that the richer mortar is in lime, the higher the values for creep and the lower the values for shrinkage. She suggested that, “[t]he amount of creep in masonry assemblies is determined by the materials used, particularly the mortar” (Sickels 1988, p.179). Her study on mortar demonstrated that creep, unlike shrinkage, greatly affected the ability of masonry to relieve pressure from stress concentrations caused by temperature change and differential stress distribution.

Brooks and Abdullah (1990) (as cited in Sayed-Ahmed et al. 1998) tested various sizes and geometries of clay and calcium silicate brickwork. They determined that the ultimate creep depended on the rate of moisture diffusion, where creep decreases as the ratio of volume to exposed surface area increases.

Sayed-Ahmed et al. (1998) noted that indeed the type of mortar selected in masonry significantly
affected the amount of creep seen in a structure. The testing program continued the results of an experimental study from 1988 and evaluate the effects of mortar type, stress level, age at loading, moisture conditions and masonry strength on the creep behavior of clay masonry. The literature suggests that the age of loading significantly affects the amount of creep; clay masonry loaded at 7 days of age will creep 1.2 to 1.7 times as much as masonry loaded at 14 days and 28 days of age. Regarding the strength of the masonry, when the long-term strength of the mortar is higher, than the amount of creep is lower; the stronger Type S mortar crept only approximately 55 to 75% as much as the lower Type N mortar. Using specific creep as a comparative unit, the literature noted that an approximate increase in the prism strength of 50 to 140% reduced the specific creep by about 40 to 75%.

### 3.2 Creep in Historical Masonry Structures

The collapses of some monumental buildings over the past fifteen years have produced records noting the measurements of large continuous internal movement in masonry, particularly in cases where the stability of the foundation is known to remain intact. Creep behavior and the creep-fatigue interaction have proven to strongly influence the mechanical behavior of not only concrete materials but also masonry. The viscoelastic nature of masonry is of great interest, particularly with respect to massive historical buildings and towers, where studies show that the masonry acting under persistent loading over relatively long periods of time can potentially collapse at lower stress values, around 45-50% of the nominal material strength (Binda et al. 2001). Due to creep, internal stresses are continuously redistributed throughout the loaded masonry. Additionally, an associated redistribution of stress from external load compounds the event. When combined with this secondary reduction in strength, an increase in the stress resultant also may lead to collapse (Papa et al. 1994).

#### 3.2.1 Early and Delayed Deformations

An historical masonry wall is typically composed of an internal rubble core sandwiched between two face layers of organized masonry units; a plaster layer is sometimes applied as a smooth exterior finish. The various layers of the wall are identified by the materials and, henceforth,
changing stiffness values. This composition induces a three part, tri-axial stress-strain field that varies over time and space. Historical masonry buildings are therefore often plagued with discontinuities in their structure, identified by anisotropic behavior and consequently a non-uniform distribution of stress (Binda et al. 1991). Misalignments from irregular sized materials or weak portions of a structure, synergistic environmental effects from cyclic wind action and large differentials in seasonal temperature are also considered contributing failure variables. These external forces influence behavior, but in general, the self-weight and dead loads on the structure outweigh these factors.

Based on the carbonation effects of lime-based mortars, early deformations can be attributed to the delay of a hardening process that can last for a relatively long period of time. This often occurs in structures characterized by thick walls or thick mortar joints. Accordingly, multiple leaf walls can be substantially affected by differentials in creep displacement during the initial set up phase. (Anzani et al. 2002).

In general, time plays a crucial role in the short-term and long-term states of masonry; a masonry structure is continually subjected to modifications in its geometry due to increasing deformations.

3.2.2 Long-Term Behavior, Creep before Collapse

The deformability that today characterizes historical masonry structures may have occurred in the past, or may still presently be active, or may eventually lead to progressive collapse. The high state of damage seen in historical structures today certainly the response of various outside influences, coupled synergistic effects of climatic changes, seismic activity and environmental responses. But certainly, a discrete influence on deformation is the intrinsic degradation of the material over time.

The damage process is negligible during the primary and secondary phases of creep. At the tertiary phase, the influence of material damage on the deformation response is pronounced; the creep strain rate increases almost without limitation. As the damage disperses, stresses are redistributed throughout the structures. Stress transfers from the most deteriorated regions to regions with the greater load carrying capacity, thus forming a damage front that propagates throughout the structure until failure takes place. (Gonçalves Filho 2005).
3.2.3 Observed Damage on Historical Masonry Structures

The influence of time and the incurrence of deformation on historical masonry became evident after the sudden failure of the Civic Tower of Pavia on March 17, 1989. With no particular warning signs for the collapse, several hypotheses have attempted to explain the collapse; soil settlement, jet vibrations, even the presence of a bomb. Shrive and Huizer (1991) postulated that creep was a primary factor in the collapse of the medieval tower. Binda et al. (1992) confirmed this hypothesis through an extensive study. Binda et al. (1992, 2002) ruled out the most obvious causes including settlement of the ground or chemical and/or physical degradation of the materials. Evidence from the investigation allowed Binda et al (1992, 2002) to relate the failure of the tower to time-dependent behavior, the long-term creep, likely coupled with synergistic cyclic loads. Numerous tests to prove this proposal on recovered masonry from the collapse site were conducted. The team performed fatigue tests, monotonic tests, creep and pseudo-creep tests, uniaxial tests and unloading/reloading cyclical experiments. The high levels of stress fatigue found in the ruins could only be attributed to the substantial dead load of the structure. The results clearly showed evidence of the three phases of creep behavior in the masonry. (Binda et al. 2002).

In surveying the literature, the Civic Tower of Pavia is not an isolated case. Many other examples indicate similar structural flaws, where damage states are potentially induced from time-dependent behavior. The bell tower of San Marco in Venice is a particularly famous case and, more recently, the collapse of the Cathedral of Noto in Italy. The characteristics of long-term behavior are still being quantified; there is however no disagreement on the extensive state of damage seen in many historical masonry buildings today. Anzani et al. (2002) have surveyed approximately 60 ancient Italian towers intending to collect vital information on the symptoms of their degradation, particularly taking note of crack patterns. Some of these examples are presented herein. Buildings are also presented where high deformability may well be explained by the creep phenomenon. [See Table 3.2-A].

The case histories often describe structural damage caused by vertical compression, mainly instigated by heavy persistent dead loads. Towers and slender elements, such as piers and columns, often see an excess of compressive stress. Moreover, portions of material are found to have high concentrations of stress due to non-uniform distributions of internal forces. Further, cracks and fissures are often seen to propagate through both mortar joints and stone units of the
structure. All of the above are presently identified as properties of long-term viscoelastic behavior, as noted through the studies conducted by the authors Anzani and Binda.

The Cathedral of Noto, Italy: An on site laboratory confirmed that the disaster of 1996 at Noto, although indeed worsened by the 1990 Sicilian earthquake, was inevitable. The identification of spalling and pre-existing diffuse vertical cracks indicated serious progressive damage. Hidden behind plaster from a 1960’s repair, the cracks were filled with gypsum mortars. Furthermore, the internal rubble structure of the piers had substantially decreased in strength. Originally constructed with mortars composed of hydrated lime and weak calcareous aggregates, the load bearing elements of the core, irregular courses of round river stone, lacked structural transverse ties to the external leaf of regular limestone blocks. While collapse is seemingly evident because of tertiary creep levels, a compressive stress test conducted on the calcarenite stone indicated that at the secondary creep phase, present at low stress levels, there was a severe reduction in the strength capability of the stone, averaging approximately 28 N/mm² (4061 psi).

The Church of SS. Crocifisso at Noto: Constructed from the widely used calcarenite stone, the 1715 Sicilian church also saw damage from the 1990 earthquake; the transept, dome and vaults of the lateral nave sustained heavy deterioration and, at the time, were supported by a provisional structure. During the diagnostics, surface plaster not associated with the original construction was removed from piers that appeared unaltered. However, the investigation revealed an alarming series of complex vertical cracks that were not only tracing mortar joints but also passing directly through the structural rubble. The cracks were widely diffuse. In numerous cases, the fissures were filled with plaster, an indication of their existence well before the material was applied and providing evidence toward the pre-existence of progressive creep in the mortar and in the stonework. (Binda et al. 2001).

The Church of SS. Annunziata at Ispica: Founded in 1703, the church has survived several events that have induced damage. In particular, a 1727 earthquake that shifted the piers of the main arcade out of plumb; the repairs never materialized for economic reasons. In 1869, the facade collapsed and recently the 1990 earthquake caused further damage. As at SS Crocifisso, the lime-based plaster concealed much of the visual cracking. Again, the plaster had seeped into pre-existing cracks; compressive damage clearly existed before the execution of the plastered surface. (Binda et al. 2001).
The Madeleine of Vézelay: While the Madeleine of Vézelay stands today as restored by Viollet-le-Duc, its recent preservation work conducted by Robert Vassas in 1968 revealed extensive horizontal displacement at the top of the nave pier and vertical deformation at the crowns of arches. Large gapping voids remain between the nave walls and the main vaulting system, with some irregularity in the stonework. (Navedo 1998).

The Civic Tower of Pavia: The collapse of the tower in late March of 1989 was the final motivating force for last decade’s effort to save the leaning Tower of Pisa from a similar fate. Prior to collapse, the slender tower exhibited vertical capillary flaws. This observation was made after careful examination of old photographs. Studies carried out after the collapse of the tower show that the mortar exhibits good compactness and satisfactory mechanical properties. Further investigation on the piers determined poor consistency in the morphology of the internal core where the mortar and its adhesion to the stone cobbles developed feebly. This situation was, however, less critical than the consistency of adhesion observed at the Cathedral of Noto. (Anzani et al. 2002).

Table 3.2-A: Observations on the State of Historical Masonry Structures

<table>
<thead>
<tr>
<th>Structure</th>
<th>Date</th>
<th>Summary of Damage</th>
<th>Comments / Perceived Causes of Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>S. María del Pero Bell</td>
<td>10th c.</td>
<td>Extensive crack patterns. (Binda and Anzani, 1997).</td>
<td>N.A.</td>
</tr>
<tr>
<td>Tower [Monastier, Treviso, Italy]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coronata Family Tower</td>
<td>13th c.</td>
<td>Extensive crack patterns, observed in 1997 study. (Binda and Anzani, 1997).</td>
<td>N.A.</td>
</tr>
<tr>
<td>[Bologna, Italy]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tarazona Cathedral</td>
<td>1235,</td>
<td>Cracks at mid-span of arches of nave; degradation of the stone in the base of the</td>
<td>No experimental testing conducted at present; Settlement of piers with respect to the buttressing may have</td>
</tr>
<tr>
<td>[Tarazona, Spain]</td>
<td>choir</td>
<td>piers; in some bays, cracks developing in transverse ribs of the arches; vertical</td>
<td>generated crack growth; reduction of the section of piers from restoration work and additions caused excessive</td>
</tr>
<tr>
<td></td>
<td>transept.</td>
<td>cracks separating arches from walls; vertical cracks separating the backing of the</td>
<td>compression and tension zones.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>nave vault from walls. (Lourenço 2001).</td>
<td></td>
</tr>
<tr>
<td>Barcelona Cathedral</td>
<td>1298</td>
<td>Extensive cracking and crushing in the original limestone piers and arches of the</td>
<td>From numerical FE analysis, when dead load is applied, large compressive damage initially appears in the</td>
</tr>
<tr>
<td>[Barcelona, Spain]</td>
<td></td>
<td>nave. (Lourenço 2001).</td>
<td>crowns of the vaults. With additional dead load, damage appears in the haunches of the aisle arches and at the base of the piers.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Location</td>
<td>Date</td>
<td>Condition</td>
<td>Notes</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>-------</td>
<td>---------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>Mallorca Cathedral [Mallorca, Spain]</td>
<td>1350</td>
<td>Large cracks and deformations; Collapse of an arch nave (1490); re-</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>construction of vaults (17th and 18th c.); deformation in the piers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>showing curvature and lateral displacement both longitudinally, transversally;</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>vertical crack at the base of some pier. (Lourenço 2001).</td>
<td></td>
</tr>
<tr>
<td>Civic Tower of Pavia [Pavia, Italy]</td>
<td>14th c.</td>
<td>Collapsed on March 17, 1989; perceived high stress values at tower</td>
<td>Experimental testing shows strong correlation of creep behavior; initial linearly elastic stress strain diagram, failure subsequently occurs shortly after tertiary creep is reached.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>base, caused by uneven loading. Estimated dead weight of the tower was</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>120,000kN, while the weight of the belfry alone was 30,000kN.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Binda and Anzani, 1997).</td>
<td></td>
</tr>
<tr>
<td>The Tower of the Church of St. Marco [Venice, Italy]</td>
<td>1514</td>
<td>_COLLapsed in 1902.</td>
<td>N.A.</td>
</tr>
<tr>
<td>The Tower of Monza [Monza, Italy]</td>
<td>16th c.</td>
<td>Extensive crack patterns. (Binda and Anzani, 1997)</td>
<td>Experimental testing shows strong correlation of creep behavior; initial linear-elastic stress strain diagram</td>
</tr>
<tr>
<td>The Church of SS. Annunziata in Ispica [Ispica, Italy]</td>
<td>1703</td>
<td>Piers tilt (1727); Façade collapsed (1869); diffused vertical cracks on</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>pillars, inserted through stone and mortar. (Binda and Anzani, 1997.)</td>
<td></td>
</tr>
<tr>
<td>Church of SS. Crocifisso in Noto, [Noto, Italy]</td>
<td>1715</td>
<td>Series of crack patterns, inserted through stone and mortar in base of</td>
<td>N.A.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>piers. (Binda and Anzani, 1997.)</td>
<td></td>
</tr>
</tbody>
</table>

### 3.3 Analysis of Creep, Time-Dependency in Historical Masonry

Regarding historical masonry, creep is thought to mainly be induced by its sustained self-weight. Extensive investigations have only recently begun to explore apparent viscosity beyond the elastic range of the historical masonry materials.

#### 3.3.1 In-Situ Experimental Studies

Long-term creep strains in masonry were found or estimated to range from about 0.3 to about 5.0 times the elastic initial strain. With the exception of Hughes and Harvey (1995) who collected
data for over 6000 days, long-term creep effects have typically been reported on short-term trials, at most 400 days (Sayed-Ahmed et al. 1998). Studies that address the long-term performance of historical masonry have typically done so from a holistic standpoint, examining recovered brickwork and block-work from collapse sites. (Shrive et al. 1997).

Of significant importance to this thesis, Binda et al. (1991, 1992, 2001a, 2001b, 2001c), along with others (Papa et al. 2000; Anzani et al. 2005), carried out experimental long-term creep tests and accelerated tests by incremental constant load analyses on recovered masonry assemblies. The studies made substantial conclusions about the long-term performance of masonry, graphically representing and empirical modeling three phases of creep. Findings published from the sudden collapse of the Civic Tower of Pavia stimulated further studies about ‘time-dependent mechanical behavior’ in masonry. It has become important to include time-dependent response in the design and analysis of masonry structures. Papa et al. (1994) loaded masonry up to 80% of its peak monotonic strength and held that load level. Failure of the prism occurred in the next three hours or less. Indeed, an increase in the stress resultant due to creep could be an instigator to collapse.

3.3.2 Analytical Studies, Parametric Theory Applications

Progress is being made to apply constitutive laws to the behavior of ancient masonry structures, aiming to predict the long-term damage, understand construction sequences and allow preventive repair interventions. Analysts find that solving complex problems with advanced numerical formulations or iterative procedures more accurately describe the behavior of material and their application to masonry analysis is proving invaluable.

This adaptation to greater sophisticated simulations is promising; the lack of experience is evident as compared to the advancements in soil, rock or concrete research. To achieve better correlations, a mathematical description of the material behavior, yielding a relationship between the stress and strain tensors, is employed. Typically named a constitutive model, these advanced tools make it possible to fully understand the limit state of masonry and its failure mechanisms. (Lourenço 2001).
Various experimental programs have been devised to predict the magnitude of creep in concrete and masonry (e.g. Lenczner 1965, 1969, 1970, 1974, 1979; Wyatt and Morgan 1974; Shrive and England 1981; Anand and Gandhi 1983; Ameny et al. 1984; Brooks 1986; Taneja et al. 1986; Harvey and Hughes 1995; Shrive et al. 1996) (as cited in Sayed-Ahmed et al. 1998). Literature shows that constitutive models or equations to describe elastic to inelastic/plastic behavior of materials are mainly based upon the theory of plasticity, the concepts of smeared cracking, or on continuum damage models (Ayala et al. 2001). [See Table 3.3-A].

**Table 3.3-A:** Common Theories and Models applied to Masonry as Constitutive Laws.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Implementation</th>
<th>Classic Formulation</th>
<th>Found In Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mohr-Coulomb yield criterion;</td>
<td>commonly used to model shear failure at joint, shear strength as a function of the normal stress.</td>
<td>( F(\sigma, \tau) =</td>
<td>\tau</td>
</tr>
<tr>
<td>Coulomb frictional law;</td>
<td>commonly used to describe stress-strain relationships, and predict tangential frictional forces at joints.</td>
<td></td>
<td>(Boothby et al. 1999), (Ma et al., 2001).</td>
</tr>
<tr>
<td>Drucker-Prager yield surface model;</td>
<td>relates stresses to hydrostatic pressures and material cohesion; commonly used as a damage variable to implement plastic mechanisms of failure and to simulate basic deformations.</td>
<td>( 3k_1 \sigma_{\text{eq}} + \sigma_\text{d} - k_2 = 0 ) where, ( \sigma_{\text{eq}} = \frac{\sigma_i}{3} = -p \cdot \sigma = \sqrt{\frac{1}{2} \sigma_{ij} \sigma_{ij}} = \frac{q}{\sqrt{3}} ) k_1 = \frac{2 \sin \phi_v}{\sqrt{3(1 - \sin \phi_v)}} ) k_2 = \frac{6 \cos \phi_v}{\sqrt{3(1 - \sin \phi_v)}}^c</td>
<td>(Boothby et al. 1999), (Pina-Henriques and Lourenço, 2003), (Lourenço and Zucchini, 2006).</td>
</tr>
<tr>
<td>Rankin-type damage variable surface;</td>
<td>commonly used to model tensile damage of an isotropic material.</td>
<td></td>
<td>(Lourenço and Zucchini, 2006).</td>
</tr>
<tr>
<td>Burger's viscoelastic model;</td>
<td>used to simulate overall creep behavior from primary stages to the final tertiary phase; implements other basic linear elastic models, the Kelvin and Maxwell bodies.</td>
<td>( \epsilon(t) = \frac{2 \alpha_1}{9K} + \frac{\sigma_1}{3G_1} + \frac{\sigma_1}{3G_1} e^{-(t/\eta_1)} + \frac{\sigma_1}{3\eta_1} t ) where, ( K = E/(3(1-2v)) ); the bulk modulus ( G_1, G_2, \eta_1, \eta_2 ) are constants describing properties of the rock.</td>
<td>(Papa, Talierno and Mirabella- Roberti, 2000), (Anzani, Binda and Talierno, 2005).</td>
</tr>
</tbody>
</table>

When structural designers and analysts attempt solutions to describe creep rupture, they typically encounter three physical sets of criteria to define: first, the nonlinearity growth of creep strain and creep damage described in evolution equations; second, the stress distribution induced by the material deterioration; and third, the redefinition of the problem scope with the propagation of the
damage front. The stiffness represented by the constitutive equations is expressed as the discrete influence on the damage process of the deformation. (Gonçalves Filho 2005).

Marais et al. (1986) proposed a solution algorithm for the analysis of bodies susceptible to creep under constant or variable loads. Adopting a law into finite element formulations, the model is represented in terms of displacement and creep strains as internal variables. The creep strains are carried out as equations of displacement with a modified stiffness matrix and force vector. Time discretization is carried out with a trapezoidal time integration scheme. Iterative steps of finite element programming are achieved by adopting a Newton-Raphson scheme for nonlinear creep stress-strain rate relations. The algorithms are implemented in a computer program as two-dimensional, plane continuum structural problems. A Von Mises type function is used to govern multi-axial creep constitutive relationships.

Binda et al. (1992) and Binda et al. (2001b) suggest that traditional stress-strain analysis is not prepared to take into account the particular critical effect of cyclical persistent loads. Analyzing results of accelerated creep tests on materials collected from the Civic Tower of Pavia, Binda et al. (2001c) model mechanical behavior by employing the theory of viscoelasticity coupled with two anisotropic damage variables or tensors. The second order tensors allow the model to access multiple directions of induced strain in the anisotropic composition. Using finite element modeling techniques, the aim was to predict creep behavior to assess the safety and reliability of existing ancient masonry. Given a set of compressive parameters to describe the deviatoric and volumetric viscoelasticity, the rheological procedure imitates known creep behavior by monotonically increasing the stress levels. The model is able to represent induced damage and predict uniaxial creep time to failure. The simulations to date have been successful; their implementation as a structural analysis is proving effective (Binda and Saisi 2004). Furthermore, the cylindrical testing predicted that the sequence of failure can begin at very low stress levels; about 45-50% of nominal material strength (Binda et al. 2001a).

Lourenço (1997, 2001, 2002, 2004, 2006) has developed at least two numerical models used to describe the complex behavior of masonry under plane stress conditions. Lourenço and Rots (1997) describes an ‘complete’ interaction of masonry units and respective mortar joints in a constitutive model that includes all potential failure modes, namely the actions of cracking and crushing of joint and unit. The model includes inelastic behavior in tension, in shear and in compression.
Lourenço et al. (1997) recognize that, for larger structures, the simplified knowledge of joint and unit interaction cannot necessarily be adopted as a global characterization in models. Hence, a second model is introduced combining plasticity concepts with anisotropic material behavior inputs that include various hardening/softening behaviors of the material. The constitutive formulations describe uniaxial tension, defined by a Rankine-like yield surface, and uniaxial compression, defined by Hill-like yield surface theories. Both constitutive models are proving useful in their application to real masonry structural analysis problems.

Garavaglia et al. (2003) worked on a probabilistic approach to interpret creep through pseudo-creep tests carried out on the ancient masonry. Physical results of the creep phenomenon have been shown in the residual life of the masonry material. The aim of the study is to develop a mathematical model able to predict possible failures of heavy masonry structures based on a critical value of the strain-rate to describe the long-term damage. As identified by Anzani et al. (2000, 2002) and Taliercio et al. (1996), the probabilistic model by Garavaglia et al. (2003) describe the three descriptive phases of long-term creep and a material dilation under severe compressive stress that corresponded to high strain prior to failure. A family of theoretical distribution curves is created to model the deformation process as a stochastic process of the random variable $\xi$ of the strain-rate. For each theoretical distribution, a unique ‘fragility curve’ can be plotted (Binda et al. 1999; Garavaglia et al. 2002 (as cited in Garavaglia et al. 2003)). Since the experimentally measured strain-rates only refer to a discrete number of stress levels, the plots are treated from the standpoint of a classic reliability problem in order to describe an overall system behavior. This procedure is found to be a good interpretation of the physical phenomenon, capturing the passage between viscoelastic and viscoplastic behavior. The model is applied to the bell tower of Monza with results showing good correlations and possible future success and refinement.

Papa et al. (2001) introduce a theoretical constitutive model employing the theory of viscoelasticity coupled with two anisotropic damage variables. The theoretical model is designed to simulate experimental results as observed on accelerated creep tests on materials collected from the Civic Tower of Pavia. Given a set of compressive constraints to describe the deviatoric and volumetric viscoelastic behavior of masonry, the rheological model imitates known creep behavior by monotonically increasing stress levels. The model is able to monitor induced damage and predict uniaxial creep time to failure. The numerical simulations of masonry completed to
date have been successful.

There are numerous publications to describe the application of a modified Burger’s rheological model to describe the primary, secondary and tertiary creep phrases using similar parametric practices to that of Papa et al. (2001). In fact, the publication by Papa et al. (2001) cites previous work by Anzani et al. (2000) as a clear source for the origination of the model and there are clear similarities between the model presented by Papa et al. (2001) and the model described by Anzani et al. (2005) which interprets creep using employed constants of the Burger’s’ body.

The approach of the theoretical models by Anzani et al. (2000, 2005) and Papa et al. (2001) is to employ the classic Burger’s rheological body as a representation of masonry mechanical behavior, including but not limited to all stages of creep. The basic modified model with damage variables consists of a Kelvin body (of stiffness $E^k$) that describes primary phase of creep and acts in series with a Maxwell-Bingham combined body (of stiffness $E^m$) that accounts for secondary creep. Tertiary creep is induced by introducing damage variables taken from theories of thermodynamics and laws originally proposed by Chan et al. (cited in Anzani, et al. 2005) within the Maxwell element; a ‘static damage’ reduced the elastic modulus of the Maxwell spring to account for decreases in stiffness and a ‘viscous damage’ variable affect the relaxation time of the spring to allow the tertiary stage of creep to precede failure of the model. The success of the model is evident from successful correlations between pseudo creep tests carried out on masonry specimens retrieved from the Belfry Tower site and numerical predictions with the modified Burger’s model. (Papa et al. 2000).

Shrive et al. (1997) made significant progress to implement the modified Burger’s model as an appropriate means to predict the creep coefficient in masonry prisms of Type N and Type S mortar. Over a period of 8 years, Shrive et al. (1997) monitored a series of wet and dry prisms in a creep test frame taking measurements of the creep compliance $\phi(t,t_0)$ and the creep coefficient $\phi(t,t_0)$ using the Demec system. Stress applied ranged from 2.43 MPa (353 psi ) to 4.86 MPa (704 psi). The results produce more significance that that previously conducted (Lenczner 1990, Shrive and England 1981; Ananda and Gandhi 1983; Ameny et al. 1984; Taneja et al. 1986; (as cited in Shrive et al. 1997)) because the experimental methods delineate other factors contributing to the overall performance of creep such as the effects of the age at loading, temperature variations, mortar type, strength of units and moisture content. The study suggests that
international codes, with the exception of the CSA-S304.1-94 Canadian code, should be re-evaluated. Shrive et al. (1997) propose the following equation, based on the modified Burger’s model to predict the creep ratio in masonry:

\[ \phi(t, t_0) = R t^{0.3} + \left(1 - e^{-Rt}\right) \]

where, \( R = 0.112 - 3.35 \times 10^{-6}E \)
\( E \) is the modulus of elasticity of the masonry assemblage in MPa
\( t \) is the time in days

It is important to note that Garavaglia et al. (2003) identified a material dilation / damage state phase and presented an acceptable probabilistic approach as a prediction tool for creep behavior by creating a family of strain curves noting their theoretical distribution. As a classic reliability problem, the study only considered a good interpretation of behavior from viscoelasticity to viscoplasticity. The prominent authors, Anzani et al. (2000, 2002, 2005), Binda et al. (1991, 1992, 2001a, 2001b, 2001c, 2004) and Papa et al. (2000, 2001, 2003), have presented multi-axial models with damage variables and have found this method to be highly successful to predict creep that has been re-created on samples of historical masonry. The works of each author has certainly been an influence on the other, and often, a collaboration; the models of each study appear to be derivatives of each other. By applying ‘static’ and ‘viscous’ damage variables as based on thermodynamic theories in the secondary and the tertiary phases creep, these authors, in particular Anzani et al. (2005), have been most successfully in creating and continually refining a constitutive model that predicts creep behavior of historical masonry as a whole. Formalistically, the damage states, in arrangement with Kelvin and Maxwell-Bingham viscoelastic models, combine to form a modified Burger’s body. Indeed, the strain data from these models show good correlation between hard-to-assess negative volumetric strains and the onset of fracture mechanics in the secondary phase of which closely align to the point of transition into the final tertiary phase of creep.
Chapter 4

Experimental Analysis

The following chapter describes the completion of Objective A, to experimentally determine the creep characteristics over time of historic mortars of the middle ages. In particular, this objective sets out standards for historic mortars created with lime as the main binding ingredient. The study also looks to establish relevance and extend the knowledge base for the addition of more natural and organic additives in lime-based mortars used today. Further, the results of the experimental work as described in this chapter introduce Objective B, to analyze the experimental data for behavioral trends and to identify trends that can be averaged into a singular description of the long-term performance of the mortars.

4.1 Significance and Use

This test method describes the determination of creep in mortar subjected to a sustained compressive load. This test method is limited to stacked masonry assemblies constructed with high strength, nominally sized solid bricks and appropriately proportioned lime-based mortars similar to those used during the historical time period, 1200 to 1500 AD. The experimental work completed in this thesis measures the load-induced compressive axial strains at selected ages of mortar under a non-specific set of environmental conditions. The testing is used to evaluate the primary and secondary creep in historic mortars. Further, a numerical mathematical model is extracted to represent the associated creep strain values over time. The test method takes into account initial strains and shrinkage of the mortar between the brick units. In the absence of a satisfactory study about long-term creep in historic lime-based mortars, the experimental set up is designed keeping in mind examples already available in the literature (Lenczner 1965; Sickels 1988; and Shrive et al. 1997) and attempts to follow modern procedures for mortar and cementitious materials as presented in the ASTM standards.
4.2 Experimental Apparatus

The testing is conducted in a creep frame designed specifically for the scope of this thesis. Two frames are created, each frame having the capability of applying and maintaining a consistent load onto six stacked masonry prisms. The set-up is developed by studying examples in literature as given in Shrive et al. (1997) and Sickels (1988). A one inch sized steel threaded rod is placed between three braced steel angles [see Figure 4.2-1]. The six prisms are held between steel plates, where all parts of the assembly are centrally threaded onto the rod. Full contact between each component is achieved through the applied load; no other material is used to create contact, as is the case in masonry. Three rubber washers are used to counteract the predicted relaxation of the load due to high stiffness in the system.

The threaded rod is separated into two sections at the load cell. Here, the load cell is set in place as a device to read tensile forces. To one side, the threaded rod is strung through an anchored L shape and attached into the load cell. On the other, the longer threaded rod is attached to the opposite face of the load cell and strung through another anchored steel shape. This longer rod is supported at its far end by a similar sized L-shape that, while braced in the out-of-plane direction of the load, has the ability to slide along the length of the rod. The partial restraint of this steel shape accommodates small out-of-plane alignments, whilst also ensuring that the load is applied axially. Each end of the threaded rod is capped with a hex nut.

The apparatus is designed to induce a uniaxial load. To achieve this, a constant compressive stress is transferred to the string of masonry prisms through an applied tensile force in the rod. The tensile force is achieved through the action of tightening the hex nut placed on the far end, against the partially retrained steel shape. In turn, the tensile force applies pressure to the steel shape which can slide against the masonry prisms and apply a dissipated load, equal but opposite to the force in the rod. Under load, the masonry prisms are slightly elevated, approximately 0.125” (3.20 mm), from the surface of the concrete base to counteract frictional forces. An oiled plastic liner is also placed between the prisms and the concrete footers to create a frictionless surface.

List of Equipment and Supplies: concrete pad, six (6) fabricated steel angles, 1"-14 thread B7 plain alloy steel rod, anchor bolts, four (4) 1-14 hex nuts, six (6) rubber washers, 24” wide plastic liner (20ml UHMPE), motor oil, two (2) load cells (max load, 10 000lbs (10 kips)), computer-based data acquisition and control system.
Figure 4.2-I: Test setup, a schematic of the creep apparatus without masonry prisms
4.3 Test Prisms

Tests performed on twelve stacked masonry prisms classified into two series. Prisms of each series were treated in the same manner, the intention being that from the results, a single homogenized behavior may be assessed.

4.3.1 Prism Size and Organization

A prism consists of four (4) standard modular bricks 0.092 m (3 5/8”) x 0.057 m (2 1/4”) x 0.203 m (7 5/8”) [Figures 4.3-I and 4.3-II]. [See Appendix B for more images depicting the twelve prisms tested in this study]. Each brick was an extruded high-strength unit with three (3) central core holes of approximately 35.0 x 45.0 mm (1 3/8” x 1 3/4”) [Figure 4.3-I [b]]. Each prism was constructed with three (3) 6 mm (0.236”) mortar joints, namely a, b and c. Bricks are stacked with a full mortar bed. Overall, the dimensions of each prism are approximately 92.0 x 203 x 247 mm (3 5/8” x 7 5/8” x 9 3/4”). Prisms are named under the following designation: Prism Series 0#.Prism 00#.Mortar Joint a/b/c. For example, in Figure 4.3-II, the mortar joint in the prism is labeled as “01.004.b.” Six prisms, collectively combined to form a series, are simultaneously loaded in one
(1) creep apparatus; prism series 01 contains prisms 001 through 006 [01.001-01.006] and prism series 02 contains prisms 001 through 006 [02.001-02.006].

Figure 4.3-I: Standard bricks: [a] Three-dimensional view; [b] Brick measurements (mm).
4.3.2 Mortar

As previously detailed in Chapter 2, the mortar composition includes agents that are frequently used in medieval mortars. The majority of buildings found to be susceptible to long-term damage or collapse have been constructed approximately 500 years before, during the Middle Ages when lime was well-known for its binding characteristics. Regardless, an historical mortar is often referenced as a ‘lime-based’ composite with sand and (or) pozzolan as the balancing constituents, its formulae inherited from historic times. (Sickels 1988).

A standard volumetric mortar proportion of 1 part lime and 2 parts pozzolan and 9 parts sand is used throughout this thesis. A lime-sand numerical material proportion (1to 3) is typically found in the literature as a possible standard for historical mortars and is also found to most accurately replicate historic mortars. Other selected constituents were added based on a 5% volumetric ratio of the mortar. This percentage is taken in accordance to examples presented in the literature (Velosa and Vega 2001, Bosiljkov 2001) and is again introduced here to establish the value as a standard in regards to the creation of lime-based mortars.
Table 4.3-A: A Summary of the Mortar Constituents.

<table>
<thead>
<tr>
<th>Material</th>
<th>Role</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lime</td>
<td>Non-hydraulic binding ingredient, containing calcium magnesium hydroxide; reacts with atmospheric carbon dioxide to form calcium carbonate (process commonly called 'carbonation').</td>
<td>Density (pcf)</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>calcium magnesium hydroxide, CaMg(OH)₄</td>
</tr>
<tr>
<td>Sand</td>
<td>aggregate, retains some hydraulicity properties.</td>
<td>80</td>
</tr>
<tr>
<td>Pozzolan</td>
<td>hydraulicity: sets in water, as it contains particles of clay, silica and alumina; feebly to moderately eminently hydraulic; accelerator; enables lime mortar to set more rapidly; improves compressive strength, due to highly vitreous nature, when combined with calcium hydroxide forms calcium silicates which exhibits cementitious properties.</td>
<td>65.24 *</td>
</tr>
<tr>
<td>Eggs</td>
<td>emulsifier / stabilizer; egg yolk stabilizes a n emulsion (use in small quantities); modifier; egg whites alters existing solution; plasticizer / mollifier; egg whites impart plasticity and reduce brittleness.</td>
<td></td>
</tr>
<tr>
<td>Beeswax</td>
<td>shrinkage agent; prevents / counteracts shrinkage.</td>
<td></td>
</tr>
<tr>
<td>Beer</td>
<td>air entrainer; improves durability.</td>
<td></td>
</tr>
</tbody>
</table>

* Calculated based on weight ratio to sand, 80pcf x 0.8156 = 65.24pcf
References for Table: Sickels (1988) and Taylor (2000)

4.3.2.1 Lime Selection

A mature, non-hydraulic wet lime putty is selected for this study. Its properties are considered to more closely align with the nature of limes used during the middle ages. Builders in ancient roman typically selected rich limestone and slaked the material by immersing egg-sized clumps in water, thus maintaining plasticity until the material was to be used in a mortar (Sickels 1988). In business for over 50 years, Graymont produces and supplies lime putty in a similar manner and manages facilities on sites that have been in operation for nearly 200 years. The company is
focused on providing quality high calcium and dolomitic lime products such as specialty hydrates and precipitated calcium carbonates. The Graymont putty product selected for this study is of a high purity dolomitic lime chalk collected from a stone deposit in northwestern Ohio. The material is fully slaked, screened and packaged for immediate use. This specific lime complies with ASTM C1489, Lime Putty for Structural Purposes, and can be used to formulate mortars in accordance to ASTM C270. [See Table 4.3-A for further information on this material along with its subsequent properties and roles.]

4.3.2.2 Sand Conformity

The sand selected for use in this study is a basic construction grade granular mix. It is modified to comply with ASTM C144, Standard Specification for Aggregate for Masonry Mortar. According to the standard, a natural sand for use in masonry mortar shall contain aggregates with a percent passing as described in Table 4.3-B. The sand was found to conform to ASTM C144 if the aggregate was passed through a no.8 (2.36mm) sieve. [See Table 4.3-A for further information on this material along with its subsequent properties and roles.]

<table>
<thead>
<tr>
<th>Table 4.3-B: Sand Conformity by ASTM C144</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sieve Size</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4.75-mm (No. 4)</td>
</tr>
<tr>
<td>2.36-mm (No. 8)</td>
</tr>
<tr>
<td>1.18-mm (No. 16)</td>
</tr>
<tr>
<td>600-µm (No. 30)</td>
</tr>
<tr>
<td>300-µm (No. 50)</td>
</tr>
<tr>
<td>150-µm (No. 100)</td>
</tr>
<tr>
<td>75-µm (No. 200)</td>
</tr>
<tr>
<td>Tray</td>
</tr>
</tbody>
</table>

4.3.2.3 Pozzolan, ‘crushed brick’

Pozzolan, also known as pozzolanic ash, is a fine, sandy volcanic ash, originally discovered in Pozzuoli, Italy in the region around Vesuvius. Vitruvius often spoke of four pozzolan types,
black, white, grey and red, dug from the various volcanic areas of Italy. Typically it was mixed with a two-to-one ratio with lime. Modern pozzolan is often a mix of natural and industrial pozzolan; include certain powdered brick, heat-treated clay, silica fume, fly ash, and volcanic materials. [See Table 4.3-A for further information on this material along with its subsequent properties and roles.]

In this study, a pozzolan of ‘crushed brick’ was used in six of the twelve mortar prisms. Pozzolanic reactions and the process of carbonation both impart important overall effects on the strength evolution of the mortar. Pozzolan, however, has both reactive and non-reactive phases. For these reasons, the pozzolan was added in a volumetric ratio of 1:2 (pozzolan:lime). This selection also resulted in a comparative 1 to 3 ratio of reactive ingredients to aggregates. Generally, lime mortars are commonly executed with a 1:3 (lime:aggregate) ratio. The 100-year-old ‘crushed brick’ was selected as a viable alternative to unavailable volcanic ash; the material is similar, identified as a heat-treated clay containing particles of silica and alumina. While adding to the hydraulicity of the mix, the pozzolan was used with the intention of strengthening the mix and accelerating the set, thus enabling the lime mortar to harden more rapidly.

4.3.2.4 Additives

Beer, eggs and beeswax are three of the five most popular additives used in mortars during the 1200AD to 1500AD [see Table 2.1-A]. Eggs were universal additives, used in small quantities as a popular emulsifier, modifier, stabilizer and / or mollifier. The yolk, containing the chemical lecithin, stabilizes the emulsion of the mortar. Egg whites were also known to alter and improve existing solutions in the mix by imparting plasticity, reducing brittleness and increasing workability of the mortar. Beer, too, was used an air entraining agent and was found to improve durability of the hardened set. In this study, as would have been in medieval times, the beer, a handcrafted and aged select, was purchased at a local brewery. Beeswax was popularly employed during the as an agent to prevent and counteract shrinkage. [See Table 4.3-A for further information on this material along with its subsequent properties and roles.]
4.3.3 Mortar Matrix

In order to complete a study that can identify an homogenized behavior of medieval mortars, the research addresses a variety of possible historic mortars. The range of mortars encompasses a selection of mixtures that were viable during the middle ages. The combination of additives has been carefully addressed by considering [1] how the role of each additive would interact with other materials, and [2] the rationale for the use of the additives in the construction process.

**Table 4.3-C:** The Mortar Matrix, a list of the Mortar Mixes

<table>
<thead>
<tr>
<th>Series and Mortar #</th>
<th>Base Materials</th>
<th>Additives *</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pozzolan</td>
<td>Lime</td>
</tr>
<tr>
<td>01:001</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>01:002</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>01:003</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>01:004</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>01:005</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>01:006</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>02:001</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>02:002</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>02:003</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>02:004</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>02:005</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>02:006</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

*Additives - those most commonly used during the middle ages, 1200AD to 1500AD.

4.3.4 The Fabrication Process

The prisms were fabricated under ambient laboratory conditions. Six prisms were constructed per day. Of the twelve prisms, six (6) were constructed with 1 part pozzolan, 2 parts lime and 9 parts sand, batched by weight, and the remaining six were constructed with 1 part lime, 3 parts sand, batched by weight. [See Table 4.3-D on the following page for a detailed summary of the proportions used in each mortar mix]. Mortar was prepared in accordance to ASTM C270, Standard Specification for Mortar for Unit masonry and with reference to the mixing procedures provided by Graymont, the provider company of Niagara Mature Lime Putty.
Figure 4.3-III: An Illustration of the Mortar Matrix

Table 4.3-D: Mortar Batch Factors and Weight of Materials.

<table>
<thead>
<tr>
<th>Series and Mortar #</th>
<th>H₂O Volume Consistency</th>
<th>Batch Factor *</th>
<th>Weight of Material (lbs)**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pozzolan</td>
</tr>
<tr>
<td>01:001</td>
<td>65%</td>
<td>0.007</td>
<td>0.41</td>
</tr>
<tr>
<td>01:002</td>
<td>70%</td>
<td>0.007</td>
<td>0.41</td>
</tr>
<tr>
<td>01:003</td>
<td>65%</td>
<td>0.006</td>
<td>0.33</td>
</tr>
<tr>
<td>01:004</td>
<td>70%</td>
<td>0.006</td>
<td>0.33</td>
</tr>
<tr>
<td>01:005</td>
<td>64%</td>
<td>0.006</td>
<td>0.33</td>
</tr>
<tr>
<td>01:006</td>
<td>69%</td>
<td>0.006</td>
<td>0.33</td>
</tr>
<tr>
<td>02:001</td>
<td>65%</td>
<td>0.021</td>
<td>1.67</td>
</tr>
<tr>
<td>02:002</td>
<td>70%</td>
<td>0.021</td>
<td>1.67</td>
</tr>
<tr>
<td>02:003</td>
<td>63%</td>
<td>0.017</td>
<td>1.33</td>
</tr>
<tr>
<td>02:004</td>
<td>70%</td>
<td>0.017</td>
<td>1.33</td>
</tr>
<tr>
<td>02:005</td>
<td>62%</td>
<td>0.017</td>
<td>1.33</td>
</tr>
<tr>
<td>02:006</td>
<td>68%</td>
<td>0.017</td>
<td>1.33</td>
</tr>
</tbody>
</table>

* Batch Factor was determined from the ASTM standard, C 270-06 where Batch Factor = Weight of Sand (lbs) / (density (pcf) x Volumetric Proportion). For example, 5/(80 x 0.0069) = 0.0069.

** Weight of each material was determined from the ASTM standard, C 270-06, where the Weight of Material = Volumetric Proportion x Density (pcf) x Batch Factor.
The Demec points, later used to measure strain across each joint, are glued in place before assembly and used during the fabrication process to ‘set’ the bricks at the desired mortar joint thickness. Demec points were placed on both sides of the bricks to account for out-of-plane bending; a total of six pairs of points are used to assess the displacement across each mortar joint. [See Figures 4.3-I, 4.3-II, & 4.4-IV]. Bricks were buttered with a full-face mortar bed; the middle core hole is cleared of mortar for easy in placing the prism in the testing apparatus. The prisms were then set aside to cure in a isolated, temperature-controlled location. An initial strain reading was taken on the day of fabrication and every other day thereafter for eight days. On the ninth day, the prisms were moved to an on-campus location and placed under load. The short timeframe over which specimens were allowed to cure is a result of a desire to monitor the maximum potential creep behavior of the mortars and the need to closely follow a medieval building process where compressive loads would have taken effect without purposeful curing time.

For each mortar mixture, a control sample was taken from the batch. The samples were cured and stored in ASTM approved steel shrinkage moulds 25.4 x 254 x 25.4 mm (1 x 10 x 1 inch.) in dimensions Originally, the intent was to set the mortar in the mold, later remove the sample and measure shrinkage in accordance to the ASTM C490-08 Standard, Practice for Use of Apparatus for the Determination of Length Change of Hardened Cement Paste, Mortar, and Concrete. This removal process proved to be very difficult due to the low strength of the mortar. It was decided to leave the control samples in the molds and measure shrinkage across two recessed Demec points set at a 100 mm gauge length. These control specimens were kept under the same ambient laboratory conditions as the masonry prisms.

4.4 Testing Procedure

The masonry prisms were classified into two series. Prisms were loaded after 9 days of curing. Testing on the first series began on February 22, 2008. The second series was loaded two days later, February 24, 2008. The test frame used has been described in Figure 4.2-I; an individual apparatus is constructed for each series. Figure 4.4-I shows six (6) prisms loaded into prism series 01 apparatus. Each prism is carefully strung onto the threaded rod with a steel plate in between.
A small survey was conducted to assess an appropriate stress level at which to conduct the test [see Table 4.4-A]. The value of 2.02 MPa (294 psi) was used as a minimum indicator of the stress level needed to access three phases of creep in the masonry prisms. The self-contained apparatus was loaded to an approximate constant stress value of 2.13 MPa (309 psi) or 35900 N (8080 lb) with a less than 1% error. The full-applied load is immediately maintained by the testing apparatus [see Figure 4.4-II]. Through the DAQ system, the load cell provides a constant reading of the maintained load. The system is monitored and adjusted as previously described in Section 4.2 to keep the system within the desired load range. Over the first 24 hours, the load was checked, when possible, on an hourly basis; the system required constant attention to maintain desired load range as the materials settled into a stable configuration. Thereafter, the load was monitored daily for the first 28 days and, for the remainder of the test, on a 3-5 day schedule. To date, the load error for prism series 01 and 02 is approximately 0.59% and 0.33%, respectively.

### Table 4.4-A: Observed Range of Compressive Stresses in Historical Masonry Structures

<table>
<thead>
<tr>
<th>Historic Masonry Structure Under Observation</th>
<th>Observed Range of Compressive Stresses [psi]</th>
<th>Location of Observation (if known)</th>
<th>Found in Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civic Tower of Pavia, Italy</td>
<td>Low 247, High 319, Average 283</td>
<td></td>
<td>(Anzani et al 2005)</td>
</tr>
<tr>
<td>Coronata Family Tower, Italy.</td>
<td>36.3, 109, 72.5</td>
<td></td>
<td>(Binda et al. 1997)</td>
</tr>
<tr>
<td>Church of SS. Crocifisso, Italy.</td>
<td>105, 117, 111</td>
<td></td>
<td>(Binda et al. 1997)</td>
</tr>
<tr>
<td>Tarazona Cathedral, Spain.</td>
<td>305, 305</td>
<td></td>
<td>(Lourenço 2001).</td>
</tr>
<tr>
<td>Mallorca Cathedral, Spain.</td>
<td>450, Main Arch</td>
<td>Pier Base</td>
<td>(Lourenço 2001).</td>
</tr>
<tr>
<td>Mallorca Cathedral, Spain.</td>
<td>377, 377</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MEAN Comp. Stress</strong></td>
<td>264</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MEDIAN Comp. Stress</strong></td>
<td>294</td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

* value used to determine experimental boundary conditions, based on the observations that larger numerical values occur more frequently.
Strain is measured via the Demec system, using a spring-loaded Demec gauge with a pin-needle measuring dial. To record strain, the Demec gauge is placed across a pair of points, one end held constant and the other end adjustable to 2.54 Microns or 100 Micro inches. A measurement of the distance between the points is taken and recorded in a spreadsheet. Over time, differences in this measurement provide change in length values that were converted to strain by noting the initial distances between the points. Two gauge lengths are employed, 100 mm (4”) and 50.8 mm (2”) [see Figure 4.4-IV]. The 100 mm gauge lengths provide measurements of the mortar joint strain, covering all six prisms. The 50 mm gauge lengths provide measurements for the mortar joints 01:001:a, 01:006:a, 02:001:c, and 02:006:a. These particular points are positioned on the end prisms in the series and required a smaller gauge length.

The strain in the mortar joint was recorded every second day during the unloaded time period, that being for the first nine (9) days. Once under load on day nine, the strain was recorded on the 10th day and every third day up to the 28 days. Thereafter, the strain was recorded on a 3-5 day schedule. Twelve unloaded control mortar samples were used to account for strains induced by shrinkage in the mortar and by the environmental changes in the laboratory (the room is kept at ambient temperature, 21°C (69.8°F)) [see Figure 4.4-III] Again, these specimens utilize the Demec system and a gauge length of approximately 100mm (4”). Demec points for the control specimens were set in the mortar itself, as previously described in Section 4.3.4, the Fabrication Process. Strains resulting from the loaded prisms were corrected by means of measurements taken at the same time from the control samples.

4.4.1 Compression Tests

Compressive tests were conducted to provide an additional means of validation by comparing results from this study to those available in the literature. The procedure followed was that presented in the ASTM C109 Standard, Test Method for Compressive Strength of Hydraulic Cement Mortars. Four mortar types were selected for testing: the two basic mortars of each series, lime-sand mortar 01:001 and lime-sand-pozzolan mortar 02:001, and those containing only the admixture, beer. Three (3) 50 mm (2”) cubes of each selection were placed under load. The results were recorded noting variability in the strength between those mortars containing pozzolan.
to those without. An overall comparative strength value was extracting by averaging the results of the all twelve samples.

**Figure 4.4-II:** Application of initial load onto the second series, February 24, 2008

**Figure 4.4-III:** Twelve control mortar specimens
Figure 4.4-IV: Layout of Demec System using 50mm (2") and 100mm (4") gauge lengths
4.5 Results and Discussion

While it is intended that the experiment will continue to be monitored after the completion of this study, the results thus far are promising. Results are presented and discussed for over a time period of 120 days; during this time, the prisms were loaded for 111 days. The corrected strains are plotted in Figure 4.5-I and Figure 4.5-II along with averaged trend lines. [See Appendix B for more detailed graphic data of each individual prism depicting their respective joint strains.] An initial elastic response to the load occurred at day nine (9), followed by a successful completion of the primary creep phase, and the beginnings of a secondary phase identified by a constant steady-state rate (at approximately 27 to 31 days after load inception).

4.5.1 Mortar Compressive Strength Results with Comparative Literature

To review, Bosiljkov (2001) closely studied plain lime-based mortars created using a variety of traditional, hydrated and industrial lime putties. He discovered a variable range of compressive strengths, from 0.95 MPa (138 psi) to 2.09 MPa (303 psi). Velosa and Veiga (2001) explore the use of pozzolans as additives in lime mortars to be used for rehabilitation. On average, the compressive strength of the mortars was 0.5 MPa (72 psi) at 28 days and 0.8 MPa (116 psi) at 90 days.

A sand-lime mix develops strength through the chemical process of carbonation. For sand-lime-pozzolanic mixtures, strength development is based upon the reaction between hydrated lime and finely ground amorphous silica. Conversely, it is found that the complex chemistry of a wet lime-pozzolan match closely the chemical formula of modern concrete bonding gels, C-S-H or tobermorite gel in modern concrete and hence increases the performance of cementitious materials with pozzolan as a binding ingredient (Kirc 2005).

Comparative results from Bosiljkov (2001) showed that the Azores pozzolans did not contribute towards the mechanical properties of the lime-based mortars; its compressive strengths was similar to the plain mix, from 0.6 MPa (88 psi) at 31 days and decreasing to 0.5 MPa (72 psi) at 90 days. The Cape Verde pozzolans, however, did significantly increase the strength of the lime-based mortars, showing values from 1.3 MPa (190 psi) to 2.0 MPa (290 psi).
Figure 4.5-I: Graphic Results of the Pozzolan Lime Sand Mortars, Prism Series 01
[a] Individual Results of Prisms [01:001] to [01:006]
[b] Average Performance from Six Pozzolan Lime Sand Prisms
Figure 4.5-II: Graphic Results of the Lime Sand Mortars, Prism Series_02
[a] Individual Results of Prisms [02:001] to [02:006]
[b] Average Performance from Six Lime Sand Prisms
It was initially thought that the four mortars selected for testing would provide an average assessment of the overall compressive strength of all twelve mortars. In the end, if we note the initial elastic strain results [see Figure 4.5-I and Figure 4.5-II on the previous pages] it could be inferred that the selections possibly indicated a lower average strength capacity as might be seen if the remain eight mortars were also considered. This statement, however, needs validation through further testing.

**Table 4.5-A: Results and Comparisons of Lime-based Mortar Strength Properties**

<table>
<thead>
<tr>
<th>Literature</th>
<th>Strength of Mix at 90d (MPa)</th>
<th>MOE (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bosiljkov (2001)</td>
<td>0.95 to 2.09</td>
<td>&quot;</td>
</tr>
<tr>
<td>Gimbert (2008)</td>
<td>0.67</td>
<td>0.70</td>
</tr>
<tr>
<td>Velosa and Veiga (2001)</td>
<td>0.80</td>
<td>0.5 to 2.0</td>
</tr>
</tbody>
</table>

* Values assessed at 28 days.

In general, the strengths between the pozzolanic and the non-pozzolanic mortars of this study did not statistically differ. The replicated historic lime-sand mortars at 90 days exhibited an overall compressive strength of 0.67 MPa (97 psi) and those mortars with pozzolan as a binding ingredient were found to exhibit only a slightly higher compressive strength of 0.70 MPa (102 psi). From Table 4.5-A, it can also be seen that the Young’s modulus for both sets of replicated mortars, with and without pozzolan, are more closely aligned to that of a Lime-Sand mortar Young’s modulus as presented by Velosa and Veiga (2001). For lime-sand mortars, this comparison shows a promising correlation. On the other hand, this further indicates that the pozzolan did not play a significant role in adding strength to the mortar mix. Overall, the values presented in Table 4.5-A are consistent with those found by Bosiljkov (2001) and Velosa and Veiga (2001) and helps to support the validity of the mortars used in this study.

4.5.2 The Influence of Observed Variables

Figure 4.5-I [a] and Figure 4.5-II [a] summarize the overall results of the experiment. In Figure 4.5-I [a], the creep strain for Prism Series 01 with mortars of the base formula pozzolan-lime-sand are
shown. In Figure 4.5-II [a], the creep strain for Prism Series 02 with mortars of the base formula lime-sand are shown. In comparing these figures, numerous trends are in evidence.

It can be seen that the initial strain values of the mortars and the rate at which the primary and secondary creep strains (particularly in the secondary phase) progressed did not significantly differ. It is important to note that a peak in the graphs occurs around 10 days, at the initial strain. This trend can be seen in the prisms of both series. The peak is highlighted between the initial and the primary phase of creep in Figure 4.5-I and Figure 4.5-II. This effect is probably due to an unforeseen fault in a load cell that displayed a smaller load than that actually being applied. The actual load was estimated to have reached just over 10 kips, however, because the maximum load reading on the load cell is 10 kips, this number was not verified. This oversight was corrected on the first day of loading and the sequence of loading resumed at the desired lower level (approximate 2.13MPa (309 psi) or 3.60 x 10^4 N (8080 lbs). The initial strain response to the original load, however, provides an important and valid set of information to discuss the overall performance of the mortars. Further, whether the initial load was higher or not, it can be seen that the initial strain values strongly influenced the magnitude of the maximum strain seen over time. [See Table 4.5-B for a summary and comparison of initial strains in the mortars.].

Additionally, an apparent dilation behavior appeared over the first seven days of the experiment, where the mortar appears to be in tension. This situation is near impossible since the prisms were simply curing under normal, ambient laboratory environment with no loading condition, only basic gravitational forces. This can also be seen in Figures 4.5-I and 4.5-II. Evidently, the shrinkage control specimens showed a higher shrinkage rate than the test specimens. A possible explanation is found within the parameters of the Demec system when used to monitor the shrinkage specimens. Set directly in the wet mortar, the Demec points marked the initial gauge length of the shrinkage specimens. The Demec points on the test specimens were glued to the bricks and immediately solidified in place; the change in the gauge length was calculated as change in the mortar joint, where deformation in the bricks was deemed inadmissible. In the control specimens, when the mortar was not completely set, the force exerted through the spring-loaded Demec gauge may well have ‘pushed’ the Demec points farther away from each other during the act of measuring; the initial distance between the points was not maintained. Other explanations involve chemical reactions regarding the beer with the lime and / or the pozzolan. The trend is so clearly indicated in all mortar prisms that it is believed to not be a human error. In general, the reason for the dilation behavior is not clear but it is logical to conclude that the phenomenon occurred in part
because of the general difficulty in making shrinkage measurements with the Demec system. More specifically, by selecting a dissimilar approach to measure the shrinkage data (as compared to the method by which overall strains were garnered), there evolved an inconsistency; an improved procedure is needed to better compensate for shrinkage.

The experiment was designed to monitor only the strains seen in the mortars. Henceforth, the selection of high strength bricks was important. The bricks were also monitored for compressive strain during the first 10-28 days of loading and were proven to show insignificant strain [see Figure 4.5-III]. The strains were recorded using the Demec system, where two points were placed across the width of a brick. The monitored strain, therefore, is assumed to only be representative of the mortar joint itself. Over the application of load, cracks appeared in the bricks surrounding joints 01.006.a, 01.006.b, 01.005.c, 01.004.a, 02.003.c, 02.006.a [see Appendix B for images]. This discovery, however, did not adversely affect the creep results. The performance of these joints followed similar, and more often, the same, trends as those mortar joints surrounded by intact bricks. The cracking was not surprising. Due to the large load, the relative stiffness of the system and the larger stiffness of the bricks, stress concentrations within the higher strength units were foreseeable.

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Figure 4.5-III: The Experimental Creep Behavior; An Averaged Performance Trend
4.5.3 The Influence of Additives on the Initial Strain

The data shows that, even independently of the main binding ingredients, an additive has a strong influence on the overall long-term performance of an historic mortar. Those mortars, for example, containing the additives eggs and beeswax exhibited the largest initial strains and, therefore, larger creep strain values over time. Further, those mortars containing only beer showed a smallest response to the applied load; the initial strains exhibited a range from 0.00043 in/in to 0.00175 in/in. The two base mortars without additives also exhibited response to the initial load similar to those mortars with the beer; mortar 02.001 containing lime and sand initially strained only 0.00099 in/in and mortar 01.001 containing lime, sand and pozzolan initially strained 0.00179 in/in.

Table 4.5-B: Summary of the Percentage of Shrinkage to Overall Strain

<table>
<thead>
<tr>
<th>Series, Mortar #</th>
<th>Mortar Constituents</th>
<th>Initial Strain (in/in)</th>
<th>Max Creep Strain Prim. + Sec. (in/in)</th>
<th>Max Overall Strain (in/in)</th>
<th>Max Shrinkage Strain (in/in)</th>
<th>% of Shrinkage to Overall Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>02:002</td>
<td>Lime, Sand, Br</td>
<td>4.30E-04</td>
<td>8.70E-04</td>
<td>1.30E-03</td>
<td>6.40E-04</td>
<td>49.23%</td>
</tr>
<tr>
<td>01:002</td>
<td>Lime, Sand, Pozzolan, Br.</td>
<td>1.09E-03</td>
<td>7.50E-04</td>
<td>1.84E-03</td>
<td>7.90E-04</td>
<td>42.93%</td>
</tr>
<tr>
<td>02:001</td>
<td>Lime, Sand</td>
<td>9.90E-04</td>
<td>1.32E-03</td>
<td>2.31E-03</td>
<td>6.90E-04</td>
<td>29.82%</td>
</tr>
<tr>
<td>01:001</td>
<td>Lime, Sand, Pozzolan</td>
<td>1.79E-03</td>
<td>8.40E-04</td>
<td>2.63E-03</td>
<td>1.37E-03</td>
<td>52.09%</td>
</tr>
<tr>
<td>01:004</td>
<td>Lime, Sand, Pozzolan, Eggs, Br</td>
<td>1.37E-03</td>
<td>8.30E-04</td>
<td>2.20E-03</td>
<td>3.90E-04</td>
<td>17.73%</td>
</tr>
<tr>
<td>02:004</td>
<td>Lime, Sand, Eggs, Br</td>
<td>1.75E-03</td>
<td>1.18E-03</td>
<td>2.93E-03</td>
<td>1.81E-03</td>
<td>61.77%</td>
</tr>
<tr>
<td>02:006</td>
<td>Lime, Sand, Eggs, Bwx, Br</td>
<td>1.95E-03</td>
<td>7.30E-04</td>
<td>2.68E-03</td>
<td>1.69E-03</td>
<td>63.06%</td>
</tr>
<tr>
<td>01:006</td>
<td>Lime, Sand, Pozzolan, Eggs, Bwx, Br</td>
<td>2.16E-03</td>
<td>5.20E-04</td>
<td>2.68E-03</td>
<td>2.30E-04</td>
<td>8.58%</td>
</tr>
<tr>
<td>02:003</td>
<td>Lime, Sand, Eggs</td>
<td>3.06E-03</td>
<td>2.58E-03</td>
<td>5.64E-03</td>
<td>1.35E-03</td>
<td>23.94%</td>
</tr>
<tr>
<td>01:003</td>
<td>Lime, Sand, Pozzolan, Eggs,</td>
<td>3.11E-03</td>
<td>1.05E-03</td>
<td>4.16E-03</td>
<td>6.80E-04</td>
<td>16.35%</td>
</tr>
<tr>
<td>02:005</td>
<td>Lime, Sand, Eggs, Bwx</td>
<td>5.21E-03</td>
<td>6.70E-04</td>
<td>5.88E-03</td>
<td>2.60E-03</td>
<td>44.22%</td>
</tr>
<tr>
<td>01:005</td>
<td>Lime, Sand, Pozzolan, Eggs, Bwx</td>
<td>5.89E-03</td>
<td>5.50E-04</td>
<td>6.44E-03</td>
<td>1.95E-03</td>
<td>30.28%</td>
</tr>
</tbody>
</table>

4.5.4 The Influence of Additives on the Shrinkage Rate

Table 4.5-B summarizes the maximum shrinkage strain exhibited to date in the control specimens as a percentage of the averaged maximum strain exhibited by the mortar in each prism. The results indicated that dry shrinkage in mortar 01:006 accounted for only 8.58% of the overall strain behavior. [See Appendix C for Shrinkage results of the control specimens.]
There is evidence that the combination of beeswax and eggs in the mortar was highly effective in counteracting shrinkage; note in Table 4.5-B that those mortars containing eggs and beeswax show lower relative percentages of the shrinkage to the overall strain. It can also be understood by observing the shrinkage strain rate and the magnitude of creep over which the primary creep phase occurs. The rate at which the creep strain decreases in this phase depends on; [1] the creep rate, and [2] the dry shrinkage in the material used as a daily correction factors to access creep from the overall strain value and of which is more influential to the creep results during the first 28 days of loading. The mortars containing beeswax and/or eggs show a smaller absolute magnitude of creep strain over the primary phase occurs [see Figure 4.5-II]. While initial strain of those mortars may have been large, overall this phenomenon hints at smaller changes in the shrinkage strain (hence, slower shrinkage rates) since the value of creep is calculated by [creep strain = overall strain – shrinkage strain]. For example, the mortar 01:005 (containing lime, sand, pozzolan, eggs and beeswax) began its primary creep phase at an approximate strain of 0.00395 in/in; its secondary creep phrase beginning around 0.00431 in/in. The absolute differential in the magnitude of these values is 0.00036 in/in. Additionally, mortar 01:006 (containing lime, sand, pozzolan, eggs, beer, and beeswax) showed an absolute differential in creep strain of 0.00033 in/in. The same can be said for those mortars in prism series 02. The primary creep phase of mortar 02.005 (containing lime, sand, eggs and beeswax) began with an initial strain value of approximately 0.00291 in/in and ends at approximately 0.00350 in/in. The absolute differential in the magnitudes of these values is 0.00039 in/in. In contrast, those mortars without beeswax showed a larger change in strain during the primary creep phrase, hence a faster decreasing rate of creep and a larger correction value of shrinkage. Mortar 01:001 (containing lime, sand and pozzolan) passes through the primary creep phrase over an absolute differential creep strain value of approximately 0.00060 in/in; mortar 02:004 (containing lime, sand, eggs and beer) indicated an absolute differential of creep strain across the primary phase of 0.00088in/in. Both differentials are much larger by nearly 200% of those mortars containing beeswax. [See Table 4.5-C for further iteration of this discussion.]
Figure 4.5-IV: Examples of shrinkage rates over the primary phase.
[a] Results for Mortar 01.001
[b] Results for Mortar 01.006
4.5.5 The Influence of Additives on the Magnitude of Creep

From Table 4.5-B, by realizing that the overall strain is derived from the competing shrinkage, creep and initial strain values, we can assess an overall magnitude in the creep strains for each mortar type.

When we compare the numbers of Table 4.5-B and Table 4.5-C, we can see those mortars showing large initial strains also exhibited large magnitudes of creep. For example, mortar 01:005 containing lime, sand, pozzolan, eggs and beeswax had an initial elastic response to the load of 0.00589 in/in and a differential magnitude of creep strain over time of approximately 0.00550 in/in. In contrast, mortar 02:002 containing lime, sand and beer had an initial elastic response to the load of 0.000430 in/in and a differential magnitude of creep strain over time of 0.000870 in/in.

In general, the magnitude of creep and the rate at which the primary and secondary phase progress for each mortar does not significantly differ. When we pair up the mortars of the prism series with their counterparts, there are some identifiable trends based on the additives in use. Table 4.5-C on the following page provides a good summary of these trends.

‘Crushed Brick’ Pozzolan:
Whether the mortar contained pozzolan or not, the use of pozzolan did not have a significant overall effect; a non-pozzolanic mortar did not differ significantly in the rate or the magnitude from a pozzolanic mortar. In general, the use of an additive changed the rate and the magnitude of creep more often than the use of pozzolan. In one instance, when no other additives were used, the use of pozzolan did help the rate at which creep progressed; the pozzolanic mortar [01:001] the pozzolanic and the non-pozzolanic mortar [02:001] crept at 0.000026 in/in/day and 0.000051 in/in/day, respectively.

Eggs and Beeswax:
The combined use of eggs and beeswax reduced the rate of creep through the primary and secondary creep phase. The pozzolanic mortar [01:005] with eggs and beeswax passed through the primary phase with a creep rate of 0.000016 in/in per day.; the non-pozzolanic mortar [02:005] with eggs and beeswax passed through the primary phase with a creep rate of 0.000017
in/in per day. Through the secondary phase, the pozzolanic and non-pozzolanic mortar on average crept at 0.000003 in/in/day and 0.000002 in/in/day, respectively.

**Eggs:**
A mortar with eggs as it s admixture crept at the same rate, 0.00002 in/in/day through the primary phase no matter whether pozzolan was also added or not. The secondary phase was not as consistent; the pozzolanic and non-pozzolanic mortar crept on average at 0.000007 in/in/day and 0.000003 in/in/day, respectively.

**Beer:**
The use of beer most always inhibited the mix and caused the mortar to creep at a faster rate. There was only one instance when beer was used that we can see the rate of creep through the primary phase, 0.000014 in/in/day, was low. In fact, this instance was the slowest creep rate of all twelve mortars. This pozzolanic mortar [01:006] also contained eggs and beeswax, which have shown that, when used in combination, significantly help to reduce the rate of creep.

**Table 4.5-C:** Summary of Absolute Differential in Creep and Phase Creep Rates

<table>
<thead>
<tr>
<th>Series, Mortar #</th>
<th>Mortar Constituents</th>
<th>Primary Phase Absolute Differential Creep Strain (in/in)</th>
<th>Primary Phase Creep Rate (in/in/day)</th>
<th>Secondary Phase Absolute Differential Creep Strain (in/in)</th>
<th>Secondary Phase Creep Rate (in/in/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01:001</td>
<td>Lime, Sand, Pozzolan</td>
<td>6.00E-04</td>
<td>0.000026</td>
<td>2.40E-04</td>
<td>0.000003</td>
</tr>
<tr>
<td>02:001</td>
<td>Lime, Sand</td>
<td>1.16E-03</td>
<td>0.000051</td>
<td>1.60E-04</td>
<td>0.000002</td>
</tr>
<tr>
<td>01:002</td>
<td>Lime, Sand, Pozzolan, Br</td>
<td>5.50E-04</td>
<td>0.000024</td>
<td>2.00E-04</td>
<td>0.000002</td>
</tr>
<tr>
<td>02:002</td>
<td>Lime, Sand, Br</td>
<td>6.20E-04</td>
<td>0.000027</td>
<td>2.50E-04</td>
<td>0.000003</td>
</tr>
<tr>
<td>01:003</td>
<td>Lime, Sand, Pozzolan, Eggs</td>
<td>4.50E-04</td>
<td>0.000020</td>
<td>6.00E-04</td>
<td>0.000007</td>
</tr>
<tr>
<td>02:003</td>
<td>Lime, Sand, Eggs</td>
<td>4.50E-04</td>
<td>0.000020</td>
<td>3.00E-04</td>
<td>0.000003</td>
</tr>
<tr>
<td>01:004</td>
<td>Lime, Sand, Pozzolan, Eggs, Br</td>
<td>4.80E-04</td>
<td>0.000021</td>
<td>3.50E-04</td>
<td>0.000004</td>
</tr>
<tr>
<td>02:004</td>
<td>Lime, Sand, Eggs, Br</td>
<td>8.80E-04</td>
<td>0.000038</td>
<td>3.00E-04</td>
<td>0.000003</td>
</tr>
<tr>
<td>01:005</td>
<td>Lime, Sand, Pozzolan, Eggs, Bwx</td>
<td>3.60E-04</td>
<td>0.000016</td>
<td>1.90E-04</td>
<td>0.000002</td>
</tr>
<tr>
<td>02:005</td>
<td>Lime, Sand, Eggs, Bwx</td>
<td>3.90E-04</td>
<td>0.000017</td>
<td>2.80E-04</td>
<td>0.000003</td>
</tr>
<tr>
<td>01:006</td>
<td>Lime, Sand, Pozzolan, Eggs, Bwx, Br</td>
<td>3.30E-04</td>
<td>0.000014</td>
<td>1.90E-04</td>
<td>0.000002</td>
</tr>
<tr>
<td>02:006</td>
<td>Lime, Sand, Eggs, Bwx, Br</td>
<td>4.30E-04</td>
<td>0.000019</td>
<td>3.00E-04</td>
<td>0.000003</td>
</tr>
</tbody>
</table>
4.5.6 The Effectiveness of the Additives

**Crushed Brick Pozzolan:**

When one pairs up and compares the mortars of Prism Series 01 and Prism Series 02 containing the same additives and only differentiating by the addition of the pozzolan binder, the following conclusions can be made:

- 5 out of 6 times, mortars containing only lime (Prism Series 02) as a main binding ingredient showed lower initial strain values as compared to mortar counterparts with pozzolan (Prism Series 01).
- 4 out of 6 times, mortars containing only lime (Prism Series 02) as a the main binding ingredient show higher shrinkage values as compared to mortar counterparts with pozzolan (Prism Series 01). These four instances aligned with those mortars also having a higher initial strain. Hence, mortars with only lime as a main binding ingredient shrunk further over time as compared to those mortars with lime and pozzolan.

The intention of the pozzolan was to increase the strength of the mix. The comparative initial strain values indicate the Young’s modulus values for those mortars with pozzolan is higher than those without, 1508MPa and 784MPa respectively. To state the effect of the pozzolan in regards to long-term durability, at this stage, would be premature. It appeared that those mortars with pozzolan initially withstood a larger response to the load. However, the findings from the compressive testing prove that the addition of the pozzolan did not significantly improve the overall strength of the mortars. [See Table 4.5-A].

**Beer:**

The beer was to expected to increase the durability of the mortar and also to improve its workability and flow during preparation. An air entrainer works by forming tiny air bubbles throughout the mortar mix. These bubbles work like a lubricant for sticky solutions, ‘aerating’ the mortar and making it lighter to lift and spread. The bubbles also ensure more water is held in the mix. The amount of air entrainer per mix is not well known; what is commonly thought for mortars created with portland cement is that about 10% air in the mortar is acceptable, above that the mortar is seriously weakened.
During the compression testing, on average those mortars with beer had a lower compressive strength, 88.6 psi (0.61 MPa), than those without beer, 111 psi (0.76 MPa). On the other hand, mortars containing only beer as an additive exhibited the smallest initial strain values.

In terms of workability, the use of beer created difficulties. The carbonation created a very wet, uncontrollable mix where the more one stirred the mix, the more saturated the mix became. This was not desirable for good workability in placing the mortar on the brick bed.

**Beeswax:**

Literature describes the role of beeswax in mortar as an agent to reduce and prevent shrinkage. This observation is not precisely confirmed by the results in this study, but some interesting conclusions do appear.

In summary from Section 4.5.3, those mortars with beeswax were shown to exhibit larger initial strain values and yet smaller overall percentages of shrinkage over time. The rate at which the primary phase occurs with mortars containing beeswax also appears to be slower, a hint that indeed the beeswax was helping to counteract shrinkage.

When the beeswax was added to the mortar, the process by which this was done was never previously established. In this study, the beeswax added based on volume, melted to a liquid and poured into the mix. On contact with the mixture, however, the beeswax returned to a hardened state. This created chunks in the mortar and hence may be an explanation as to why those mixes with beeswax appeared to have the highest strain values; the “aggregate” unloaded created space that when loaded was easily eliminated. A material’s ability to resist shrinkage is also further substantiated in freeze-thaw cycles, since temperature and environmental changes can influence not only the change in axial strain due to shrinkage but also the rate and the magnitude of shrinkage cracking. As a final comment, it would be interesting to attempt the use of beeswax with mortars composed of slaked ‘hot’ quick lime where the wax would have the ability to remain in a liquid state when mixed.

**Eggs:**

Literature suggests that eggs act as a plasticizer and a stabilizer, imparting plasticity and reducing brittleness. The results of this study show that indeed eggs significantly aided the mix to counteract shrinkage [see Table 4.5-B]. In fact, when used in conjunction with the beeswax, the combination proved the most effective, showing the lowest relative percentage of shrinkage.
Additionally, the eggs improved the workability of the mix; there was less water required to create a ‘solid’ mix and hence the mix certainly appeared more coagulated.

4.6 Summary of Experimental Results

The following is a list of preliminary conclusions from the empirical work of this study:

- A primary phase began immediately after the application and settling of the load.
- A secondary phase appeared around the 31st day of the experiment (23rd day from the loading).
- Mortars containing pozzolan exhibited a slightly larger average initial strain value, approximately 0.00266 in/in, in response to a constant stress value (309 psi or 2.13 MPa) as compared to the lime-sand mortars of Prism Series 02 that show an average initial strain of 0.00205 in/in. Relatively, these values are not so statistically different.
- Compressive testing shows that the strength of mortars of Prism Series 02 (0.67 MPa) containing only lime as a binder were slightly lower than those mortars of Prism Series 01 (70 MPa) containing lime and pozzolan as binders. These results are in good agreement with other values presented in the literature.
- Using the initial strain value and the application of elastic laws, mortars containing only lime as a binder retain a Young’s Modulus of $E_{P902} = 784.1$ MPa (1.137 x $10^5$ psi) while those mortars containing pozzolan retain a Young’s Modulus of $E_{P901} = 1508$ MPa (2.187 x $10^5$ psi). These results differ from values found in the literature by approximately 56% and 59%, respectively. While these results are variable, if we look at an average value of the Young’s Modulus, $E_h = 1164$ MPa for the replicated historic mortars and acknowledge that the pozzolan did not significantly improve the strength of the mix, this average value is more acceptable to those presented in the literature for lime-sand mortars, differing by only 35%.
- The holistic performance of each mortar prism did not significantly differ after the initial strain response. The magnitude and the rate at which the primary and the secondary phase progress for each of the twelve prisms are similar.
A mortar containing a particular selection of additive(s) will exhibit the same behavior trends regardless of the addition of pozzolan as a binder.

In regards to shrinkage, mortars containing eggs, beeswax and a combination thereof exhibit relatively lower values of shrinkage as a percentage of the overall corrected strain. Mortars containing only the combined additives eggs and beeswax exhibit the lowest values of shrinkage as a percentage of the overall strain. On the contrary, mortars without additives (only lime and the combination of lime and pozzolan) exhibit the higher values of shrinkage as a percentage of the overall strain. Mortars containing only the additive beer exhibit the highest values of shrinkage as a percentage of the overall strain. [See Table 4.5-C]

In regards to initial strains, mortars without additives exhibit relatively low values of initial strain, ranging from 0.00099in/in to 0.00179in/in. Mortars containing only the additive beer exhibit the lowest initial strain values, from 0.00043in/in to 0.00109in/in. On the contrary, mortars containing eggs, beeswax and a combination thereof exhibit relatively high values of initial strain, ranging from 0.00175in/in to 0.00589in/in. Mortars containing only the combined additives eggs and beeswax exhibit the highest initial strain values, 0.00521in/in and 0.00589in/in. [See Table 4.5-B].

In regards to creep rates, there was not significant difference between pozzolanic and non-pozzolanic mortars. Those mortars with relative low initial strains also showed relatively low creep rates. Mortars with the combined additive beeswax and eggs performed better by creeping slower. Mortars containing only the additive beer exhibited the higher creep rates. The basic Lime-Sand [02:001] mortar showed the highest creep rate over the primary phase. The basic non-pozzolanic mortar, Lime-Sand [02:001] showed the highest creep rate over the primary phase, 0.000051 in/in/day. The pozzolanic mortar [01:003] with eggs and beer showed the highest creep rate over the secondary phase, 0.000007 in/in/day. [See Table 4.5-C].

The addition of a ‘crushed brick’ pozzolan did not statistically improve the strength of the mortar.

Beer as an additive over worked the mix, acting as an air entrainer rather than a plasticizer.
- The use of eggs in a lime-based mortar proves effective; its role as a stabilizer and plasticizer were evident in this study.
- The additive beeswax requires further study from the perspective of [1] its role as an effective shrinkage agent and [2] the manner in which the material is added to the mix.

4.6.1 Suggested Improvements for the Experiment Setup

The manner in which the experiments were conceived in this study was very much limited by a small monetary budget and a short timeframe. If the experiment was to be repeated or expanded upon, the following improvements should be made:

- Developing a frictionless testing bed. There was little discrepancy in the data from strain readings taken at the upper Demec position compared to strain readings taken at the lower Demec position, closer to the concrete footing. [See Appendix B for comparative results at each position.] This is a good indication that any possible friction from the concrete surface did not impede the bricks. However, this surface may still have had the ability to affect the overall results from a holistic standpoint, meaning that each position, from top to bottom, was affected by the results and, therefore, any friction that was present may have reduced the overall rate at which the creep was occurring.

- Providing a more accurate application of load by adapting the machinery to a hydraulic application. This will allow full consistency of the load applied rather than achieving an acceptable range of loading. From observations during the secondary phase, the strain of the mortar joint was very sensitive to the load applied. However, the application method was not conducive to dealing with this sensitivity because of its ‘low’ tech setup; with a small adjustment on the hex nut there was a larger than acceptable jump in the applied load. Often, the load was left to fall back to the desired threshold of 2.13 MPa or 8080 lbs. This is believed to create some unlikely spikes in the creep trends. While these spikes have been ‘averaged’ out, it is important to note that the average may be above what was actually being seen.
- Designing / researching an improved method to measure and calibrate shrinkage data, particularly over the first 10 days. The use of calipers to measure the amount of displacement from the edge separation of the mortar from the mold could improve the accuracy of the measurements. Another proposal would involve the creation of twelve unloaded prisms of which could be measured in the same manner as those under load. At the same time, the design of the control specimens is not ideal. The longer the mortar is allowed to set and expel moisture, the more fragile the hardening specimen becomes.

### 4.6.2 A Homogenized Creep Behavior

The purpose of the experimental phase is to assess an overall homogenized behavior, a common trend with respect to the creep performance of lime-based ‘historic’ mortars. With three (3) mortar joints replicated for every one of the twelve (12) prisms in total and each mortar joint having six (6) Demec point pairs to assess, 108 strain measurements have been noted for every reading taken over the past 120 days. In all, that amounts to a total of 6,696 measurements to characterize an homogenized behavior of the replicated mortars. Figure 4.6-I shows a summary of the experimental results. It can be seen that those mortars of Prism Series 01 containing pozzolan exhibit a slightly larger average initial strain value, approximately 0.00266 in/in as compared to the lime-sand mortars of prism series 02 that show an average initial strain of 0.00205 in/in. Relatively, these values are especially different. Moreover, when a trend line is matched to the averaged creep performance of each mortar series, the mathematical formula is comparatively the same. For lime-sand mortars, the logarithmic equation, \( y = 0.0006\ln x - 0.0007 \), is matched to describe the data trend. For pozzolanic mortars, the logarithmic equation, \( y = 0.0007\ln x - 0.0006 \), is a close correlation. When we average the pozzolanic and the non-pozzolanic mortars into two separate trend lines [see Figure 4.6-I], we can the close correlation of all twelve mortars; an overall behavior of the two series can then be derived. It too can be matched to a logarithmic trend line, \( y = 0.0006\ln x - 0.00065 \), of which is nearly identical to the aforementioned Prism Series trend lines. Figure 4.6-I successfully describes this homogenized performance based on the initial elastic response and the creep behavior for lime-based ‘historic’ mortars.
In summary:

- A primary phase began immediately after the application and settling of the load on day 9 of the experiment (also seen as the first day of the load inception).
- A secondary phase appeared around the 31st day of the experiment (also seen as the 23rd day since the load inception).
- The Lime-Sand Mortars (PrmS_02) showed a smaller strain distribution in response to the constant stress as compared to the Lime-Sand Pozzolan (PrmS_01).

**Figure 4.6-I:** The Experimental Creep Behavior; An Averaged Performance Trend
Chapter 5
Mathematical Modeling: An Analytical Model for Homogenization

The following chapter describes the second half of objective B, to relate the homogenized behavior of creep derived from the experimental results to a basic viscoelastic mathematical model. The purpose of this objective is to generate a better representation of creep behavior as a for use in computational analysis, as later demonstrated in Chapter 7.

5.1 Viscoelastic Creep Models

Many models of creep analysis rely on Boltzmann’s principles of superposition, where the strain at any time caused by a constant stress applied earlier is independent of the effects of all other stress changes applied at all other times (Shrive et al. 1997). The most common rheological models (England and Jordaan 1975; Jordaan et al. 1977 (as cited in Shrive et al. 1997)) used for general viscoelastic (creep) analysis are the classical Maxwell model, Kelvin model, and Burger’s model, which define the total strain over time t under a sustained stress σ, relying on the assumption that creep varies linearly with stress. As previously introduced in Section 3.4.2, these models can be mathematically described by the following expressions:

The Maxwell model:

$$\epsilon(t) = \frac{\sigma}{E} + \frac{\sigma}{\lambda} t$$

The Kelvin model:

$$\epsilon(t) = \frac{\sigma}{E} \left(1 - e^{-\frac{E}{\lambda} t}\right)$$

The terms of the Maxwell and the Kelvin model form to create the Burger’s model:

$$\epsilon(t) = \frac{\sigma}{E} + \frac{\sigma}{E} \left(1 - e^{-\frac{E}{\lambda} t}\right) + \frac{\sigma}{\lambda} t \tag{5.1}$$

where,

- $E$ is the Modulus of Elasticity
- $t$ is the time in days
- $\lambda$ is a parameter related to delayed strain response of the material
The analysis to follow shows that the data in this study is best represented by the Burger’s model, described above by Shrive et al. (1997) and more descriptively represented below by Jenkins (2002):

\[ \varepsilon(t) = \varepsilon_i + \varepsilon_r \left(1 - e^{kt} \right) + \dot{\varepsilon}_{ss} t \]

where,  
\( k \) is a curve fitting constant  
\( \varepsilon_i \) is the initial strain response  
\( \varepsilon_r \) is the strain accumulated over the transition from primary to secondary  
\( \dot{\varepsilon}_{ss} \) is the steady-state strain rate defining the slope of the secondary phase

### 5.2 The Strain Distribution Graphs

In the previous chapter, Figure 4.6-I shows the entirety of the elastic plus the creep strain distribution over time \( t \) in days. The general trend is an example of long-term creep, a behavior characterized by slow deformation in a material subjected to a constant stress level. Creep is represented through distinct stages; in this study the strain distribution to date depicts three stages: the initial strain, the primary creep phase and the introduction of the secondary creep phase. A tertiary phase is not yet present in the data. From data collected to date, the three visible stages of are graphically reiterated in Figures 5.2-I, II, III on the following pages.

The initial increase of the data shown in Figure 5.2-I is a manifestation of the material response to the initial load, and hence, this phase is representative of the initial strain. The linear behavior of this region is characterized by Young’s modulus; the strain is directly proportional to the stress. The accidental overload offset, between the curing and the primary phase [see Figures 5.2-I, II, III] brought the average of the initial strains to a much higher value, 0.00236 in/in. If the overload offset is considered as inconsistent data and we note that the loading occurred on the ninth day, then the initial strain \( \varepsilon_i \) is found to be 0.00186 in/in. This value is assessed from the magnitude of strain from the time of loading up to the tenth day [see Figures 5.2-I] and is far more consistent with the flow of data. This strain value is then used to evaluate the Modulus of Elasticity [MOE] and to derive the homogenized constitutive mathematical model of the replicated mortars.
Figure 5.2-I: The Initial Strain Phase, showing $\epsilon_i$.

Figure 5.2-II: The Primary Phase, showing $\epsilon_i$ and a theoretical $\epsilon_i$. 
The primary phase of the data is highlighted in Figure 5.2-II and is characterized by a decreasing strain rate, transient creep. Since the amount of the initial strain of a material is due in part to the number of interstitial gaps initially present, the primary region is strongly dependent on the history of the material. The value of the initial elastic strain, \( \varepsilon_i \), is found to be 0.00186 in/in, this value represents a yielding point in the mortar material where the primary phase begins. Primary creep is usually less than one percent of the sum of the elastic, steady state and primary strains. In Figure 5.3-I, the variable of \( \varepsilon_i \) is introduced as a value to characterizing the absolute strain over which the primary phase occurs. Two values are presented. While the smaller value \( \varepsilon_i = 0.000438 \) in/in would yield a smaller percentage, it is assessed taken into considering the error of the unforeseen load. Hence, the larger value \( \varepsilon_i = 0.000736 \) in/in is more representative of the appropriate primary strain distribution; it is assessed from the end of the initial strain \( \varepsilon_i \) and is used to evaluate the homogenized mathematical model.

The secondary region of the data is presented in Figure 5.2-III. This phase of the strain distribution is labeled the steady-state region for obvious reasons, the strain rate \( \dot{\varepsilon}_{ss} \) (also identified as the
slope) is constant at \( \dot{e}_{ss} = 0.000001 \). In this region the rate is balanced between strain hardening and recovery effects. Hardening happens because defects within the solid interact; a tiny interstitial space intersects the region of another interstitial space. The motion can be impeded or propelled to dislodge; this retardation (or recovery) is characteristic of strain hardening.

### 5.3 Derivation of an Analytical Model

Based on Jenkins’ model [Equation 5.2] and the information presented in the previous section, an equation can be matched to the creep trend of the twelve replicated mortars by the simple action of inserting the numerical variables from the experimental data into Equation 5.2 as follows [see Figure 5.3-1 for graphical confirmation]:

\[
\begin{align*}
\text{If} & \quad \epsilon_i = 0.00186 \text{ in/in} \\
\epsilon_i & = 0.000736 \text{ in/in} \\
k & = -0.1 \\
\dot{e}_{ss} & = 0.0000007 \text{ in/in per day}
\end{align*}
\]

*Therefore, for replicated historic lime-based mortars, the axial strain \( \varepsilon(t) \) with time (days) subjected to a constant axial stress \( \sigma \) is:*

\[
\varepsilon(t) = 0.00186 + 0.000736 \left( 1 - e^{-0.1t} \right) + 0.0000007t \quad \text{[5.3]}
\]

The Jenkins model is essentially a viscoelastic Burger’s body with four parts to describe the creep. Through wide application (see Anzani et al. 2000, 2005; Binda et al. 2001; Papa et al. 2001), a modified Burger’s model has through better regression results proven to be a realistic match for the viscoelastic nature of historical masonry. Here, the Burger’s model has proven to be a good match for the replicas of historic mortars. More specifically, the model can be successfully derived using material property constants, \( K, G_1, G_2, \eta_1, \eta_2 \) (Goodman 1989).

\[
\varepsilon(t) = \frac{2\sigma}{9K} + \frac{\sigma}{3G_2} + \frac{\sigma}{3G_1} \left( 1 - e^{-\left(\frac{G_1}{\eta_1}\right)t} \right) + \frac{\sigma}{3\eta_2} t \quad \text{[5.4]}
\]
Note that one can see the similarities in the forms of Equation 5.3 and Equation 5.4. Equation 5.4 is written using the viscoelastic material property constants where correlations of the terms found in Equation 5.2 and Equation 5.3 can be made:

\[
\varepsilon_1 = \frac{2\sigma}{9K} + \frac{\sigma}{3G_2} = 0.00186, \quad \varepsilon_4 = \frac{\sigma}{3G_1} = 0.000736,
\]

\[
\dot{\varepsilon}_{ss} = \frac{\sigma}{3\eta_2} = 0.000007, \quad k = \frac{G_1}{\eta_1} = 0.1
\]

The Equation 5.4 can be expanded to yield the viscoelastic form presented by Goodman (1989). In this body, the axial strain \(\varepsilon(t)\) over time \(t\) in days when subjected to a constant axial stress \(\sigma\) is:

\[
\varepsilon(t) = \frac{2\sigma}{9K} + \frac{\sigma}{3G_2} + \frac{\sigma}{3G_1} \left( e^{-\frac{G_1}{\eta_1}t} \right) + \frac{\sigma}{3\eta_2} t \tag{5.5}
\]

where, \(\sigma\) is the constant stress in lbs/in\(^2\)

\(k\) is the bulk modulus in lbs/in\(^2\)

\(G_2\) is the elastic shear modulus in lbs/in\(^2\)

\(G_1\) controls the amount of delayed elasticity

\(\eta_1\) determines the rate of delayed elasticity

\(\eta_2\) describes the rate of viscous flow

The Figure 5.3-II graphically depicts the creep strain described by Equation 5.5, where a solid has been subjected to a uniaxial compression under deviatoric stress conditions. Two variables, \(\varepsilon_o\) and \(\varepsilon_B\), are introduced to relate various portions of the data over time.
The variable, $\varepsilon_0$, is an intercept where $t = 0$ along the creep curve and can be written as:

$$\varepsilon_0 = \frac{2\sigma}{9K} + \frac{\sigma}{3G_2} \quad [5.6]$$

The variable, $\varepsilon_B$, is the intercept of a line describing the strain of large $t$, also known as the secondary phase, and can be written as:

$$\varepsilon_B = \frac{2\sigma}{9K} + \frac{\sigma}{3G_2} + \frac{\sigma}{3G_1} \quad [5.7]$$

Knowing that the data of this study successfully matches the Burger’s model and knowing that the Goodman model, essentially the Burger’s model, can be written using viscoelastic constants as described above, then a set of material properties, abbreviated as [MP.#], can be derived to describe historic replicated lime-based mortars. The properties are confirmed by re-establishing Goodman’s model.
Evaluation of the Modulus of Elasticity, $E_h$:

The derivative of the Modulus of Elasticity is assumed to be independent of time and reliant on the initial response of the material to load. It can be evaluated through Hooke’s law, where the stress within the bounds of elasticity is found to be proportional to the strain.

$$E_h = \frac{\sigma_i}{\varepsilon_i} \quad [5.8]$$

where, $\sigma = 309 \text{ psi}$ as established in Chapter 4, Section 4.4.
$\varepsilon_i = 0.001859 \text{ in/in}$ as seen in Figure 5.24.

Therefore,

$$E_h = \frac{309 \text{ psi}}{0.00186 \text{in/in}}$$

$$E_h = 1.66 \times 10^5 \text{ psi} \quad [\text{MP.1}]$$

Evaluation of the Bulk Modulus, $K_h$:

The bulk modulus is also assumed to be independent of time and can be evaluated through simple elastic laws.

$$K_h = \frac{E_h}{3(1 - 2\nu)} \quad [5.9]$$

using, $\nu = 0.1$ (Erdogmus 2004; Navedo 1995)

$$E_h = 1.66 \times 10^6 \text{ psi}$$

Therefore,

$$K_h = 6.93 \times 10^4 \text{ psi} \quad [\text{MP.2}]$$

Evaluation of the Elastic Shear Modulus, $G_2$:

The variable $\varepsilon_o$ as described in Figure 5.3-II can be set to the value of the initial strain, $\varepsilon_i = 0.001859 \text{in/in}$. A set of known variables can be substituted in as follows to evaluate $G_2$:
From Equation 5.6, we can rearrange to isolate $G_2$,

$$G_2 = \frac{1}{3 \left( \frac{e_0}{\sigma} - \frac{2}{9K_b} \right)}$$ \[5.10\]

Therefore,

$$G_2 = 1.19 \times 10^3 \text{ psi} \hspace{1cm} \text{[MP.3]}$$

Evaluation of the property, $G_1$, to control the amount of delayed elasticity:

A trend line can be matched to the secondary phase of the data. As can be seen in Figure 5.3-I, this linear model has a y intercept at $e_B$ and a slope of $\sigma_1/3\eta_2$

So if,

$$y = 0.000001t + 0.00260$$

And,

$$y = \frac{\sigma}{3\eta_2}t + e_B$$

Then,

$$e_B = 0.00260$$

From Equation 5.7 we can rearrange to isolate $G_1$:

$$G_1 = \frac{1}{3 \left( \frac{e_B}{\sigma} - \frac{2}{9K} - \frac{1}{3G_2} \right)}$$ \[5.11\]

Therefore,

$$G_1 = 1.39 \times 10^5 \text{ psi} \hspace{1cm} \text{[MP.4]}$$

Evaluation of the property, $\eta_2$, to describe the rate of viscous flow:

Again, the secondary phase can be matched to the linear trend line described above with a y intercept at $e_B$ and a slope of $\sigma_1/3\eta_2$ as can be seen in Figure 5.3-I.

So if,

$$y = 0.000001t + 0.00260$$
And,
\[ y = \frac{\sigma}{3\eta_2} t + \epsilon_B \]

Then,
\[ \frac{\sigma}{3\eta_2} = 0.000001 \]

Further, the value of \( \sigma \) is substituted to yield the value of \( \eta_2 \):

\[ \eta_2 = 103,000,000 \text{ psi/min} \]

Evaluation of the property, \( \eta_1 \), to control the amount of delayed elasticity for Historic Mortars:

To evaluate \( \eta_1 \), the variable \( q \) must be referenced through regression. Indicated in Figure 5.3-II, \( q \) is the “…positive distance between the creep curve and the line asymptotic to the secondary creep curve” (Goodman 1989, p 208).

If the line describing the secondary creep curve is linear:

\[ \epsilon(t) = \frac{\sigma}{3\eta_2} t + \left( \frac{2\alpha}{9K} + \frac{\sigma}{3G_2} + \frac{\sigma}{3G_1} \right) \]

[5.12]

with a slope of \( \frac{\sigma}{3\eta_2} \) and y intercept at \( \epsilon_B = \left( \frac{2\alpha}{9K} + \frac{\sigma}{3G_2} + \frac{\sigma}{3G_1} \right) \)

And the overall creep behavioral model is described as:

\[ \epsilon(t) = \frac{2\alpha}{9K} + \frac{\sigma}{3G_2} + \frac{\sigma}{3G_1} t + \left( \frac{\sigma}{3\eta_2} \left( e^{-\frac{\alpha}{\eta_1}} \right) + \frac{\sigma}{3\eta_2} t \right) \]

[5.13]

Then \( q \) is denoted as the exponentially decreasing distance between the primary curve and a line describing the linear trend of the secondary phase, [see Figure 5.3-I]:
\[ q = \varepsilon(t)_a - \varepsilon(t)_b \]

\[ q(t) = \frac{\sigma}{3G_1} \left( e^{-\frac{G_1}{3\eta_1}t} \right) \tag{5.14} \]

*If the \( \log_{10} \) of both sides of Equation 5.14 are taken, the following semilog plot results where \( \log_{10} q \) versus \( t \) has intercept \( \frac{\sigma_1}{3G_1} \) and slope \( -\frac{G_1}{2.3\eta_1} \):*

\[ \log_{10} q(t) = \log_{10} \left( \frac{\sigma}{3G_1} \right) - \frac{G_1}{2.3\eta_1} t \tag{5.15} \]

By retrieving a set of \( q \) values assessed from day 11 of the data from this study (the first day of loading) until around day 27 (when the secondary creep phase appears), a linear trend line[ See Figure 5.3-II] is matched to the semilog plot of Equation 5.15 and is found to be:

\[ \log q(t) = -2.93 - 0.0465t \]
Referencing Equation 5.15 then,

\[- \frac{G_1}{2.3 \eta_1} = -0.0465\]

And therefore,

\[\eta_1 = \frac{G_1}{2.3(0.0465)}\]

\[\eta_1 = 1,300,000 \text{ psi/min} \]

Using the derived viscoelastic material properties, the following mathematical model can be used to successfully depict the primary and secondary phases of creep in historic lime-based mortars:

\[\varepsilon(t) = \frac{2\sigma}{9K_h} + \frac{\sigma}{3G_2} + \frac{\sigma}{4.17 \times 10^3} \left(1 - e^{-0.107t}\right) + \frac{\sigma}{309 \times 10^6} t\]

More specifically then, following the Goodman model, the replicated mortars subjected to an approximate constant stress, \(\sigma = 309\text{ psi}\) exhibit strain \(\varepsilon\) over time \(t\) as:

\[\varepsilon(t) = \frac{2(309)}{9(6.93 \times 10^5)} + \frac{309}{3(119 \times 10^5)} + \frac{309}{4.17 \times 10^5} \left(1 - e^{-0.107t}\right) + \frac{309}{309 \times 10^6} t\]

\[\varepsilon(t) = 0.00186 + 0.000741 \left(1 - e^{-0.107t}\right) + 0.0000001t\]

It is found that Equation 5.17 can be improved by implementing a curve fitting parameter, \(k\), to the exponential term. This modification allows the initial curve of the analytical model to achieve a further ‘best fit’ result, moving the graph to appearing between the initial experimental points rather than below [See Figure 5.3-II]. In the case of the historic lime-based mortars of this study, \(k = 1.2\). Therefore, for the replicated historic lime-based mortars of this study, subjected to an approximate constant stress, \(\sigma = 309\text{ psi}\), the strain \(\varepsilon\) over time \(t\) is:

\[\varepsilon(t) = 0.00186 + 0.000741 \left(1 - e^{-0.107(1.2)t}\right) + 0.0000001t\]
\[ \varepsilon(t) = 0.00186 + 0.000741(1 - e^{-0.128t}) + 0.000001t \]  

Note the similarities between **Equation 5.18** and **Equation 5.3**; only items in the second exponential term slightly vary. In general, it is concluded that the mathematical model to describe the behavior of viscoelastic creep strain \( \varepsilon(t) \) of historic lime-based mortars subjected to a constant stress \( \sigma \) over time \( t \) is:

\[ \varepsilon(t) = \varepsilon_i + \varepsilon_1(1 - e^{\alpha t}) + \dot{\varepsilon}_{ss} t \]  

where,

\[ \varepsilon_i = \frac{2\sigma}{9K} + \frac{\sigma}{3G_2} \]

\[ \varepsilon_1 = \frac{\sigma}{3G_1} \]

\[ \dot{\varepsilon}_{ss} = \frac{\sigma}{3\eta_2} \]

\[ \alpha = -k\left(\frac{G_1}{\eta_1}\right) \]

and where,

- \( \sigma \) is the constant stress in lbs/in\(^2\)
- \( K \) is the bulk modulus in lbs/in\(^2\)
- \( k \) is a curve fitting parameter, in this case, 1.2
- \( G_2 \) is the elastic shear modulus in lbs/in\(^2\)
- \( G_1 \) controls the amount of delayed elasticity
- \( \eta_1 \) determines the rate of delayed elasticity
- \( \eta_2 \) describes the rate of viscous flow
Figure 5.3-III: The Homogenized Behavior, A Modified Burger’s Body.

5.4 Summary and Discussion of Analytical Results

The homogenized material properties used to derive Equation 5.19 and the analytical data set are summarized in Table 5.4-A.

Table 5.4-A: Summary of Homogenized Material Properties [MP] of the Mortars

<table>
<thead>
<tr>
<th>Material Property</th>
<th>in psi</th>
<th>in MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity, $E_h$</td>
<td>166,000</td>
<td>1,150</td>
</tr>
<tr>
<td>Bulk Modulus, $K_h$</td>
<td>69,300</td>
<td>478</td>
</tr>
<tr>
<td>Elastic Shear Modulus, $G_2$</td>
<td>119,000</td>
<td>819</td>
</tr>
<tr>
<td>$G_f$</td>
<td>139,000</td>
<td>958</td>
</tr>
<tr>
<td>$\eta^2$</td>
<td>103,000,000</td>
<td>710,000</td>
</tr>
<tr>
<td>$\eta^1$</td>
<td>1,300,000</td>
<td>8,960</td>
</tr>
</tbody>
</table>
It is important to note that the value of $G_2$ listed above is 57% higher than expected by linear elastic laws. Knowing Young’s modulus of the material and using a Poisson’s ratio of $\nu = 0.1$, the Shear modulus for the mortars should equate to 75,600psi when we consider the linear equation $E/2(1+\nu)$ and the fact that the shear modulus is typically half of the Modulus of Elasticity. However, in regards to the mathematical model present herein, the larger value of $G_2$ provides an analytical data set with a closer correlation to the experimental results. Henceforth, the larger value of $G_2$ is used in the derivation of the homogenized mathematical model. This may suggest the nonlinearity of the mortar material.

The analytical results from Equation 5.19 and the empirical data are summarized and compared in Figure 5.4-I above and Table 5.4-B at the end of this section. One can see the strain distribution of the non-pozzolanic mortars (Prism Series 02) and the pozzolanic mortars (Prism Series 01) listed in the third and fourth columns. Following, in the fifth column is the Homogenized (Averaged) results of these two series. This data is used to derive the comparative Strain distribution seen in the sixth Column, “Adjusted Homogenized Experimental Data.” The analytical results derived from Equation 5.19 are listed in the seventh column. There is very good correlation between the homogenized experimental strain distribution and the analytical results. This close association is clearly depicted in Figure 5.4-I.
Further, the eighth column indicates where change in the creep phases is occurring. The initiation of the secondary phase within the experimental data is indicated by the subscript \( a \), while the initiation of the secondary phase of the analytically derived data is indicated by the subscript \( b \).

From previous figures and Chapter 4, it was concluded that the secondary phase of the experimental data began around the 31st day, the 23rd day from the application of load. This is approximately 21 days before the analytical model predicts the beginning, at 42 days from the inception of the load. The results of the analytical model present a more concise analysis since it has the ability to [1] account for the accumulation of strain each day, whereas the experimental data is taken on a 3-5 day basis; [2] avoids the influence of human error in the reading and [3] disregards the fall off in the application of a constant load. On the other hand, the experimental data present a more realistic situation where stress distributions could indeed disrupt the strain distribution. An exact day for the beginning of the secondary phase cannot be confidently specified. It is said however, that the secondary phase will appear after the initial ‘set up’ the mortar, typically specified as 28 days and between 30 to 45 days following the application of load.
The Equation 5.19 allows us to conceive of strain values in the historic mortar over a significant timeframe. This study considers of long-term strains in masonry prism with approximately 8% mortar and 92% solid unit. Accordingly, the mathematical model predicts that the primary phase initiates with a strain value of 1950με and ends around the 35th days of loading at an approximate creep strain value of 2640με. At 900 days, the accumulation of strain is expected to reach 3500με. Further, Equation 5.19 suggests that at 100 years, strain values in the historic mortars reach 39000με; at 250 years, strain values reach 93800με.

Anzani et al. (2005) in conjunction with Binda et al. (2005) monotonically loaded sample masonry prisms having composite properties of approximately 20-30% mortar 80-70% solid masonry prisms from the Civic Tower of Pavia. Cyclically loading the specimens to a sonically determined static peak stress of 2.25MPa for over 1000 days, they found that vertical strain at the beginning of the primary phase were within a range of 800με to 1400με and, nearing the end of the secondary phase (around 900 days of their study), were within a range of 1500με to 2800με.

It is acknowledged that a body to represent the transition between the secondary and the tertiary creep has not been presented in the constitutive model. This transition would be inaccurate and ill conceived since the key parameter to identifying the time when tertiary creep starts, that being the accumulation of negative volumetric strain at the onset of dilation due to fracturing and crack opening, is not defined in the data set of this study. However, it is plausible to suggest that a tertiary phase began before 500 years. Accordingly, if one matches the strain values presented by Anzani et al., then it would be suggestive to say, according to Equation 5.19, that the tertiary creep phase for historic mortars appears before 250 days is reached. It is in all possible that Equation 5.19 is susceptible to inaccuracies over the secondary phase. The secondary phase has not significantly progressed and certainly, the prisms under load do not appear to be near apparent failure.

It is acknowledged that even at 100 years, the model proposes strain values that exceed what is deemed admissible in literature. At 500 years, the strains are beyond realistic, reaching over 185000με. It is important to note that Equation 5.19 does not take into account damage in the form of dilatancy as observed for brittle materials when approaching the tertiary phase. The model proposed by Anzani et al. does so through a variety of ‘damage’ variables. Furthermore, the strains presented by Anzani et al. are descriptive of holistic over-consolidated masonry prisms taken from existing building sites. Testing is not conducted on a singular entity of the masonry as in the case of this study, where only the mortar strains are recorded. All these factors contribute
to discrepancies for comparison to the literature of which, overall, there is a scare source for doing so.

In general, the experimental work of this study should be monitored over a longer timeframe, a pseudo-creep test performed rapidly and therefore more conveniently should be considered. Further, the mathematical body depicted by Equation 5.19 requires refinement in the secondary and tertiary stages. Overall, it is promising to see that the results are within an acceptable range of magnitude to those previously presented in literature.
Table 5.4-B: Numerical Comparison of Experimental and Analytical Results

<table>
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<th>Time (days)</th>
<th>Strain Distribution (in/in)</th>
<th>Length of Study</th>
<th>Loaded</th>
<th>Lime Sand</th>
<th>Lime Sand Pozzolan</th>
<th>Homogenized Experimental Data</th>
<th>Adjusted Homogenized Experimental Data*</th>
<th>Empirical Results Using Eq. 5.14</th>
<th>Percent Error %</th>
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<td>2.24%</td>
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</table>

* Adjusted by means of reflecting an initial strain of $\varepsilon_i$.

a The Experimental Secondary Phase as visual from through the graphical data and as calculated by a constant strain rate of $\dot{\varepsilon}_{ss} = 0.0000007$ in/in.

b The Analytical Secondary Phase as calculated by a constant slope rate of $a_s = 0.000001$ in/in.
Chapter 6

Finite Element Analysis Applied to Creep in Lime Mortars

Finite Element Analysis (FEA) modeling is frequently employed to view various complexities of the built form throughout the engineering field. While there are other methods to study masonry, three-dimensional finite element codes have the potential to provide accurate solutions for structures with complex mechanisms because they have the ability to implement complex constitutive models to simulate material properties and to create difficult formulations of geometry. However, for analysts assessing historical masonry using FEA, the ability to validate and develop comprehensive whole-building models is proving difficult. While input data and results are typically validated and derived from in-situ and experimental investigations, simplifications made about material behavior are often too basic to match the sophistication of computational analysis provided by FEA programs, such as the commercially available ANSYS, to deem the model valid.

6.1 The Basic Theory of Finite Element Analysis

FEA method treats the complexities of a structure as elemental volumes. Each volume contains distinct points, or nodes. Thousands of nodes can be created for one simple model. Stress levels within a model are assessed by calculating the displacements of each node based on the following linear matrix equation:

\[ [F] = [k] \cdot [d] \]

where,
- \( F \) is the force acting within the confines of the restrained model.
- \( k \) is a numerical value determined by material property, matrix size and overall geometry.
- \( d \) is nodal displacement in the x, y and z directions of model space, also through rotation.

Analysis can be applied through various defined modeling frameworks depending on the task at hand. Some common analyses include:

[1] linear elastic finite element analysis,
kinematic limit analysis,
3 limit analysis to derive building safety variable,
4 nonlinear physical finite element analysis, and
5 nonlinear combined physical / geometrical finite element analysis.

Common idealizations for FE models associated with masonry are linearly elastic in nature. Linear analysis requires knowledge about the strength and stiffness of materials, resulting in information about the deformability and failure mechanisms of the structure. However, research has found that a nonlinear analysis is the a more appropriate method because it is able to trace a complete response of the structure from elastic behavior to its nonlinear or plastic state(s). Nonlinear analysis requires the knowledge of elastic properties, strength of materials and additional stress-strain diagrams detailing nonlinear behavior. While complexity of this idealization necessitates the use of complicated mathematical equations, this time consuming task is far outweighed by the overall benefits for understanding an inclusive behavior. The analyst is able to explore the deformational behavior and the stress distribution, time dependency, ultimate strength and the failure mechanisms. Construction sequences can also be considered in a phased analysis of the model.

While methods exist to perform nonlinear analysis, typically an analyst will construct either a simple two- or three-dimensional FEA model of which is usually assumed to be of a uniform composite, regardless of any multifaceted built form. The geometry of this model is generally created by [1] manually entering key nodes with specified coordinates that reflect the accurate size and shape of the structure, or [2] the geometry is created in another user-friendly software application from which data can later be exported as a compatible file type with the modeling program. A series of assumptions are made to describe the material. In many cases, the volume is created using tetrahedral element having various degrees of freedom at each node. Generally, the base of the model is constrained in all directions, but this action is highly dependent on the conditions of the structure in question. The material is often assumed to act linearly. From here, ANSYS meshes the model using a smart sizing assumption (mesh sizes of 1 to 10), where relatively larger mesh sizes are created around simpler geometries and finer mesh sizes define more detailed areas. The output of results is depicted as a visual graphical field detailing levels of stress, or another chosen entity, across the model. (Stenman et al. 2007).
6.2 Common Applications of FEA to the study of Masonry Structures

The computational approach to studying historical masonry through FEA presently can be described as an application through two scientific-based categories, each with their own difficulties; [1] a set of principles describing hypotheses to reference physical mechanisms, and [2] a set of secondary principles to hypotheses about the geometry and properties of the modeled structure.

The first set of principles allows the selection of an “…adequate frame theory…” The formation of this theory relies on techniques used to describe the geometry and the material in relation to continuum mechanics. The application of constitutive equations to describe long-term performance is an attempt to accurately calculate the real mechanical and strength phenomena observed in a masonry building. Computational modeling continues to be subjective in nature and hence unconventionally accepted since the analysts must make significant conclusions on their own accord that ultimately inform the optimal approach to assess each particular case.

Roca (2005) suggests the difficulties when attempting to analyze historical structures using computational approaches. He presents the conditions of three gothic churches from Spain to illustrate his findings: Tarazona Cathedral, the Barcelona Cathedral, and the Mallorca Cathedral. Accordingly, the difficulty in the computer analysis of historic masonry stems from the ignorance of historical facts about the masonry including construction processes, historical genesis, and external forces acting on the structure. In addition to the theoretical concepts brought about by historical facts, the difficulty of determining material properties, construction details and internal material compositions renders the analysis of ancient masonry virtually impossible to be entirely accurate.

Since the ability to properly diagnose the performance of a historic structure is associated with the accuracy of mathematical models, it is crucial to calibrate and validate the predictions given by the model. Typically, the validation of an FE modeling is practiced by comparing acquired results with (1) experimentally-derived information, compiled and measured from specifically designed and controlled laboratory tests, and (2) in situ investigations, where particular comparative conditions, such as the states of deformation and the extent of cracking, have been recorded. Acquisition of the comparative results for validation is achieved through various technical methods and mathematical representations. A common example is that of modal
analysis. Modal analysis is a nondestructive vibration testing procedure where the user can confirm material properties and the behavioral response of the structure by comparing experimental and constructive models of the mode shapes, the frequencies associated with observed modes and an approximated stiffness variable obtained from the inverse of displacement at a zero frequency (Erdogmus et al. 2005).

The second set of principles defines more particular hypotheses about the masonry morphology – form and proportion, stability and age. Again, the analyst encounters problems; it is seemingly impossible to record the extensive amount of information about the historic buildings. Creating a holistic model also requires extensive research on the background of the building. The historical unknowns associated with masonry buildings are extensive. Often identification of long term alterations, such as cracks and deformations developing from those existing at an earlier configuration of the building, is near impossible. Roca (2005) simply states,

…no matter the effort invested in the inspection and experimentation, the amount of information will never be enough to describe the full complexity of the structure, nor will any conceivable model be able to receive it comprehensively….additional hypotheses are needed about the best way of simplifying or idealizing – facts consistent with the very essence of modeling – the features of the particular building (pp.74).

While the idealization of geometry for FEA can be accomplished in multiple ways, the success of the model is still largely dependent on the ability of the analyst to understand the conditions required to accurately represent the specific structure. Inputting the correct geometric form and representing the correct boundary conditions are only elementary steps to creating an FE model.

In 1995, José Navedo made closing suggestions in his thesis, Plastic Analysis of Unreinforced Masonry Assemblies, remarking refinements for FE models in light of plastic theories, he suggested that an element type to more appropriately simulate lime-based mortar properties was needed. The assessment of masonry as a homogenized volume in ANSYS produces poor agreement and thus, a volume element type to represent joint, block and mortar interaction is needed.

When masonry is considered a uniform composite in FEA, many factors are neglected and numerous issues arise. A medieval building is a multifaceted built form – individual stone blocks
with rubble fill and mortar connections. In most cases, the volume is created using tetrahedral element having various degrees of freedom at each node. Generally, the base of the model is constrained in all directions, even though in actuality the foundation may not be intact or the soil may have settled over the years. The uniform composite is to a large extent assumed to act linearly, although the strength of the stone and the mortar units if modeled individually create several interactive facets. The uniform composite too is regarded as isotropic, meaning it retains identical values of a property in all crystallographic directions; this disregards the fact that materials in masonry act as anisotropic elements, where its properties are directional dependent with changing stress distributions in all directions. In masonry, the mortar causes variable stress distributions; the properties of stone, depending on its orientation and the direction in which a force is applied, will vary accordingly as well. The values of Poisson’s ratio and Young’s Modulus are typically taken as compressive, \( v \approx 0.1-0.2 \) and \( E \approx 3-10 \text{ GPa} \), since compression dominates in masonry structures. Areas of high stress typically appear where volumes transition in the model – the peaks of naves, the meeting point between the nave and the side aisles, the base of piers. And while these areas may correctly show an elevated stress level, the FEA program can incorrectly analyze these areas due to difficulty in creating elements to fit these homogenized volumes. Additionally, programs such as ANSYS allow inputs to justify the behavior of materials based on static limit values. For example, the tensile strength of mortar utilized in an ANSYS model is bounded from 200 kPa to 2000 kPa. Therefore, the limiting stress the masonry can see is the tensile strength of mortar. However, minor cracking in masonry does not signal an unstable structure. It does cause the redistribution of stress levels, a plastic behavior. Hence, the ANSYS results can erroneously imply unrealistic concentrated stresses. In areas of highly elevated stress, an analyst can create refined mesh models to further trace the stress propagation. However, results indicate that increased levels of refinement in mesh geometry tend to create multiple solutions and once again, the analyst is left to subjectively interpret the results (Boothby, 2001; Ergogmus et al. 2005; Lourenço, 2005).

6.3 Advances to FEA Masonry Modeling

Improved numerical representations of masonry are achieved through two types of modeling techniques. The discrete element model type (Pagnoni, 1994) (as cited in Ma et al., 2001), also known as micro-modeling, is often computationally intensive and focuses on fracture mechanisms
of the materials. The continuum or homogenized procedure (Page 1978; Lotfi and Shing, 1991) (as cited in Ma et al. 2001), also known as macro-modeling, is applicable to large-scale masonry representations. One method is not preferred over another, since both are applicable to various circumstances; micro-modeling is useful to analyze local behavioral conditions and macro-modeling is often used to assess sufficiently larger dimensions where stresses can be assumed to act uniformly (Lourenço, 2005; Boothby, 2001).

6.3.1 Micro-Modeling, the application of FEA discrete elemental method

Micro numerical modeling focuses on the individual components of the masonry, (brick, block, mortar joint), particularly the interfaces of these components. Micro modeling can be performed from either a detailed or simplified approach: the detailed method simulates the unit and the mortar as continuum elements; the interface between the two remains discontinuous. The simplified model represents the mortar and the unit-mortar interface as merged discontinuous elements.

Boothby and Navedo (1998) developed extensive FE micro numerical models to examine the effects of allowing joints of sand-lime mortar to open rather than develop tensile capacity in ancient masonry. The study compares mechanisms and magnitudes of deformation in the piers and arches of typical ancient unreinforced masonry construction keeping in mind the systematic influence on load distribution of buttress, walls, and vaults. The authors reasonably correlated FE models of voussoir arches and piers – in terms of mechanism, deformations and stability – clarifying various analytical and numerical techniques to assess ancient masonry in light of plastic theories. Three different characteristics of joints were applied to a pier and an arch finite element model: no joint, dry joint and joints with soft sand-lime based mortar. It is noted that while lime mortars typically underwent high deformation in a nonlinear and inelastic state under compressive stress, the models used an elastic analysis with an applied spring constant. Experimentally derived, the spring constant makes two basic assumptions [1] “that lime mortars engage in plastic behavior immediately on being loaded and [2] the loading of the structure is monotonic and static.” The simplification allows one to observe the assumed plastic hardening of lime mortars while bypassing the intricacies of recreating nonlinear elasto-plastic material behavior in the models. The results of the ANSYS models verified the fundamental behaviors of
piers and arches. Models assuming a homogenous (no joint) material were inconsistent with the actual observed deformations of the structure in question, the Basilica of Sainte Madeleine of Vézelay.

Smith et al. (1997) took a critical look at the use of the micro smeared crack approach applied in masonry FE models to analyze the nonlinear effects of brittle materials. The study introduced a more sophisticated method of modeling by pointing out the importance of the individual components in masonry. While the smeared crack approach accounted for non-tensile behavior by integrating a mesh of ‘gauss elements’ that crack under a defined limited capacity, the propagation of cracking is ill defined because it does not account for the heterogeneous nature of masonry. The block unit of masonry, while retaining nearly 5 times the tensile strength of the mortar, still has a limited tensile capacity that cannot be neglected. While cracking may develop in the solid unit, the block nature of masonry dictates that the formation of cracks occurs preferentially along the bed and perpendicular to the mortar joint of the masonry. In view of this, Smith et al. make comparable FE analyses of the smeared crack approach with a modeling technique that takes into account the individual components of masonry. Validating the models with experimental data, the authors provide substantial support to the new method that essentially uses a simplified smear crack approach for each mortar joint and unit rather than assessing the masonry from a holistic, homogenous perspective. This method, refining the parameters of meshing, allows the researcher to clearly define the materials properties of the mortar joint and the block.

6.3.2 Macro-Modeling, an application of FEA homogenization method

Macro-modeling turns out to be a more practical approach to holistic structural analysis of masonry; it represents masonry as a homogenized unit, smearing out the units, mortar and unit-mortar interfaces in a continuum model. Homogenization methods can be classified into three types of analysis:

[1] The traditional approach is focused on comparing volumes, physicality and material property of mortar joints though empirical approximations (Pande et al. 1989; Hendry 1990; Gimbert 2006; Erdogmus 2004) (as cited in Ma et al. 2001).
Homogenization techniques developed to represent compact volumes as a “periodic composite” in the continuum model (Ma et al. 2001). This approach was adequate for large-scale representation but did not provide a substantial framework for stress analysis. The author Anthoine (1995) (as cited in Ma et al., 2001) is a proponent of this method.

Advanced homogenization techniques account for stress distribution by representing masonry through micromechanics and “micro structural concepts” (Ma et al. 2001). A masonry cell containing all the geometric and constitutive information is employed as the basic building block of the continuum model. (Luciano and Sacco 1997; Alpa and Monetto 1994) (as cited in Ma et al. 2001); (Lourenço and Zucchini 2006; Ma et al. 2001).

Ma et al. (2001) addressed the difficulties of performing macro-level modeling from micro or cell-based tools, noting that it is particularly difficult to determine how to transfer detailed material properties as a micro-mechanism into a homogenized continuum model. The authors have selected a typical unit of masonry to act as representative volume element (RVE). The RVE is composed of idealized isotropic brick and mortar units and is numerically simulated to calculate the elastic moduli, strength behavior and damage behavior by responding to various boundary conditions. Within the RVE finite element model, failure of individual materials is conditional on three modes: [1] tensile failure of the mortar joint through a basic facture law, [2] a Mohr-Coulomb law is employed to simulate shear failure at the interface between mortar joint and brick, and [3] brick failure is simulated by Mazar (1986) (as cited in Ma et al. 2001) concrete damage model, where induced damage is imparted from the equivalent strain. The RVE simulation results were implemented in a plastic continuum damage model and show masonry with anisotropic material behaviors. This study only considered in-plane elastic and failure behaviors of masonry.

Lourenço and Zucchini (2006) studied continuum model methods that address the difficulties of accurately reproducing experimental responses and also account for current stipulations in the codes regulations, such as ACI (as cited in Lourenço and Zucchini, 2006), that do not accept simplified theories for analysis. Regulated formulas in the codes are considered too simple and conservative. The numerical model used an iterative procedure as common to FEA analysis and a few micro-deformation mechanisms to accurately reproduce simulations incorporated into a nonlinear continuum finite element analysis with no convergence.
The adopted coupled homogenization-damage model advanced our understanding of masonry’s behavior by representing tensile and compressive damage of the block and the mortar; the model accounts for increasing deformations due to tension, plastic flow and hardening-softening effects. To achieve this sophistication, a basic masonry cell micromechanical model developed through previous research (see the authors (Pina-Henriques and Lourenço 2003) as cited in (Lourenço and Zucchini, 2006) is adopted to a Rankine type surface tensile damage surface model and extended to represent plasticity compression behavior with a nonlinear constitutive Drucker-Prager yield surface model. When the homogenized model is compared to FE derived experimental results, the correlation of graphical information is compelling and shows very little deviation. Lourenço and Zucchini (2006) have attempted to view changes in the correlations of the models by using three various mortar types: weak, stiffer and stronger formulas. Finally, a more accurate value for the masonry strength is found when using the homogenization tool as compared to other theoretical models and code processes presented in the literature.

As a very simplified macro-modeling approach, the rule of mixtures employed by Erdogmus (2004) while producing comparative FE analyses of French and Florentine Gothic structures, linearly homogenized a comprehensive masonry construct through stiffness parameters of a mortar-masonry assembly based on length of the specimen in question.

\[
\frac{\delta_t}{l} = \frac{\delta_m + \delta_i}{l_m + l_i}
\]

*If stress over the cross section remains constant, then:*

\[
\frac{\sigma}{E_{eff}} = \frac{(\sigma \cdot l_m)}{E_m} + \frac{(\sigma \cdot l_i)}{E_i}
\]

*Stress cancels and the following results:*

\[
\frac{1}{E_{eff}} = \frac{l_m}{l} \cdot \frac{1}{E_m} + \frac{l_i}{l} \cdot \frac{1}{E_i}
\]

\[
\frac{1}{E_{eff}} = \frac{\frac{l_m}{l}}{E_m} + \frac{\frac{l_i}{l}}{E_i} \text{ and, therefore;}
\]
\[ E_{\text{eff}} = \left[ \frac{l_m/l}{E_m} + \frac{l_t/l}{E_t} \right]^{-1} \]

where,  
\( E_{\text{eff}} \) = the effective modulus of the tile mortar assembly  
\( l_m \) = length of the mortar joint  
\( l \) = the total length of the tile units and the mortar joints  
\( E_m \) = the modulus of elasticity of the mortar alone  
\( l_t \) = the length of the tile unit  
\( E_t \) = the modulus of elasticity of the tile unit alone

Elements of the above approaches are adopted in the following chapters to numerically analyze the experimental and analytical data of this study. A masonry prism is re-created in finite element analysis as a micro-modeling approach. A creep implicit time integration scheme is utilized to refine the material properties and validate the results. To move into a macro-modeling phase, a series of prisms with changing material composition percentiles are analyzed.
Chapter 7

Numerical Analysis of a Masonry Prism

Linear analysis of masonry structures, while requiring limited knowledge about the strength of materials, henceforth results in limited information about the deformability and failure mechanisms of a structure. Research has found that a nonlinear analysis is an improved numerical method to study masonry because it is possible to trace a complete response of the structure from elastic behavior to its nonlinear or plastic state(s). Nonlinear analysis requires the knowledge of elastic properties, strength of materials and additional stress-strain laws detailing the nonlinear behavior. While complexities of this idealization necessitate the derivation and use of complicated mathematical equations as demonstrated in Chapter 5, this often time consuming task may be outweighed by the overall benefits for an inclusive understanding of the behavior. The analyst is able to explore the deformation behavior and the stress distribution, time dependency, ultimate strength and the failure mechanisms. Construction sequences can later be considered as a phased analysis in the model. (Lourenço et al. 2001).

The following chapter is the final installment of this study. It addresses Objective C; to simulate the ‘homogenized’ creep behavior of the mortars in preliminary FE models and to determine appropriate methods to implement the constitutive mathematical formula derived in objective B as a means to advance the numerical simulations of historical masonry in FEA models.

In Chapter 5, a basic viscoelastic model for the nonlinear creep behavior of historic mortars is presented. This model also furnishes a set of homogenized material properties.

7.1 The Finite Element Analysis Method, a Micro-Modeling Technique

The numerical analyses in this thesis are completed using the commercially available software ANSYS (Version 11.0). The approach to modeling is conducted on a micro scale, a technique where a prism of masonry is considered in detail. All simulations to follow are carried out as static analyses, where the structure is subjected to self-weight and, as necessary, applied loads. The initial simple geometric model is designed to reflect the experimental prism and was created
through various steps where geometry, element type, meshing, boundaries, material properties, and applied load conditions were defined.

### 7.1.1 Geometry

The geometry of the model reflects the exact dimensions of a single prism in the experiments described in Chapter 4. The model is composed of four bricks and three mortar joints.

![Geometry and Meshed Elements of 8%-92% Masonry ANSYS model](image)

**Figure 7.1-I**: Geometry and Meshed Elements of 8%-92% Masonry ANSYS model

The forms are created as three-dimensional (3-D) block geometries; four brick volumes of the dimensions \( l_x = 0.0572 \text{m} \times b_y = 0.0921 \text{m} \times h = 0.203 \text{m} \) (2 1/4 inches \( \times 3 5/8 \) inches \( \times 7 5/8 \) inches) and three mortar volumes of dimensions \( l_x = 0.006 \text{m} \times b_y = 0.0921 \text{m} \times h = 0.203 \text{m} \) (1/4 inches \( \times 3 5/8 \) inches \( \times 7 5/8 \) inches). Overall, the volumes are combined in an alternating fashion where the complete model had an overall length \( l_x = 0.247 \text{m} \). [See Figure 7.1-I]. The dimensions, as does the experimental work, reflect an 8% mortar - 92% solid unit masonry
composite. It is noted further on that the simple volumes of the model could be adjusted to reflect various percentages of the mortar and the solids in a masonry unit.

The model is created by defining key points of which mark the corner nodes of each volume. From here, the four points are selected in one interval to define each of the seven individual volumes. Contact behavior between the mortar joints and the bricks are defined through shared key points, which, in turn, defined a shared two-dimensional surface area.

7.1.2 Element Type and Meshing Parameter

Once the three-dimensional volumes are formed, these entities were meshed into finite elements. In this study, the element type SOLID185 was selected for all entities. This is a simple 8-node solid brick element with the ability to implement creep behavior. This element type is used for both the bricks and the mortar volumes. This kept the model on a desirable uncomplicated level. Meshing is completed where the file size, the success in the convergence of the solution, and the resultant shape of the element employed by ANSYS are the controlling factors. The length of a brick is subdivided into ten (10) divisions and the length of a mortar joint was subdivided into four(4) divisions. These mesh sizes are determined by trial and error as sufficiently fine to reach reasonable results. The resulting meshed elements are appropriately mapped across each volume, with no errors in the process found by ANSYS.

7.1.3 Boundary and Applied Load Conditions

Boundary conditions are applied to limit deflection and/or rotation of the model. The objective of this experimental phase is to replicate most nearly the experimental results as a means of validation. As in the experiment, load is employed as a constant pressure of $2.13 \times 10^6$ Pa (309psi) on the external bedding surface of the first brick in the series of the alternating forms; the last brick in the series is restrained on the external surface in all directions.
7.1.4 Material Property Parameters

Two sets of property parameters are defined in the ANSYS model to represent the bricks and the mortar materials. The definition of the mortar model is specifically based on the experimental and analytical work previously presented.

7.1.4.1 The Solid Units

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity [E] (MPa)</th>
<th>Poisson's Ratio [v]</th>
<th>Density [d] (kg/m^3)</th>
<th>Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid Units (Limestone</td>
<td>20,000-37,250</td>
<td>0.1-0.23</td>
<td>2100-2700</td>
<td>Erdogmus (2004),</td>
</tr>
<tr>
<td>characteristics)</td>
<td></td>
<td></td>
<td></td>
<td>Navedo (1999)</td>
</tr>
<tr>
<td>ANSYS Model, Bricks</td>
<td>22,000</td>
<td>0.1</td>
<td>2300</td>
<td></td>
</tr>
</tbody>
</table>

The bricks are not intended to be the focus of this numerical analysis. The resultant strains exhibited by the mortar are of greater interest. However, the bricks are important in the overall behavior of the masonry prism and an accurate material representation for these volumes has strongly been considered. The solid units of an historical masonry construct are typically limestone. Henceforth, from available literature (Erdogmus, 2004) the range of the material properties for the solid units are given in Table 7.1-A. From this information, a reasonable set of material properties were selected for use in the ANSYS model, as also can be seen for comparison in Table 7.1-A.

7.1.4.2 The Mortar Joint; application of the analytical constitutive equation

The definition of the mortar material in the ANSYS model is specifically based on the results compiled in Chapter 5. In the computational program, the historic material is described as having an initial linear isotropic response [see Table 7.1-B] to the load and thereafter, as having an ability to creep with time hardening over a primary and secondary phase.
Table 7.1-B: Linear Elastic Material Properties for Mortar

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity [E] (MPa)</th>
<th>Poisson's Ratio [ν]</th>
<th>Density [d] (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortar</td>
<td>1,150</td>
<td>0.1</td>
<td>NA</td>
</tr>
</tbody>
</table>

ANSYS analyzes creep using implicit and the explicit time integration schemes. The implicit method is suggested as the most accurate means to represent creep in ANSYS. The program supports thirteen implicit creep models along with the tools to generate and fit creep experimental data. In this case, implicit creep model no.11 is chosen as an acceptable form for an ideal fit to experimental results, where primary and secondary phases can be described as creep strain over time:

\[
\varepsilon_{cr} = \left[C_1 \sigma^C t^C t^{-1} e^{-C_3 / T} / (C_3 + 1) \right] + \left[C_2 \sigma^C t e^{-C_2 / T} \right] \tag{7.1}
\]

Each creep model is written with specific coefficients to define. ANSYS offers a curve fitting procedure is used to derive set values for these coefficients. Information regarding the material behavior is tabulated and read into ANSYS as a formatted text file. Within that file, items such as creep strain rate, time, temperature and creep rate are defined. In regards to this study, the information is developed through Equation 5.19. Using the analytical equation as a mathematical constitutive model, a refined data set to describe the creep strains of the mortars can be extracted; one can envision a set of strain values spanning over a longer period of time than just that the 120-day experiment as described in Chapter 4. Equation 5.19 describes elastic and creep strains over time model. To use the implicit creep models in ANSYS, the initial elastic strain is eliminated from the set of data so that only primary and secondary strain data remains to be curve fitted.

To generate an acceptable solution of the coefficients, ANSYS runs a nonlinear curve regression. The results can be reviewed and verified in ANSYS or by inserting the generated coefficients into the accepted implicit creep model \[7.1]\) [see Figure 7.1-II] where strain is given in inch per inch (in/in), stress is given in Pascals (Pa) and time is given in days.
Figure 7.1-II: Curve Fitting results used to define the mortar material model.

Due to the difficulty of experimentally determining the volumetric behavior of creep in a material, the curves in Figure 7.1-II address the uniaxial linear behavior of creep. It is difficult to reconcile uniaxial measurements of creep values when the computational programming requires volumetric strain inputs. When curve fitting on the creep axial strains of Equation 5.19, a scale factor of 3.34 was found that resulted in the appropriate uniaxial behavior. This factor is implemented during the curve fitting process as an adjustment parameter to the primary strain terms. Additionally, the scale factor is approximately the observed ratio between the bulk modulus and the modulus of elasticity, that is, the ratio between the volumetric and the linear strains.

By achieving a set of working curve fitting parameters demonstrated in Figure 7.1-II, the material model for the mortar can be defined by the implicit creep model no.11 using those scaled coefficients given in Table 7.1-C.
Table 7.1-C: Coefficients for Implicit Creep Equation in ANSYS

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.62E-06</td>
<td>0.25</td>
<td>-0.72</td>
<td>-572.45</td>
<td>1010.61</td>
<td>-73.42</td>
<td>1411.43</td>
</tr>
</tbody>
</table>

7.2 Validation, the results of implementing the analytical model

The successful implementation of the coefficients given in Table 7.1-C can be seen by in Figure 7.2-I where the numerical ANSYS results of the elastic and creep strains are plotted against the analytical model, Equation 5.19, the reiteration of a refined set of the observed experimental results. One can observe in Figure 7.2-II that the initial elastic response of mortar in ANSYS is in very good correlation to that observed in the experiments, 0.00182 in/in and 0.00186 in/in respectively. At 120 days, the creep strain in the mortar of the numerical model is found to be on average 0.000820 in/in [see Figure 7.2-III]. This value is also in very good correlation to the predicted creep strain value $\varepsilon = 0.000860$in/in achieved by the analytical model and as seen in the experimental results $\varepsilon = 0.000793$ in/in at 120 days.

Figure 7.2-I: Numerical v Analytical Results of 8%-92% Masonry ANSYS model
Figure 7.2-II: Average inelastic strain in the mortar along the z-axis

Figure 7.2-III: Average creep strains in the mortar along the z-axis @ 120 days
For further validation, the numerical and an analytical strain of the masonry prism over its length are compared. The total mechanical strain seen in ANSYS is found to be $\varepsilon = 0.000231 \text{ in/in}$, where the total change in length (DMX) of the masonry prism is 0.00224 inches (0.0000568m). The linear strain of a whole masonry prism with 8% mortar and 92% solid unit is derived through elastic displacement laws is found to be $\varepsilon = 0.000232 \text{ in/in}$. These values are in excellent agreement.

### 7.3 A Method for Globalizing

It is particularly difficult to transfer incremental details of the micro or elemental level to an encompassing macro-level model in computational analysis. A simplified method attempting to express an effective overall behavior of masonry materials was previous presented by Erdogmus (2004). The overall behavior of historical Gothic masonry structures was represented in FEA models by initially formulating of an effective Modulus of Elasticity, $E_{\text{eff}}$, using a rule of mixture:

$$E_{\text{eff}} = \left[ \frac{l_m/l}{E_m} + \frac{l_t/l}{E_t} \right]^{-1}$$  \[7.1\]

where,  
- $E_{\text{eff}}$ = the effective modulus of the tile mortar assembly  
- $l_m$ = length of the mortar in prism  
- $l$ = the total length of the tile units and the mortar joints  
- $E_m$ = the modulus of elasticity of the mortar alone  
- $l_t$ = the length of the tile in prism  
- $E_t$ = the modulus of elasticity of the tile unit alone

While the formula provides an admissible method to quickly assess an overall masonry behavior, the calculation assumes that historic mortars behavior as a linearly elastic materials and ultimately dismisses an increasingly important dimension in the analysis of historic masonry, time. To explore this technique on a time scale, the numerical model is expanded to represent various percentages of mortar and solid units. Five proportions of the two basic composite in masonry are explored:

[1] 8% mortar + 92% solid unit composite  
[2] 20% mortar + 80% solid unit composite
[3] 30% mortar + 70% solid unit composite
[4] 40% mortar + 60% solid unit composite
[5] 50% mortar + 50% solid unit composite

A numerical model is created for each proportion and the creep properties of the mortar as previously described in Section 7.2 are inserted into the model. For each volume fraction, the overall displacement of the masonry prism is considered and then converted to strain. Knowing the stress under which the masonry composite acts, \( \sigma = 2.13 \text{MPa} \), an overall ‘effective’ Modulus of Elasticity \( [E_{\text{ref}}] \) can be found to represent the prism of masonry. Further, the value of the \( E \) changes in response to the change in the overall displacement of the masonry prism over time, \( t \). These trends can be plotted as seen in Figure 7.3-II. The chart is proposed as an advanced reference for representing historic masonry on a macro-level, where the masonry structure can be effectively simulated as a single homogenized material

The chart is derived over a 150-year time frame, as limited by convergence of the ANSYS model. Two sets of data for each volume fraction are shown on the chart by a solid line and by a dotted line. They can be described as bounds for the effective \( E_{\text{ref}} \) trends of the various masonry fractions. These bounds do very well to follow each other over 3-5 years. In one set, the secondary creep coefficient found in Chapter 5 was added to the ANSYS creep equation. In this model, the effective Modulus of Elasticity eventually drops to zero. In the other model, no secondary creep is considered. In this model creep progresses with the primary phase, and hence the effective \( E_{\text{ref}} \) reaches an asymptote.
Figure 7.3-I: Effective Modulus of Elasticity Chart, $E_{ref}$ v Time

Figure 7.3-II: $E_{ref}$ Chart over the first 5 years
Figure 7.3-III shows the change in the effective modulus of Elasticity as compared to the volume fraction of the mortar over time. It is clear that a linear trend does not exist and, hence, indicates the need to depict individual MOE trends for the various proportions of mortar-solid masonry materials. This is an important point since, for example, an historical masonry pier may be externally composed of masonry with far less mortar, around 10-20% of the composite, than the internal leaf, which is typically found to be composed of 40-50% mortar.

![Graph showing Mortar Volume Fraction v. Effective Modulus of Elasticity](image)

**Figure 7.3-III:** Mortar Volume Fraction v. Effective Modulus of Elasticity

### 7.4 Discussion and Summary

Acknowledging the method presented by Erdogmus (2004), a masonry prism composed of 8% mortar and 92% solid unit and employing the linear elastic mortar properties as conceived through this study, the following effective Young’s modulus would result:

\[
E_{\text{eff}} = \left[ \frac{0.75/9.75}{1.66 \times 10^5} + \frac{9/9.75}{31.9 \times 10^5} \right]^{-1}
\]
\[ E_{\text{eff}} = 1.33 \times 10^6 \text{ psi} = 9160 \text{ MPa} \]

The first five years of the chart shown in Figure 7.14 are well established; the strains in the mortar are successfully described in ANSYS over the primary phase. Using this data, where time and long-term behavior are taken into account, an initial effective modulus of elasticity for historic masonry of an 8% mortar 92% solid unit is approximately 9289MPa. The chart suggests that the value found by a purely elastic analysis, \( E_{\text{eff}} = 9160\text{MPa} \), is in good agreement to the initial effective \( E \), as it should be. Time has not progressed. However, knowing that this value decrease over time and even more so when we consider masonry morphology with 20%, 30%, 40% and 50% mortar content (at 50%, the approximate elastic \( E_{\text{eff}} = 2200\text{MPa} \)), the decrease in the effective \( E \) as influenced by creep behavior shows that, for example, at 5 years, the use of the rule of mix formula overstates an effective \( E \) by approximately 50.1%.

The Effective Modulus of Elasticity Chart is herein suggested to be only a reference, valid for analysts using FEA to model masonry as a singular material. Certainly, it is acknowledge that improvements can be made. The chart also has the potential for use in modern design applications in masonry structures built today using lime-based mortars.
Chapter 8

Conclusions

This study has investigated creep behavior in twelve lime-based historic mortars from an experimental, analytical and numerical perspective. The twelve mortars are studied in two (2) sets of six (6) stacked masonry prisms. The first set, Prism Series 01 (PrmSr_01), contains sand-lime mortars with a pozzolan made of crushed bricks as the main reactive binder. The second set, Prism Series 02 (PrmSr_02), contains only sand-lime mortars where the lime acts as the primary binding ingredient. The mortars are replicated from examples provided in the literature and incorporate three (3) popular additives from the middle ages: beer, beeswax and eggs. Particular attention has been given to using the cementitious materials with reference to standard procedures as specified in the ASTM standards. Over 120 days, the mortars were placed under a constant compressive stress of 309psi (2.13MPa). Elastic and creep strains were monitored over this time. Shrinkage was also noted as a competing variable to the magnitude of creep. A viscoelastic analytical model is extracted to describe the creep behavior of the mortars over the experimental time frame. The model also furnishes a set of homogenized material properties. The analytical results are implemented as a constitutive law in the commercial program ANSYS by using the implicit creep analysis tools provided by the program. Once a successful numerical representation of the nonlinear mortar was formulated in ANSYS, five preliminary masonry prisms with varying volume fractions were created implementing the previously established material model of the historic mortars. The numerical models are assessed over a lengthened time span of 150 years with the intention of recording trends of the effective moduli of elasticity over time with the change in material percentage. The resulting chart is introduced as a technique to refine the manner in which one simulates the homogenization of masonry in computational analyses.

The following is a detailed summary of the conclusions from this study.

8.1 Experimental Results

A successful long term testing methodology has been established. This study has refined the definition of an historic mortar by providing a process to create the mix used in the Middle Ages
through proportionality and material constituents. Compressive tests of selective mortar samples show good correlation to strength tests in the literature. The results suggest that the lime-based mortars with and without additives or with a change in the primary binder do not significantly differ in their overall performance and, henceforth, this general lack of variance allows one to homogenized the behavior into a singular descriptive trend.

A primary phase of creep began in the mortars immediately after the application and settling of the load on day 9 of the experiment. A secondary phase appears approximately 23 days after the application of the load, around the 31st day of the experiment and here after continues at a steady strain rate. This trend is more accurately described in the analytical portion of this study. The initial strains seen in the mortars and, in particular, the rate at which the primary and the secondary creep strains progress does not significantly differ. Still, significant conclusions were reached regarding the use of the pozzolan, beer, beeswax and eggs within the mortar mix.

### 8.1.1 Trends regarding the Prism Series

It is noted that the finely crushed 100-year-old brick when used as a pozzolan did not have a significant effect on the strength of mortar. Those mortars of PrmSr_01 employing crushed brick were found to be only approximately 5% stronger.

Regarding the influence of the binding ingredients on the initial strain value, it was noted that when one compares a series 01 mortar to its counterpart in PrmSr_02, 5 out of 6 of the sand-lime mortar with the same additives exhibited lower initial strain values and two thirds exhibited higher percentages of shrinkage to the overall strain.

Those mortars showing large initial strains also exhibited large magnitudes of creep. The magnitude of creep and the rate at which the primary and secondary phase progress for each mortar across the each series does not significantly differ. When we pair up the mortars of the prism series with their counterparts, there are identifiable creep performance trends based on the additives in use. In general, the use of an admixture changed the rate and the magnitude of creep more often than the use of the pozzolanic binder.
8.1.2 Trends regarding the Additives

Disregarding the primary binder, those mortars with the same additives performed in a very similar manner. For example, a sand-lime mortar of PrmSr_02 with eggs and beer performed very similarly to its counterpart in PrmSr_01, the mortar with sand, lime, pozzolan, eggs and beer. This may indicate a lack of reactivity in the crushed brick.

A mortar containing eggs and a mortar containing beeswax showed evidence of being effective against shrinkage as compared to those other mortars regardless of the presence of beer. Those mortars with a combination of eggs and beeswax were highly resistant to shrinkage over time compared to all other mortar types studied; their shrinkage rate over the primary phase was significantly less, at times almost half, than that exhibited by other mortars. On the contrary, those same mortars with beeswax and eggs exhibited the largest initial response to the applied load and the largest magnitudes of creep strain over time. A mortar containing beer or no additive showed the smallest initial strain value and, in turn, the smallest magnitudes of creep over time. In general, a mortar exhibiting large initial strain values, over time showed large magnitudes of creep strain. While the response of the mortars to shrinkage was more varied, in general, if a mortar exhibited a large initial strain value and a large magnitude of creep over time, then to follow, the percentage of shrinkage as compared to the overall strain was small.

8.1.3 Performance Results of the Individual Additives

The following is a summary detailing the performance results of each additive used in this study.

8.1.3.1 Summary Remarks about the Performance of Pozzolan

The pozzolan did not have a significant effect on the strength of mortar. The use of the pozzolanic binder did little to influence the rate and the magnitude of creep. A majority of those mortars with pozzolan fared better against the initial load application. By observation, those mortars with pozzolan appear to have better durability and far better against crumbling around the mortar joint, particularly after the initial set up phase.
8.1.3.2 Summary Remarks about the Performance of Beer

On average, those mortars with beer exhibited a lower compressive strength, 88.6psi (0.61MPa) as compared to those without beer, 111psi (0.76MPa). On the other hand, as has been previously noted, those mortars with beer showed smaller initial strain values. The use of beer most always inhibited the mix and caused the mortar to creep at a faster rate. It is suggested that the carbonation of the beer ensured the mix retained more water and, as was expected, improved the initial set up of the mortar. Hence, the beer performed like an air entrainer as expected. On the other hand, the beer did not improve workability and the initial wet mix may have allowed the mortar to retain a higher than desirable percentage of air over time.

8.1.3.3 Summary Remarks about the Performance of Beeswax

While the use of the use of the beeswax on its own did not significantly reduce the shrinkage in the mortar, when used in combination with eggs, the material proved to be highly effective against shrinkage, particularly to slow the rate at which shrinkage occurred during the primary phase of creep behavior. The beeswax acted more like an aggregate as a whole and not as a coagulating liquid. This also hindered its ability to improve the workability of the mortar. Instead, a mortar with beeswax was simply put chunky and hence aerated. In fact, since the mortar was less dense, those mortars with beeswax exhibited the largest initial strain values. A mortar with beeswax showed above average performance of creep. Those mortars with beeswax had the highest frequency of a low creep rate as compared to the other admixtures and the combined use of eggs and beeswax proved to more consistently reduced the rate of creep through the primary and secondary phase than any other combination.

8.1.3.4 Summary Remarks about the Performance of Eggs

The eggs were highly effective additives for counteracting shrinkage. This study has also found that eggs indeed act as a good stabilizer to the mix, imparting plasticity and reducing the brittleness of the mortar, as concluded from observations during the course of the experiment.
A mortar with only eggs as its admixture exhibited an average creep rate, while a mortar with eggs and beeswax crept less through the primary and the secondary phase.

8.2 Analytical Results

The analytical model is selected as an average representation of the replicated mortars over time. The model successfully takes into account the initial elastic response of the material and thereafter successfully presents two phases of creep behavior, the primary and the secondary phase. The model is based on a simple viscoelastic mathematical body, the Burger’s model, with slightly modifications with a curve fitting parameter. The initial elastic strain of the analytical model can be exactly correlated to the experimentally observed averaged initial elastic strain of the mortars, that of 0.00186 in/in. Thereafter, the analytical model is derived by furnishing a set of material properties from the experimental results.

8.2.1 The Viscoelastic Material Properties

The viscoelastic properties including Young’s modulus of Elasticity, \( E_b \); the Bulk modulus, \( K_b \); the Elastic Shear modulus, \( G_2 \); and other suitable variables to describe the rate of viscosity; \( G_1 \), \( \eta_1 \), and \( \eta_2 \), and are summarized in Table 5.4-A on page 76. A Poisson’s ratio of \( \nu = 0.1 \) is used. The modulus of elasticity, \( E \), prescribed by the analytical model is well within an admissible range for sand-lime mortars although it is approximately 34% less than those in the literature. The bulk modulus, \( K \), suggested by the analytical model is also consistent with published data. Note, that a comparative value for \( E \) is limited by availability for historic replicated mortars. The shear modulus, \( G_2 \), is 57% higher than expected by linear elastic laws. This may suggest further nonlinearity of the material.

8.2.2 Short-term Response of the Analytical Model

The model is activated immediately after the elastic response of the material and shows a primary phase for approximately 42 days. This varies from the experimental trend in which the primary
phase appears to end on the 23rd day after loading. The secondary phase cannot confidently be specified in the experimental results and so it is concluded that the analytical model may suggest a more precise beginning. Overall, the values of the analytical model are in very good agreement to those of the experimental results over the primary phase. The secondary phase has variable correlations and is in need of further research, as can be observed in the numerical phase of this study.

8.2.3 Long-term Response of the Analytical Model

The analytical model suggests a steady strain rate of \( \dot{\varepsilon}_{ss} = 0.000001 \) in/in per day in the secondary creep phase under a stress of 2.13 MPa (309 psi). However, given that the experiment has only been in progress for approximately 140 days to date, one cannot confidently state a true value of \( \dot{\varepsilon}_{ss} \). Further, the viscoelastic model proposes that the secondary phase is linear in nature due to the steady strain rate. This may also be inadmissible, but such a conclusions requires further experimental data to confidently define any nonlinearity that may exist. The model does not incorporate a tertiary phase since this experimental trend is not available. The model allows us to conceive of strain values in the historic mortar over a significant period of time. The creep strains are roughly comparable to values published in the literature. The model by Anzani et al. (2005) where the comparative values are extract from is produced from cyclical testing over 1000 days. For Anzani et al. (2005), the beginning of failure starts at 900 days. In comparison, the correlative strains achieved in the analytical model suggest that the beginning of failure as Anzani has seen at 90 days should for this study appear around 250 days after loading. The analytical model of this study does not define a point of transition from secondary to tertiary creep, nor does the experimental observations indicate such an event. In general, the strain values are within a comparable magnitude to as those presented by Anzani et al. (2005). A complementary relationship requires further research.

8.3 Numerical Results

Following the derivation of the analytical model, a refined set of data is extracted and applied as a constructive mathematical body to describe the behavior of the mortar in finite element analysis. The numerical analysis is completed using the commercially available software ANSYS (version
The program employs a curve fitting procedure to successfully match experimental data to implicit creep models as provided within the framework of ANSYS. The analytical data, as derived from observed experimental data, is successfully matched to creep model no. eleven (11), a mathematical body employing primary and secondary phases of creep with time hardening. Seven constants are extracted by ANSYS to define the body and are summarized in Table 7.1-C. These constants are specific to the data provided by the user.

The experimental work in this study observed the results of masonry prisms with volume fractions at 8% mortar and 92% solid unit. An initial numerical model is constructed in ANSYS employing the same geometry and loading scheme as seen in the experimental phase. The behavior of the mortar is successfully employed using the implicit integration scheme with a working set of curve fitting parameters as can be seen in Figure 7.1-II. A scale factor of 3.34 was implemented with the seven coefficients to refine the material model. This was necessary after it became increasingly difficult to reconcile uniaxial measurements of strains to inputs that are assumed volumetric by ANSYS. Accordingly, it is noted that the observed scale factor is found to be approximately the ratio between the bulk modulus and the modulus of elasticity as defined by linear elastic laws.

This initial 8%-92% numerical model is analyzed over a period of 150 years. The initial elastic strains rates of the mortar described by the numerical model and the analytical model exhibit are show great correlation, 0.00182 in/in and 0.00186 in/in respectively. At 120 days, the predicted numerical creep strain reaches 0.000820 in/in. This value is also in very good correlation with the results of the analytical model where \( \varepsilon_c = 0.000860 \) in/in and the experimental results where \( \varepsilon_c = 0.000793 \) in/in. Following, the numerical results are found to be in good agreement over the first 3 years or 1000 days of the analysis. Here after, the numerical results waver from what is expected by the analytical model by indicating a markedly stiffer material through lower strain values.

Using the results of the numerical analysis, a chart to describe the trends of an effective modulus of elasticity is successfully created. [See Figure 7.3-I] The overall change in length of the numerical masonry prism is recorded over time and converted into a strain. Following, an effective \( E \) is extracted as a parameter to globalize the performance of the masonry material. Four (4) additional numerical models are developed to explore the change in the effective \( E \) trend within masonry composed of 20%, 30%, 40% and 50% mortar volume fractions. Since the long-term
behavior of the numerical model cannot be confidently stated as a viable response, two sets of data to describe the effective $E$ are presented in the chart as bounds for an acceptable trend of each masonry volume fraction.

Further study is required to refine the secondary phase of the numerical results, as was also found to be the case with the analytical results. The results of the chart, however, clearly indicate that the linear elastic rule of mix used previously to homogenize masonry as a material in computational simulations is overstated as an effective material property.

### 8.4 Recommendations for Future Study

#### 8.4.1 Experimental Outcomes

The short time frame of this study certainly limited our understanding of the secondary creep phase. The experimental data recorded to date does not provide substantial results to confidently suggest a prediction for the secondary trend. Nor are there any clues to account for a transition between the secondary and the tertiary phase. The results of the analytical and the numerical model highlight this fact. A longer experimental phase is desirable for a creep study of this magnitude.

Adapting the test set up to hydraulic machinery would provide a more accurate and consistent application of the load, providing more consistent data and in turn reducing the amount of time spent monitoring the experiment. A method that more closely compliments the test set up is required for the shrinkage control specimens. The results of these readings can be improved by doing so.

It is not suggested that a future study regarding other material possibilities in mortar be completed. However, regarding the use of the beeswax, this material could be better dispersed if used in combination with a quick lime that can remain ‘hot’ during the mixing phase. Regarding the beer, this liquid should be added at a lower than 5% volumetric ratio.
8.4.2 Analytical and Numerical Outcomes

It is understood that the analytical and numerical models of this study only take into account limited linear uniaxial creep strains to define the long-term behavioral response of the mortar. The numerical procedures projected for use from this study are considered a substantial improvement to the manner in which one can simulate masonry materials as a globalized entity in finite element analysis. Still, the secondary phase requires further attention to confidently define the steady state rate and hence, a future look at the combined effects between cracking, crumbling, and linear elastic response in the mortar would benefit the analytical and numerical models for long-term predictability.

Regarding a tri-axial approach to the mortar, a more extensive look at applying damage variables to describe degradation of the material in the analytical model should be considered. This, however, would in turn require more advanced monitoring of the experimental work in order to record the volumetric response of the material.

Further research into the manner in which ANSYS implements creep behavior may also prove to be invaluable. Imbedding the analytical data as a user defined creep model rather than fitting it to a preset body in ANSYS could substantially change the outcomes of numerical model.

8.5 Transmissibility of Results

The results of this study certainly aid to increase the database of knowledge regarding lime-based mortars. The experimental work has further explored the manner in which historic mortars were created and has provided a substantial database to reiterate the significance of three popular historic additives and, more importantly, why and how they were used.

8.5.1 Lime-based Mortars as a viable “Green” alternative

It is herein appropriate to suggest that this study can be extended beyond just the scope of historical preservation and into the realm of sustainability. Environmental consciousness is talked about with enthusiastic exclaim where technologies can improve and creative paradigms can
intelligently solve our worldly issues. What is most interesting about our process to ‘green’ the
world, however, is that often we find the old, historic or ancient (where practicality triumphed) is
so often appropriately becoming the ‘new’ means to ‘green.’ This study supports the later criteria.

As the engineering world explores the processes of the building industry from an energy
standpoint, lime is quickly becoming a viable alternative to portland cement. Low in embodied
energy and cost, companies are striving to develop what the industry is fast referring to as “eco-
mortars,” a hydraulic lime-based mortar. Lime is often used as an environmentally conscious
material in foundations, slabs and mortars.

The results of this study not only provide a means to analyze historic masonry but also provide a
means to monitor over time lime-based mortars used in modern applications. The effective
modulus of elasticity chart is a viable reference in the analysis modern structures using lime-
based mortars, particularly considering the fact that the chart is limited in data to 150 years. In
fact, the entirety of this thesis provides a large database of information for further success in the
integration of lime-based products in the construction industry.

### 8.5.2 Extended Finite Element Application

The breadth of this study, in particularly, the Effective Modulus of Elasticity chart will in fact
provide a good first pass analysis for whole building finite element models of masonry. It is
certainly intended to use the chart. Beverley Minster is one fantastic example where the
applicability of the chart is warranted. [See excerpt in Appendix A for further history on the
church]. Speculations regarding the current state of deformability in historical masonry are
centered on the geometric displacement, the foundation settlement and, more recently, the
performance of creep. Therefore, it would certainly be very useful to garner results of a numerical
simulation depicting a bay of Beverley using the linear globalizing effective \( E \) and comparing
these results to the same model re-evaluated using the nonlinear effective \( E \) presented in this
study. The second model would introduce deformability related specifically to creep in the
mortar of which can be comparable to [1] the initial model and [2] the foundation to understand
the severity of creep in historic masonry constructs.
References


http://library.witpress.com/pdfs/abstracts/STR03/STR03010AU.pdf April 2007


Appendix A

An Brief Excerpt on the History of Beverly Minster

Figure A-I: The nave of the Beverley Minster, Beverley, England.
The nave has 11 bays, each about 127-129” in width; a typically ‘church’ of this size usually has 7 to 8 bays.

In northern England sits the magnificent Beverley Minster. Originally founded as a monastery in the 8th century, the existing church was built during the 13th and 14th century and represent mainly three gothic styles: Early English, Decorated and Perpendicular. Today, the pilgrimage church is regarded as one of the finest Gothic examples in Europe; its harmonious unity of all three architectural styles is easily comparable to the great Westminster. While its beauty is undeniable, the record of its numerous renovations speaks modestly of its structural integrity.

A humbling location, the site at Beverley was once a budding trade center. Since its discovery by St. John in 714, the land has undergone numerous building programs to accommodate the passage of pilgrims and, as time passed, the surrounding town. In 1188, the original Anglo-Saxon Minster was damage by fire. And while it seemed plausible to repair the building, the collapse of the central tower in 1213 ultimately cleared the foundation for a new church.

Like all medieval churches, the new construction began at the east end and progressed westward. The east and the transept are mainly Early English Gothic, characterized by pointed arches, lancet windows and Purbeck marble. In the first half of the 14th century, the nave was reconstructed in the Decorated style; the curved tracery and intricate carvings clearly define this style and accompany a series of quadripartite vaults supported by piers and columns. The west facade of the Minster is of the Perpendicular style; its spatial verticality emphasized by large pointed windows and two soaring towers and to compliment, in 1416, the great east window was replaced in the Perpendicular-style. The skillful and creative combination of all three architectural styles
indicates the care and detail taken to design the Minster; the conscious unity can clearly be seen in the continuous vaulting throughout the nave and the choir.

While splendid in its aesthetic endeavors, an overall structural crisis remained. It is well noted in literature that the early 13th century architects approached the design with lofty ambitions; the 14th and 15th century programs followed suit to achieve the daring perception of verticality in the space. The originally design intended to eliminate the external flying buttresses, an uncluttered approach for a small church whose vaults reach only to a relatively small height of 66ft (20.6m). Subsequent wrought iron tie-bars were set vertically at column capitals and the clearstory level of the transept and choir to resist any buckling from lateral thrust that may be caused by their omission. The discovery that several existing buttresses were not built structurally integral to the walls they seemingly support indicate their later addition, perhaps as early as the 13th century. The construction of the nave in the 14th century did not eliminate buttresses; in addition to the lateral support, tie rods were used within the thick walls for further stability. While the nave appears visually sturdy, its space comprised of more bays divisions than was typically seen over its short length, the proportions of column shafts are notably smaller in diameter, having a 1:2 ratio with the space between columns bases, whereas in other churches this value typically reaches 1:3 ½ or 1:4.

Over the centuries, the nave walls of Beverley have undergone significant displacement, causing the vaults to flatten. In the 18th century, the parish church had major restoration work completed by Nicholas Hawksmoor, who attempted to remedy the movement by placing ties at the roof level. He also realigned the north wall of the north transept that had been leaning four feet out onto the street. A similar program of work undertaken by Sir George Gilbert Scott at the end of the 19th century added more ties to the building with little success of stopping the movement. In the 1970s, it was decided to remove the ties placed by Hawksmoor; while the iron connections of the rods at the nave wall were causing the old limestone to spall, the system was also deemed redundant to the additions placed by Scott.

By 1989, the consulting engineering firm of Price & Myers was asked to conduct an inspection to determine the cause of movement. More spalling of the stone was found beneath the wall tie beams; it was concluded that measuring rods should be used to monitor further displacement of the structure. The devices were placed in 1991 and after 11 years, indicate an additional 5mm of movement between the Nave walls. The spreading of the walls at Beverley can be seen quite clearly at the crowning of the Nave where the vaults are loosing curvature. This flattening indicates a serious danger and if movement were allowed to continue indefinitely collapse of the Nave could easily occur.

In July of 2004, Price & Myers reported on their findings at Beverley. The investigation entailed plumbing and aligning the nave piers, capitals, clearstory windows and aisle walls. While Price & Myers discovered large out-of-plumb situations (see Appendix 1.1) in the walls of the Nave, more so to the south than the north, their analysis concludes that the spreading of the nave walls is due in part to the construction of the foundation. Here, Price & Myers concluded that at the occasional contact points between chalk block, the stress in the masonry is well above the average stress in the foundation of 0.3N/mm². Price & Myers conclude that “it very likely that there has been gradual crushing at these local hard spots” (Price et al., 2004) caused by voids and un-mortared chalk construction of the foundation.

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1 The nave at Beverley has 11 bays, each about 127-129" in width. Typically churches of this size had 7 to 8 bays.
Price & Myer have proven that the existence of excessive stress from unpredicted point loads exists in the foundation. Long term creep within the bounds of the church may very well exist and progressively add to the spreading of the walls. Price and Myer conclude that the north wall sees less movement because of its restrained situation – the buttressing effects of the Highgate porch – but this is a possibility and not a proven circumstance. Intuitively, the report also indicates that the West end, restrained by the west tower and the East end, built against the walls of the transepts find enough stiffness to resist the turning out of the nave walls; hence much of the spreading is occurring at the center of the walls. Are these walls however, falling in with the spreading of the nave?
Appendix B

Creep Strain Results for Prism Series

[Graph showing creep strain results over time for different joints in a prism series.]

[Images of the prism series showing different sections labeled as a, b, and c.]
Appendix C

Shrinkage Results of Control Specimens

The following graphs show shrinkage readings from twelve samples of the replicated mortars, each deemed a control specimen for strain correction.
Strain results of Shrinkage Control Specimens

[a] Results of mortar specimens [01:001] to [01:006]

[b] Results of mortar specimens [02:001] to [02:006]
Appendix D

Numerical files for Application to ANSYS

MACRO FILE to extract average creep strains in mortar elements

esel,s,mat,2  !selecting all the elements of mat=2
/post1
set,last
*get,nsb,active,0,solu,ncmss  !get the total number of loadsteps
*get,nls,active,0,solu,ncmss  !get the total number of substeps
*dim,zcs,table,nsb+nls,2
zcs(0,1) = 1  !initializing
zcs(0,2) = 2
k = 1

*do,i,1,nls,1  !loop for loadsteps
  set,i,first  !set first substep number
  *get,fss,active,0,set,sbst  !get first substep number
  set,i,last  !similarly, for last substep of that loadstep
*do,j,fss,ss,1  !loop over substeps
  set,i,j
  allsel,below,elem  !select all the entities
  etable,tab1,epcr,2
  ssum  !create an element table
  *get,nnum,elem,0,count  !counting the number of elements
  *get,nsum,ssum,0,item,tab1
  avg=nsum/nnum  !get the sum and store it in nsum
  *get,ts,active,0,set,time
  zcs(k,1) = ts  !time of that substep
  zcs(k,2) = avg
  k = k+1
*endo
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DATA FILE for implicit creep analysis and curve fitting procedure

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Numerical Model of Masonry Prism with 8%-92% Volume Fraction

LOG FILE 7.1.1

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KEYW, PR_SET, 1
KEYW, PR_STRUCT, 1
KEYW, PR_THERM, 0
KEYW, PR_FLUID, 0
KEYW, PR_ELMAG, 0
KEYW, MAGNOD, 0
KEYW, MAGEDG, 0
KEYW, MAGHFE, 0
KEYW, MAGELC, 0
KEYW, PR_MULTI, 0
KEYW, PR_CFD, 0

! c***DEFINE KEYPOINTS
K.1, -0.046812, 0.096837, 0.012699
K.2, 0.046812, 0.096837, 0.012699
K.3, 0.046812, -0.096837, 0.012699
K.4, -0.046812, -0.096837, 0.012699
K.5, -0.046812, 0.096837, 0.069850
K.6, 0.046812, 0.096837, 0.069850
K.7, 0.046812, -0.096837, 0.069850
K.8, -0.046812, -0.096837, 0.069850
K.9, -0.046812, 0.096837, 0.076200
K.10, 0.046812, 0.096837, 0.076200
K.11, 0.046812, -0.096837, 0.076200
K.12, -0.046812, -0.096837, 0.076200
K.13, -0.046812, 0.096837, 0.133350
K.14, 0.046812, 0.096837, 0.133350
K.15, 0.046812, -0.096837, 0.133350
K.16, -0.046812, -0.096837, 0.133350
K.17, -0.046812, 0.096837, 0.139699
K.18, 0.046812, 0.096837, 0.139699
K.19, 0.046812, -0.096837, 0.139699
K.20, -0.046812, -0.096837, 0.139699
K.21, -0.046812, 0.096837, 0.196850
K.22, 0.046812, 0.096837, 0.196850
K.23, 0.046812, -0.096837, 0.196850
K.24, -0.046812, -0.096837, 0.196850
K.25, -0.046812, 0.096837, 0.203200
K.26, 0.046812, 0.096837, 0.203200
K.27, 0.046812, -0.096837, 0.203200
K.28, -0.046812, -0.096837, 0.203200
K.29, -0.046812, 0.096837, 0.260350
K.30, 0.046812, 0.096837, 0.260350
K.31, 0.046812, -0.096837, 0.260350
K.32, -0.046812, -0.096837, 0.260350
C***CREATE VOLUMES
V, 1,2,3,4,5,6,7,8
V, 5,6,7,8,9,10,11,12
V, 9,10,11,12,13,14,15,16
V, 13,14,15,16,17,18,19,20
V, 17,18,19,20,21,22,23,24
V, 21,22,23,24,25,26,27,28
V, 25,26,27,28,29,30,31,32

C***DEFINE ELEMENT TYPES
! C***element type block and mortar
ET,1,SOLID185
MPTEMP,.......,
MPTEMP,1,295.37
MPDATA,EX,1,,2.2e+10
MPDATA,PRXY,1,,0.1
MPTEMP,.......,
MPTEMP,1,295.37
MPDATA,EX,2,,1.15e+9
MPDATA,PRXY,2,,0.1

TB,CREEP,2,1,7,11
TBTEMP,295.37
TBDATA,,1.67E-007,0.253398054,-0.723676415,-572.4462374,1010.6142496,-73.4153224
TBDATA,,1411.4267819,.....

! C***element attributes #1: block
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM_,Y,VOLU
VSEL,,,,P51X
CM_,Y1,VOLU
CMSEL,S_,Y
!
CMSEL,S_,Y1
VATT, 1, , 1, 0
CMSEL,S_,Y
CMDELE_,Y
CMDELE_,Y1
!

! C***element attributes #2: nonlinear mortar
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM_,Y,VOLU
VSEL,,,,P51X
CM_,Y1,VOLU
CMSEL,S_,Y
!
CMSEL,S_,Y1
VATT, 2, 1, 0
CMSEL,S_,Y
CMDELE_,Y
CMDELE_,Y1
!
! C***MESH ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM_,Y,LNE
LSEL,,.,P51X
CM_,Y1,LNE
CMSEL,,Y
!*
LESIZE_,Y1,,10,,1
!*
FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM_,Y,LNE
LSEL,,.,P51X
CM_,Y1,LNE
CMSEL,,Y
!*
LESIZE_,Y1,,4,,1
!*
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM_,Y,VOLU
VSEL,,.,P51X
CM_,Y1,VOLU
CHKMSH,"VOLU"
CMSEL,S_,Y
!*
VSWEEP_,Y1
!*
CMDELE_,Y
CMDELE_,Y1
CMDELE_,Y2
!*
! VPLOT
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM_,Y,VOLU
VSEL,,.,P51X
CM, Y1, VOLU
CHKMSH, VOLU
CMSEL, S, Y
!*
VSWEET, Y1
!*
CMDELE, Y
CMDELE, Y1
CMDELE, Y2
!*
FINISH
/SOL
FLST, 2, 1, 5, ORDE, 1
FITEM, 2, 1
!*
/GO
DA, P51X, ALL, 0
FLST, 2, 1, 5, ORDE, 1
FITEM, 2, 36
FLST, 2, 1, 5, ORDE, 1
FITEM, 2, 36
/GO
!*
SFA, P51X, 1, PRES, 2130480
!*
/SOLU
RATE, OFF
TIME, 1.0E-8
SOLVE

LOG FILE 7.1.2 with secondary creep

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/PREP7
/NOPR
/title, NONLINEAR: 8-92 masonry prism 4.1.2
/PMETH, OFF, 0
KEYW, PR_SET, 1
KEYW, PR_STRUC, 1
KEYW, PR_THERM, 0
KEYW, PR_FLUID, 0
KEYW, PR_ELMAG, 0
KEYW, MAGNOD, 0
KEYW, MAGEDG, 0
KEYW, MAGHFE, 0
KEYW, MAGELC, 0
KEYW, PR_MULTI, 0
KEYW, PR_CFD, 0
!
! c**DEFINE KEYPONTS
K, 1, -0.046812, 0.096837, 0.012699
K, 2, 0.046812, 0.096837, 0.012699
K, 3, 0.046812, -0.096837, 0.012699
K, 4, -0.046812, -0.096837, 0.012699
K.5, -0.046812,0.096837,0.069850
K.6, 0.046812,0.096837,0.069850
K.7, 0.046812,0.096837,0.069850
K.8, -0.046812,-0.096837,0.069850
K.9, -0.046812,0.096837,0.076200
K.10, 0.046812,0.096837,0.076200
K.11, 0.046812,-0.096837,0.076200
K.12, -0.046812,-0.096837,0.076200
K.13, -0.046812,0.096837,0.133350
K.14, 0.046812,0.096837,0.133350
K.15, 0.046812,-0.096837,0.133350
K.16, -0.046812,-0.096837,0.133350
K.17, -0.046812,0.096837,0.139699
K.18, 0.046812,0.096837,0.139699
K.19, 0.046812,-0.096837,0.139699
K.20, -0.046812,-0.096837,0.139699
K.21, -0.046812,0.096837,0.196850
K.22, 0.046812,0.096837,0.196850
K.23, 0.046812,-0.096837,0.196850
K.24, -0.046812,-0.096837,0.196850
K.25, -0.046812,0.096837,0.203200
K.26, 0.046812,0.096837,0.203200
K.27, 0.046812,-0.096837,0.203200
K.28, -0.046812,-0.096837,0.203200
K.29, -0.046812,0.096837,0.260350
K.30, 0.046812,0.096837,0.260350
K.31, 0.046812,-0.096837,0.260350
K.32, -0.046812,-0.096837,0.260350

! C***CREATE VOLUMES
V. 1,2,3,4,5,6,7,8
V. 5,6,7,8,9,10,11,12
V. 9,10,11,12,13,14,15,16
V. 13,14,15,16,17,18,19,20
V. 17,18,19,20,21,22,23,24
V. 21,22,23,24,25,26,27,28
V. 25,26,27,28,29,30,31,32

! C***DEFINE ELEMENT TYPES
! C***element type block and mortar
ET.1,SOLID185
MPTEMP,......
MPTEMP,1,295.37
MPDATA,EX.1,.115e+9
MPDATA,PRXY.1,.01
MPTEMP,......
MPTEMP,1,295.37
MPDATA,EX.2,.115e+9
MPDATA,PRXY.2,.01

! TB,CREEP.2,1,7,11
TBTEMP,295.37
TBDATA,4.76E-007,.017,-.0723676415,-.572.4462374,.96736E-012,1,
TBDATA,.0,....
! C***element attributes #1: block
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM._Y,VOLU
VSEL,,.,P51X
CM._Y1,VOLU
CMSEL,S_,Y
*
CMSEL,S_,Y1
VATT, 1,, 1, 0
CMSEL,S_,Y
CMDELE_,Y
CMDELE_,Y1
*
CMSEL,S_,Y1
VATT, 2,, 1, 0
CMSEL,S_,Y
CMDELE_,Y
CMDELE_,Y1
*
! C***ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM._Y,LNE
LSEL,,.,P51X
CM._Y1,LNE
CMSEL.,_Y
*
LESIZE_.Y1,,10,,1
*
FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM._Y,LNE
LSEL,,.,P51X
CM._Y1,LNE
CMSEL,:,Y
!*  
LESIZE,:,Y1,:,4,:,4,:,1  
!*  
FLST,5,3,6,ORDE,3  
FITEM,5,2  
FITEM,5,4  
FITEM,5,6  
CM,:,Y,VOLU  
VSEL,, ,,P51X  
CM,:,Y1,VOLU  
CHKMSH,'VOLU'  
CMSEL,S,:,Y  
!*  
VSweep,:,Y1  
!*  
CMDELE,:,Y  
CMDELE,:,Y1  
CMDELE,:,Y2  
!*  
! Vplot  
FLST,5,4,6,ORDE,4  
FITEM,5,1  
FITEM,5,3  
FITEM,5,5  
FITEM,5,7  
CM,:,Y,VOLU  
VSEL,, ,,P51X  
CM,:,Y1,VOLU  
CHKMSH,'VOLU'  
CMSEL,S,:,Y  
VSweep,:,Y1  
CMDELE,:,Y  
CMDELE,:,Y1  
CMDELE,:,Y2  
FINISH  
/SOL  
FLST,2,1,5,ORDE,1  
FITEM,2,1  
/GO  
DA,P51X,ALL,0  
FLST,2,1,5,ORDE,1  
FITEM,2,36  
FLST,2,1,5,ORDE,1  
FITEM,2,36  
/GO  
SFA,P51X,1,PRES,2130480  
/SOLU  
RATE,OFF  
TIME, 1.0E-8  
SOLVE
Numerical Model of Masonry Prism with 20%-80% Volume Fraction

LOG FILE 7.2.1

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KEYW,PR_STRUC,1
KEYW,PR_THERM,0
KEYW,PR_FLUID,0
KEYW,PR_ELMAG,0
KEYW,MAGNOD,0
KEYW,MAGEDG,0
KEYW,MAGHFE,0
KEYW,MAGELC,0
KEYW,PR_MULTI,0
KEYW,PR_CFD,0
!
! c**DEFインKEYPOINTS
K.1,-0.046812,0.096837,0.012699
K.2, 0.046812,0.096837,0.012699
K.3, 0.046812,-0.096837,0.012699
K.4,-0.046812,-0.096837,0.012699
K.5,-0.046812,0.096837,0.069850
K.6, 0.046812,0.096837,0.069850
K.7, 0.046812,-0.096837,0.069850
K.8,-0.046812,-0.096837,0.069850
K.9,-0.046812,0.096837,0.08885
K.10, 0.046812,0.096837,0.08885
K.11, 0.046812,-0.096837,0.08885
K.12,-0.046812,-0.096837,0.08885
K.13,-0.046812,0.096837,0.14600
K.14, 0.046812,0.096837,0.14600
K.15, 0.046812,-0.096837,0.14600
K.16,-0.046812,-0.096837,0.14600
K.17,-0.046812,0.096837,0.16500
K.18, 0.046812,0.096837,0.16500
K.19, 0.046812,-0.096837,0.16500
K.20,-0.046812,-0.096837,0.16500
K.21,-0.046812,0.096837,0.22215
K.22, 0.046812,0.096837,0.22215
K.23, 0.046812,-0.096837,0.22215
K.24,-0.046812,-0.096837,0.22215
K.25,-0.046812,0.096837,0.24115
K.26, 0.046812,0.096837,0.24115
K.27, 0.046812,-0.096837,0.24115
K.28,-0.046812,-0.096837,0.24115
K.29,-0.046812,0.096837,0.29830
K.30, 0.046812,0.096837,0.29830
K.31, 0.046812,-0.096837,0.29830
K.32,-0.046812,-0.096837,0.29830
! C***CREATE VOLUMES
V, 1,2,3,4,5,6,7,8
V, 5,6,7,8,9,10,11,12
V, 9,10,11,12,13,14,15,16
V, 13,14,15,16,17,18,19,20
V, 17,18,19,20,21,22,23,24
V, 21,22,23,24,25,26,27,28
V, 25,26,27,28,29,30,31,32
!
! c***DEFINE ELEMENT TYPES
! C***element type block and mortar
ET,1,SOLID185
MPTEMP,,,,,,
MPTEMP,1,295.37
MPDATA,EX,1.,2,2e+10
MPDATA,PRXY,1.,0.1
MPTEMP,,,,,,
MPTEMP,1,295.37
MPDATA,EX,2.,1.15e+9
MPDATA,PRXY,2.,0.1
!
TB,CREEP,2,1.7,11
TBTEMP,295.37
TBDATA,,16.17E-007,0.253398054,-0.723676415,-572.4462374,1010.6142496,-73.4153224
TBDATA,,1411.4267819,,
!
! C***element attributes #1: block
FLST,5,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM,,Y,VOLU
VSEL,,P51X
CM,,Y1,VOLU
CMSEL,S,,Y
!
CMSEL,S,,Y1
VATT,,1.,1.,0
CMSEL,S,,Y
CMDELE,,Y
CMDELE,,Y1
!
! C***element attributes #2: nonlinear mortar
FLST,5,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM,,Y,VOLU
VSEL,,P51X
CM,,Y1,VOLU
CMSEL,S,,Y
!
CMSEL,S_,Y1
VATT, 2, 1, 0
CMSEL,S_,Y
CMDELE_,Y
CMDELE_,Y1
!
! C***MESH ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM_,Y,LINELN
LSEL,,.,P51X
CM_,Y1,LINELN
CMSEL,,_Y
!
LESIIZE_,Y1,,10,,1
!
FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM_,Y,LINELN
LSEL,,.,P51X
CM_,Y1,LINELN
CMSEL,,_Y
!
LESIIZE_,Y1,,6,,1
!
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM_,Y,VOLU
VSEL,,.,P51X
CM_,Y1,VOLU
CHKMSH,"VOLU"
CMSEL,S_,Y
!
VSWEEP_,Y1
!
CMDELE_,Y
CMDELE_,Y1
CMDELE_,Y2
!
! VPLOT
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM_,Y,VOLU
VSEL,,.,P51X
LOG FILE 7.2.1 with secondary creep

/FILNAME,NONLINEAR_20-80_4.2.2_masonryprism,0
/PREP7
/NOPR
/title, NONLINEAR: 20-80 masonry prism 4.2.2
/PMETH,OFF,0
/KEYW,PR_SET,1
/KEYW,PR_STRUCT,1
/KEYW,PR_THERM,0
/KEYW,PR_FLUID,0
/KEYW,PR_ELMA,0
/KEYW,MAGNOD,0
/KEYW,MAGEDG,0
/KEYW,MAGHFE,0
/KEYW,MAGELC,0
/KEYW,PR_MULTI,0
/KEYW,PR_CFD,0
!
! c**DEFINE KEYPOINTS
K.1, -0.046812, 0.096837, 0.012699
K.2, 0.046812, 0.096837, 0.012699
K.3, 0.046812, -0.096837, 0.012699
K.4, -0.046812, -0.096837, 0.012699

CM,Y1,VOLU
CHKMSH,"VOLU"
CMSEL,S,Y!
!* VSWEEP,Y1
!* CMDELE,Y!
CMDELE,Y CMDELE,Y1 CMDELE,Y2
!* FINISH
/SOL
FLST,2,1,5,ORDE,1
FITEM,2,1
!* /GO
DA,P51X,ALL,0
FLST,2,1,5,ORDE,1
FITEM,2,36
FLST,2,1,5,ORDE,1
FITEM,2,36
/GO
!* SFA,P51X,1,PRES,2130480
!* /SOLU
RATE,OFF
TIME, 1.0E-8
SOLVE
K.5, -0.046812,0.096837,0.069850
K.6, 0.046812,0.096837,0.069850
K.7, 0.046812,-0.096837,0.069850
K.8, -0.046812,-0.096837,0.069850
K.9, -0.046812,0.096837,0.08885
K.10, 0.046812,0.096837,0.08885
K.11, 0.046812,-0.096837,0.08885
K.12,- 0.046812,-0.096837,0.08885
K.13,- -0.046812,0.096837,0.14600
K.14, 0.046812,0.096837,0.14600
K.15, 0.046812,-0.096837,0.14600
K.16,- 0.046812,-0.096837,0.14600
K.17,- -0.046812,0.096837,0.16500
K.18, 0.046812,0.096837,0.16500
K.19, 0.046812,-0.096837,0.16500
K.20,- 0.046812,-0.096837,0.16500
K.21,- -0.046812,0.096837,0.22215
K.22, 0.046812,0.096837,0.22215
K.23, 0.046812,-0.096837,0.22215
K.24,- 0.046812,-0.096837,0.22215
K.25,- -0.046812,0.096837,0.24115
K.26, 0.046812,0.096837,0.24115
K.27, 0.046812,-0.096837,0.24115
K.28,- 0.046812,-0.096837,0.24115
K.29,- -0.046812,0.096837,0.29830
K.30, 0.046812,0.096837,0.29830
K.31, 0.046812,-0.096837,0.29830
K.32,- 0.046812,-0.096837,0.29830

! C***CREATE VOLUMES
V, 1,2,3,4,5,6,7,8
V, 5,6,7,8,9,10,11,12
V, 9,10,11,12,13,14,15,16
V, 13,14,15,16,17,18,19,20
V, 17,18,19,20,21,22,23,24
V, 21,22,23,24,25,26,27,28
V, 25,26,27,28,29,30,31,32

! C***DEFINE ELEMENT TYPES
! C***element type block and mortar
ET,1,SOLID185
MPTEMP,.......  
MPTEMP,1,295.37
MPDATA,EX,1.,2.2e+10
MPDATA,PRXY,1.,0.1
MPTEMP,.......  
MPTEMP,1,295.37
MPDATA,EX,2.,1.15e+9
MPDATA,PRXY,2.,0.1

! 
TB,CREEP,2,1,7,11
TBTEMP,295.37
TBDATA,4.76E-007,0.17,-0.723676415,-572.4462374,9.6736E-012,1,
TBDATA,0,.....
! C***element attributes #1: block
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM_Y,VOLU
VSEL,.,P51X
CM_Y1,VOLU
CMSEL,S_Y
!
CMSEL,S_Y1
VATT,1,1,0
CMSEL,S_Y
CMDELE_Y
CMDELE_Y1
!
! C***element attributes #2: nonlinear mortar
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM_Y,VOLU
VSEL,.,P51X
CM_Y1,VOLU
CMSEL,S_Y
!
CMSEL,S_Y1
VATT,2,1,0
CMSEL,S_Y
CMDELE_Y
CMDELE_Y1
!
! C***MESH ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM_Y LINE
LSEL,.,P51X
CM_Y1 LINE
CMSEL,Y
!
LESIZE_Y1,,10,,1
!
FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM_Y LINE
LSEL,.,P51X
CM_Y1 LINE

CMSEL,,_Y
!*
LESIZE,,_Y1,,6,,1
!*
FLST,5,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM,,_Y,VOLU
VSEL,,P51X
CM,,_Y1,VOLU
CHKMSH,'VOLU'
CMSEL,,_Y
!*
VSweep,,_Y1
!*
CMDELE,,_Y
CMDELE,,_Y1
CMDELE,,_Y2
!*
! VPlot
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM,,_Y,VOLU
VSEL,,P51X
CM,,_Y1,VOLU
CHKMSH,'VOLU'
CMSEL,,_Y
VSweep,,_Y1
CMDELE,,_Y
CMDELE,,_Y1
CMDELE,,_Y2
!*
Finish
/SOL
FLST,2,1,5,ORDE,1
FITEM,2,1
!*
/GO
DA,P51X,ALL,0
FLST,2,1,5,ORDE,1
FITEM,2,36
FLST,2,1,5,ORDE,1
FITEM,2,36
/GO
!*
SFA,P51X,1,PRES,2130480
/SOLU
RATE,OFF
TIME,1.0E-8
Solve
NUMERICAL MODEL OF MASONRY PRISM WITH 30%-70% VOLUME FRACTION

LOG FILE 7.3.1

/FILNAME,NONLINEAR_30-70_4.3.1_masonryprism,0
/PREP7
/NOPR
/title, NONLINEAR: 30-70 masonry prism 4.3.1
/PMETH,OFF,0
KEYW,PR_SET,1
KEYW,PR_STRUCT,1
KEYW,PR_THERM,0
KEYW,PR_FLUID,0
KEYW,PR_ELMAQ,0
KEYW,MANGD,0
KEYW,MAGHE,0
KEYW,MAGELL,0
KEYW,PR_MULTI,0
KEYW,PR_CFD,0
!
!c***DEFINE KEYPOINTS
K.1,-0.046812,0.096837,0.012699
K.2, 0.046812,0.096837,0.012699
K.3, 0.046812,-0.096837,0.012699
K.4,-0.046812,-0.096837,0.012699
K.5,-0.046812,0.096837,0.069850
K.6, 0.046812,0.096837,0.069850
K.7, 0.046812,-0.096837,0.069850
K.8,-0.046812,-0.096837,0.069850
K.9,-0.046812,0.096837,0.10245
K.10, 0.046812,0.096837,0.10245
K.11, 0.046812,-0.096837,0.10245
K.12,-0.046812,-0.096837,0.10245
K.13,-0.046812,0.096837,0.15960
K.14, 0.046812,0.096837,0.15960
K.15, 0.046812,-0.096837,0.15960
K.16,-0.046812,-0.096837,0.15960
K.17,-0.046812,0.096837,0.19220
K.18, 0.046812,0.096837,0.19220
K.19, 0.046812,-0.096837,0.19220
K.20,-0.046812,-0.096837,0.19220
K.21,-0.046812,0.096837,0.24935
K.22, 0.046812,0.096837,0.24935
K.23, 0.046812,-0.096837,0.24935
K.24,-0.046812,-0.096837,0.24935
K.25,-0.046812,0.096837,0.28195
K.26, 0.046812,0.096837,0.28195
K.27, 0.046812,-0.096837,0.28195
K.28,-0.046812,-0.096837,0.28195
K.29,-0.046812,0.096837,0.33910
K.30, 0.046812,0.096837,0.33910
K.31, 0.046812,-0.096837,0.33910
K.32,-0.046812,-0.096837,0.33910
! C***CREATE VOLUMES
V, 1,2,3,4,5,6,7,8
V, 5,6,7,8,9,10,11,12
V, 9,10,11,12,13,14,15,16
V, 13,14,15,16,17,18,19,20
V, 17,18,19,20,21,22,23,24
V, 21,22,23,24,25,26,27,28
V, 25,26,27,28,29,30,31,32
!
! C***DEFINE ELEMENT TYPES
! C***element type block and mortar
ET,1,SOLID185
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX,1,,2.2e+10
MPDATA,PRXY,1,,0.1
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX,2,,1.15e+9
MPDATA,PRXY,2,,0.1
!
TB,CREEP,2,1,7,11
TBTEMP,295.37
TBDATA,1,,16,17E-007,0.253398054,-0.723676415,-572.4462374,1010.6142496,-73.4153224
TBDATA,1,,1411.4267819,......
!
! C***element attributes #1: block
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM,,Y,VOLU
VSEL, , , ,P51X
CM,,Y1,VOLU
CMSEL,S,,Y
!*
CMSEL,S,,Y1
VATT, 1, 1, 0
CMSEL,S,,Y
CMDELE,,Y
CMDELE,,Y1
!
! C***element attributes #2: nonlinear mortar
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM,,Y,VOLU
VSEL, , , ,P51X
CM,,Y1,VOLU
CMSEL,S,,Y
!*
CMSEL,S,_Y1
VATT, 2, 1, 0
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
! C***MESH ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM,_Y.LINE
LSEL,,P51X
CM,_Y1.LINE
CMSEL,,_Y
!* LESIZE,_Y1,,10,,1
!* FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM,_Y.LINE
LSEL,,P51X
CM,_Y1.LINE
CMSEL,,_Y
!* LESIZE,_Y1,,10,,1
!* FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM,_Y,VOLU
VSEL,,P51X
CM,_Y1,VOLU
CHKMSH,"VOLU"
CMSEL,S,_Y
!* VSWEEP,_Y1
!* CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
!* ! VPLOT
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM,_Y,VOLU
VSEL,,P51X
CM,_Y1,VOLU
CHKMSH,"VOLU"
CMSEL,S,_Y
!* VSWEEP,_Y1
!* CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
!* FINISH
/SOL
FLST,2,1,5,ORDE,1
FITEM,2,1
!* /GO
DA,P51X,ALL,0
FLST,2,1,5,ORDE,1
FITEM,2,36
FLST,2,1,5,ORDE,1
FITEM,2,36
!*/ GO
!* SFA,P51X,1,PRES,2130480
!* /SOLU
RATE,OFF
TIME, 1.0E-8
SOLVE

LOG FILE 7.3.2 with secondary creep

/FILNAME,NONLINEAR_30-70_4.3.2_masonryprism,0
/PREP7
/NOPR
/title, NONLINEAR: 30-70 masonry prism 4.3.2
/PMETH,OFF,0
KEYW,PR_SET,1
KEYW,PR_STRUCT,1
KEYW,PR_THERM,0
KEYW,PR_FLUID,0
KEYW,PR_ELMAG,0
KEYW,MAGNOD,0
KEYW,MAGEDG,0
KEYW,MAGHFE,0
KEYW,MAGELC,0
KEYW,PR_MULTI,0
KEYW,PR_CFD,0
! c**DEFINE KEYPOINTS
K,1,-0.046812,0.096837, 0.012699
K,2, 0.046812,0.096837, 0.012699
K,3, 0.046812,-0.096837, 0.012699
K,4,- 0.046812,-0.096837, 0.012699
K.5, -0.046812, 0.096837, 0.069850
K.6, 0.046812, 0.096837, 0.069850
K.7, 0.046812, -0.096837, 0.069850
K.8, -0.046812, -0.096837, 0.069850
K.9, -0.046812, 0.096837, 0.10245
K.10, 0.046812, 0.096837, 0.10245
K.11, 0.046812, -0.096837, 0.10245
K.12, -0.046812, -0.096837, 0.10245
K.13, -0.046812, 0.096837, 0.15960
K.14, 0.046812, 0.096837, 0.15960
K.15, 0.046812, -0.096837, 0.15960
K.16, -0.046812, -0.096837, 0.15960
K.17, -0.046812, 0.096837, 0.19220
K.18, 0.046812, 0.096837, 0.19220
K.19, 0.046812, -0.096837, 0.19220
K.20, -0.046812, -0.096837, 0.19220
K.21, -0.046812, 0.096837, 0.24935
K.22, 0.046812, 0.096837, 0.24935
K.23, 0.046812, -0.096837, 0.24935
K.24, -0.046812, -0.096837, 0.24935
K.25, -0.046812, 0.096837, 0.28195
K.26, 0.046812, 0.096837, 0.28195
K.27, 0.046812, -0.096837, 0.28195
K.28, -0.046812, -0.096837, 0.28195
K.29, -0.046812, 0.096837, 0.33910
K.30, 0.046812, 0.096837, 0.33910
K.31, 0.046812, -0.096837, 0.33910
K.32, -0.046812, -0.096837, 0.33910
!
! C***CREATE VOLUMES
V. 1,2,3,4,5,6,7,8
V. 5,6,7,8,9,10,11,12
V. 9,10,11,12,13,14,15,16
V. 13,14,15,16,17,18,19,20
V. 17,18,19,20,21,22,23,24
V. 21,22,23,24,25,26,27,28
V. 25,26,27,28,29,30,31,32
!
! C***DEFINE ELEMENT TYPES
! C***element type block and mortar
ET.1,SOLID185
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX.1,2,2e+10
MPDATA,PRXY.1,0.1
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX.2,1.15e+9
MPDATA,PRXY.2,0.1
!
TB,CREEP,2,1,7,11
TBTEMP,295.37
TBDATA,4.76E-007,0.17,-0.723676415,-572.4462374,9.6736E-012,1,
TBDATA,0,.....
! C***element attributes #1: block
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM_.Y,VOLU
VSEL_ , , , , P51X
CM_.Y1,VOLU
CMSEL,S_.Y
!
CMSEL,S_.Y1
VATT, 1, , 1, 0
CMSEL,S_.Y
CMDELE_.Y
CMDELE_.Y1
!
! C***element attributes #2: nonlinear mortar
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM_.Y,VOLU
VSEL_ , , , , P51X
CM_.Y1,VOLU
CMSEL,S_.Y
!
CMSEL,S_.Y1
VATT, 2, , 1, 0
CMSEL,S_.Y
CMDELE_.Y
CMDELE_.Y1
!
! C***MESH ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM_.Y,LINE
LSEL_ , , , , P51X
CM_.Y1,LINE
CMSEL,.Y
!
LESIZE_.Y1_ , , 10, , 1
!
FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM_.Y,LINE
LSEL_ , , , , P51X
CM_.Y1,LINE
CMSEL,_,Y
!*  
LESIZE,_,Y1,.,10,.,1 
!*  
FLST,5,3,6,ORDE,3  
FITEM,5,2  
FITEM,5,4  
FITEM,5,6  
CM,_,Y,VOLU  
VSEL,.,P51X  
CM,_,Y1,VOLU  
CHKMSH,'VOLU'  
CMSEL,S,_,Y 
!*  
VSWEEP,_,Y1 
!*  
CMDELE,_,Y  
CMDELE,_,Y1  
CMDELE,_,Y2 
!*  
! VPLOT  
FLST,5,4,6,ORDE,4  
FITEM,5,1  
FITEM,5,3  
FITEM,5,5  
FITEM,5,7  
CM,_,Y,VOLU  
VSEL,.,P51X  
CM,_,Y1,VOLU  
CHKMSH,'VOLU'  
CMSEL,S,_,Y  
VSWEEP,_,Y1  
CMDELE,_,Y 
CMDELE,_,Y1 
CMDELE,_,Y2  
FINISH  
/SOL  
FLST,2,1,5,ORDE,1  
FITEM,2,1  
/GO  
DA,P51X,ALL,0  
FLST,2,1,5,ORDE,1  
FITEM,2,36  
FLST,2,1,5,ORDE,1  
FITEM,2,36  
/GO  
SFA,P51X,1,PRES,2130480  
/SOLU  
RATE,OFF  
TIME, 1.0E-8  
SOLVE
LOG FILE 7.4.1

/FILNAME,NONLINEAR_40-60_4.4.1_masonryprism,0
/PREP7
/NOPR
/title, NONLINEAR: 40-60 masonry prism 4.4.1
/PMETH,OFF,0
KEYW,PR_SET,1
KEYW,PR_STRUCT,1
KEYW,PR_THERM,0
KEYW,PR_FLUID,0
KEYW,PR_ELMAG,0
KEYW,MAGNOD,0
KEYW,MAGEDG,0
KEYW,MAGHE,0
KEYW,MAGELC,0
KEYW,PR_MULTI,0
KEYW,PR_CFD,0

! c**DEFINE KEYPOINTS
K.1,-0.046812,0.096837,0.012699
K.2,0.046812,0.096837,0.012699
K.3,0.046812,-0.096837,0.012699
K.4,-0.046812,-0.096837,0.012699
K.5,-0.046812,0.096837,0.069850
K.6,0.046812,0.096837,0.069850
K.7,0.046812,-0.096837,0.069850
K.8,-0.046812,-0.096837,0.069850
K.9,-0.046812,0.096837,0.12065
K.10,0.046812,0.096837,0.12065
K.11,0.046812,-0.096837,0.12065
K.12,-0.046812,-0.096837,0.12065
K.13,-0.046812,0.096837,0.17780
K.14,0.046812,0.096837,0.17780
K.15,0.046812,-0.096837,0.17780
K.16,-0.046812,-0.096837,0.17780
K.17,-0.046812,0.096837,0.22860
K.18,0.046812,0.096837,0.22860
K.19,0.046812,-0.096837,0.22860
K.20,-0.046812,-0.096837,0.22860
K.21,-0.046812,0.096837,0.28575
K.22,0.046812,0.096837,0.28575
K.23,0.046812,-0.096837,0.28575
K.24,-0.046812,-0.096837,0.28575
K.25,-0.046812,0.096837,0.33655
K.26,0.046812,0.096837,0.33655
K.27,0.046812,-0.096837,0.33655
K.28,-0.046812,-0.096837,0.33655
K.29,-0.046812,0.096837,0.39370
K.30,0.046812,0.096837,0.39370
K.31,0.046812,-0.096837,0.39370
K.32,-0.046812,-0.096837,0.39370
! C***CREATE VOLUMES
V, 1,2,3,4,5,6,7,8
V, 5,6,7,8,9,10,11,12
V, 9,10,11,12,13,14,15,16
V, 13,14,15,16,17,18,19,20
V, 17,18,19,20,21,22,23,24
V, 21,22,23,24,25,26,27,28
V, 25,26,27,28,29,30,31,32
!
! C***DEFINE ELEMENT TYPES
! C***element type block and mortar
ET,1,SOLID185
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX,1,,2.2e+10
MPDATA,PRXY,1,,0.1
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX,2,,1.15e+9
MPDATA,PRXY,2,,0.1
!
TB,CREEP,2,1,7,11
TBTEMP,295.37
TBDATA,,16.17E-007,0.253398054,-0.723676415,-572.4462374,1010.6142496,-73.4153224
TBDATA,,1411.4267819,,
!
! C***element attributes #1: block
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM,_,Y,VOLU
VSEL,,P51X
CM,_,Y1,VOLU
CMSEL,S,_,Y
*!
CMSEL,S,_,Y1
VATT,,1,,1,,0
CMSEL,S,_,Y
CMDELE,_,Y
CMDELE,_,Y1
!
! C***element attributes #2: nonlinear mortar
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM,_,Y,VOLU
VSEL,,P51X
CM,_,Y1,VOLU
CMSEL,S,_,Y
*!
CMSEL,S,_Y1
VATT,2,1,0
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!
! MESH ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM,_Y,LINE
LSEL,,P51X
CM,_Y1,LINE
CMSEL,,Y
*
LESIZE,,Y1,,10,,,1
*
FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM,_Y,LINE
LSEL,,P51X
CM,_Y1,LINE
CMSEL,,Y
*
LESIZE,,Y1,,14,,,1
*
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM,Y,VOLU
VSEL,,,P51X
CM,Y1,VOLU
CHKMSH,"VOLU"
CMSEL,S,Y
*
VSWEEP,Y1
*
CMDELE,Y
CMDELE,Y1
CMDELE,Y2
*
! VPLOT
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM,Y,VOLU
VSEL,,,P51X
CM, Y1, VOLU
CHKMTH, "VOLU"
CMSEL, S, Y
!*
VSHEET, Y1
!*
CMDELE, Y
CMDELE, Y1
CMDELE, Y2
!*
FINISH
/SOL
FLST, 2, 1, 5, ORDE, 1
FITEM, 2, 1
!*
/GO
DA, 51X, ALL, 0
FLST, 2, 1, 5, ORDE, 1
FITEM, 2, 36
FLST, 2, 1, 5, ORDE, 1
FITEM, 2, 36
/GO
!*
SFA, 51X, 1, PRES, 2130480
!*
/SOLU
RATE, OFF
TIME, 1.0E-8
SOLVE

LOG FILE 7.4.2 with secondary creep

/FILNAME, NONLINEAR_40-60_4.4.2_masonryprism, 0
/PREP7
/NOPR
/title, NONLINEAR: 40-60 masonry prism 4.4.2
/PMETH, OFF, 0
KEYW, PR_SET, 1
KEYW, PR_STRUCT, 1
KEYW, PR_THERM, 0
KEYW, PR_FLUID, 0
KEYW, PR_ELMAG, 0
KEYW, MAGNOD, 0
KEYW, MAGEDG, 0
KEYW, MAGHFE, 0
KEYW, MAGELC, 0
KEYW, PR_MULTI, 0
KEYW, PR_CFD, 0
!
! c**DEFINE KEYPOINTS
K, 1, -0.046812, 0.096837, 0.012699
K, 2, 0.046812, 0.096837, 0.012699
K, 3, 0.046812, -0.096837, 0.012699
K, 4, -0.046812, -0.096837, 0.012699
K.5, -0.046812, 0.096837, 0.069850
K.6, 0.046812, 0.096837, 0.069850
K.7, 0.046812, -0.096837, 0.069850
K.8, -0.046812, -0.096837, 0.069850
K.9, -0.046812, 0.096837, 0.12065
K.10, 0.046812, 0.096837, 0.12065
K.11, 0.046812, -0.096837, 0.12065
K.12, -0.046812, -0.096837, 0.12065
K.13, -0.046812, 0.096837, 0.17780
K.14, 0.046812, 0.096837, 0.17780
K.15, 0.046812, -0.096837, 0.17780
K.16, -0.046812, -0.096837, 0.17780
K.17, -0.046812, 0.096837, 0.22860
K.18, 0.046812, 0.096837, 0.22860
K.19, 0.046812, -0.096837, 0.22860
K.20, -0.046812, -0.096837, 0.22860
K.21, -0.046812, 0.096837, 0.28575
K.22, 0.046812, 0.096837, 0.28575
K.23, 0.046812, -0.096837, 0.28575
K.24, -0.046812, -0.096837, 0.28575
K.25, -0.046812, 0.096837, 0.33655
K.26, 0.046812, 0.096837, 0.33655
K.27, 0.046812, -0.096837, 0.33655
K.28, -0.046812, -0.096837, 0.33655
K.29, -0.046812, 0.096837, 0.39370
K.30, 0.046812, 0.096837, 0.39370
K.31, 0.046812, -0.096837, 0.39370
K.32, -0.046812, -0.096837, 0.39370

! C***CREATE VOLUMES
V, 1,2,3,4,6,7,8
V, 5,6,7,8,9,10,11,12
V, 9,10,11,12,13,14,15,16
V, 13,14,15,16,17,18,19,20
V, 17,18,19,20,21,22,23,24
V, 21,22,23,24,25,26,27,28
V, 25,26,27,28,29,30,31,32

! C***DEFINE ELEMENT TYPES
! C***element type block and mortar
ET,1,SOLID185
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX,1,2.2e+10
MPDATA,PRXY,1,0.1
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX,2,1.15e+9
MPDATA,PRXY,2,0.1

TB,CREEP,2,1,7,11
TBTEMP,295.37
TBDATA,4.76E-007,0.17,-0.723676415,-572.4462374,9.6736E-012,1,
TBDATA,0,.....
! C***element attributes #1: block
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM._Y,VOLU
VSEL,,,,P51X
CM._Y1,VOLU
CMSEL,S._Y
*
CMSEL,S._Y1
VATT, 1., 1., 0
CMSEL,S._Y
CMDELE._Y
CMDELE._Y1
*
CMSEL,S._Y1
VATT, 2., 1., 0
CMSEL,S._Y
CMDELE._Y
CMDELE._Y1
!
! C***element attributes #2: nonlinear mortar
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM._Y,VOLU
VSEL,,,,P51X
CM._Y1,VOLU
CMSEL,S._Y
*
CMSEL,S._Y1
VATT, 2., 1., 0
CMSEL,S._Y
CMDELE._Y
CMDELE._Y1
!
! C***MESH ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM._Y,LINE
LSEL,,,,P51X
CM._Y1,LINE
CMSEL,.Y
*
LESIZE_.Y1,,10,,1
*
FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM._Y,LINE
LSEL,,,,P51X
CM._Y1,LINE
CMSEL,_Y
!*    
LESIZE,_Y1,,14,,1
!*    
FLST,5,3,6,ORDE,3    
FITEM,5,2    
FITEM,5,4    
FITEM,5,6    
CM_,_Y,VOLU    
VSEL,,P51X    
CM_,_Y1,VOLU    
CHKMSH,'VOLU'    
CMSEL,S,_Y    
!*    
VSweep,_Y1    
!*    
CMDELE,_Y    
CMDELE,_Y1    
CMDELE,_Y2    
!*    
! Vplot    
FLST,5,4,6,ORDE,4    
FITEM,5,1    
FITEM,5,3    
FITEM,5,5    
FITEM,5,7    
CM_,_Y,VOLU    
VSEL,,P51X    
CM_,_Y1,VOLU    
CHKMSH,'VOLU'    
CMSEL,S,_Y    
VSweep,_Y1    
CMDELE,_Y    
CMDELE,_Y1    
CMDELE,_Y2    
finish    
/sol    
FLST,2,1,5,ORDE,1    
FITEM,2,1    
/go    
DA,P51X,ALL,0    
FLST,2,1,5,ORDE,1    
FITEM,2,36    
FLST,2,1,5,ORDE,1    
FITEM,2,36    
/go    
SFA,P51X,1,PRES,2130480    
/solu    
rate,off    
time,1.0E-8    
solve
Numerical Model of Masonry Prism with 50%-50% Volume Fraction

LOG FILE 7.5.1

/FILNAME, NONLINEAR_50-50_4.5.1_masonryprism, 0
/PREP7
/NOPR
/title, NONLINEAR: 50-50 masonry prism 4.5.1
/PMETH, OFF, 0
KEYW, PR_SET, 1
KEYW, PR_STRUCT, 1
KEYW, PR_THERM, 0
KEYW, PR_FLUID, 0
KEYW, PR_ELMAG, 0
KEYW, MAGNOD, 0
KEYW, MAGTED, 0
KEYW, MAGHFE, 0
KEYW, MAGEC, 0
KEYW, PR_MULTI, 0
KEYW, PR_CFD, 0

! c***DEFINE KEYPOINTS
K, 1, -0.046812, 0.096837, 0.012699
K, 2, 0.046812, 0.096837, 0.012699
K, 3, 0.046812, -0.096837, 0.012699
K, 4, -0.046812, -0.096837, 0.012699
K, 5, -0.046812, 0.096837, 0.069850
K, 6, 0.046812, 0.096837, 0.069850
K, 7, 0.046812, -0.096837, 0.069850
K, 8, -0.046812, -0.096837, 0.069850
K, 9, -0.046812, 0.096837, 0.14615
K, 10, 0.046812, 0.096837, 0.14615
K, 11, 0.046812, -0.096837, 0.14615
K, 12, -0.046812, -0.096837, 0.14615
K, 13, -0.046812, 0.096837, 0.20330
K, 14, 0.046812, 0.096837, 0.20330
K, 15, 0.046812, -0.096837, 0.20330
K, 16, -0.046812, -0.096837, 0.20330
K, 17, -0.046812, 0.096837, 0.27960
K, 18, 0.046812, 0.096837, 0.27960
K, 19, 0.046812, -0.096837, 0.27960
K, 20, -0.046812, -0.096837, 0.27960
K, 21, -0.046812, 0.096837, 0.33675
K, 22, 0.046812, 0.096837, 0.33675
K, 23, 0.046812, -0.096837, 0.33675
K, 24, -0.046812, -0.096837, 0.33675
K, 25, -0.046812, 0.096837, 0.41305
K, 26, 0.046812, 0.096837, 0.41305
K, 27, 0.046812, -0.096837, 0.41305
K, 28, -0.046812, -0.096837, 0.41305
K, 29, -0.046812, 0.096837, 0.47020
K, 30, 0.046812, 0.096837, 0.47020
K, 31, 0.046812, -0.096837, 0.47020
K, 32, -0.046812, -0.096837, 0.47020
! C***CREATE VOLUMES
V, 1,2,3,4,5,6,7,8
V, 5,6,7,8,9,10,11,12
V, 9,10,11,12,13,14,15,16
V, 13,14,15,16,17,18,19,20
V, 17,18,19,20,21,22,23,24
V, 21,22,23,24,25,26,27,28
V, 25,26,27,28,29,30,31,32
!
! c***DEFINE ELEMENT TYPES
! C***element type block and mortar
ET,1,SOLID185
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX,1.,2.2e+10
MPDATA,PRXY,1.,0.1
MPTEMP,........
MPTEMP,1,295.37
MPDATA,EX,2.,1.15e+09
MPDATA,PRXY,2.,0.1
!
TB,CREEP,2,1,7,11
TBTEMP,295.37
TBDATA,.16.17E-007,0.253398054,-0.723676415,-572.4462374,1010.6142496,-73.4153224
TBDATA,.1411.4267819,.......
!
! C***element attributes #1: block
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM,_Y,VOLU
VSEL.,, ,P51X
CM,_Y1,VOLU
CMSEL,S,_Y
!
CMSEL,S,_Y1
VATT, 1., 1., 0
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!
! C***element attributes #2: nonlinear mortar
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM,_Y,VOLU
VSEL.,, ,P51X
CM,_Y1,VOLU
CMSEL,S,_Y
!
CMSEL,S,_Y1
VATT, 2, 1, 0
CMSEL,S,_Y
CMDELE,_Y
CMDELE,_Y1
!
! C***MESH ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM,_Y,LINE
LSEL,,P51X
CM,_Y1,LINE
CMSEL,,_Y
*
LESIZE,_Y1,,10,,1
*
FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM,_Y,LINE
LSEL,,P51X
CM,_Y1,LINE
CMSEL,,_Y
*
LESIZE,_Y1,,18,,1
*
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM,_Y,VOLU
VSEL,,P51X
CM,_Y1,VOLU
CHKMSH,"VOLU"
CMSEL,S,_Y
*
VSWEEP,_Y1
*
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
*
! VPLOT
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM,_Y,VOLU
VSEL,,P51X
CM_Y1,VOLU
CHKMsh:"VOLU"
CMSEL,S,_Y
!* 
VSWEEP,_Y1
!* 
CMDELE,_Y 
CMDELE,_Y1 
CMDELE,_Y2
!* 
FINISH 
/SOL 
FLST,2,1,5,ORDE,1 
FITEM,2,1
!* 
/GO 
DA,P51X,ALL,0 
FLST,2,1,5,ORDE,1 
FITEM,2,36 
FLST,2,1,5,ORDE,1 
FITEM,2,36 
/GO
!* 
SFA,P51X,1,PRES,2130480 
!* 
/SOLU 
RATE,OFF 
TIME, 1.0E-8 
SOLVE

LOG FILE 7.5. 2 with secondary creep

/FILNAME, NONLINEAR_50-50_4.5.2_masonryprism,0 
/PREP7 
/NOPR 
/title, NONLINEAR: 50-50 masonry prism 4.5.2 
/PMETH,OFF,0 
KEYW,PR_SET,1 
KEYW,PR_STRUCT,1 
KEYW,PR_THERM,0 
KEYW,PR_FLUID,0 
KEYW,PR_ELMA,0 
KEYW,MAGNOD,0 
KEYW,MAGEDG,0 
KEYW,MAGHFE,0 
KEYW,MAGELC,0 
KEYW,PR_MULTI,0 
KEYW,PR_CFD,0
!* 
! c***DEFINE KEYPOINTS 
K,1,-0.046812,0.006837, 0.012699 
K,2, 0.046812,0.096837, 0.012699 
K,3, 0.046812,-0.096837, 0.012699 
K,4,- 0.046812,-0.096837, 0.012699
K.5, -0.046812,0.096837,0.069850
K.6, 0.046812,0.096837,0.069850
K.7, 0.046812,-0.096837,0.069850
K.8, -0.046812,-0.096837,0.069850
K.9, -0.046812,0.096837,0.14615
K.10, 0.046812,0.096837,0.14615
K.11, 0.046812,-0.096837,0.14615
K.12, -0.046812,-0.096837,0.14615
K.13, -0.046812,0.096837,0.20330
K.14, 0.046812,0.096837,0.20330
K.15, 0.046812,-0.096837,0.20330
K.16, -0.046812,-0.096837,0.20330
K.17, -0.046812,0.096837,0.27960
K.18, 0.046812,0.096837,0.27960
K.19, 0.046812,-0.096837,0.27960
K.20, -0.046812,-0.096837,0.27960
K.21, -0.046812,0.096837,0.33675
K.22, 0.046812,0.096837,0.33675
K.23, 0.046812,-0.096837,0.33675
K.24, -0.046812,-0.096837,0.33675
K.25, -0.046812,0.096837,0.41305
K.26, 0.046812,0.096837,0.41305
K.27, 0.046812,-0.096837,0.41305
K.28, -0.046812,-0.096837,0.41305
K.29, -0.046812,0.096837,0.47020
K.30, 0.046812,0.096837,0.47020
K.31, 0.046812,-0.096837,0.47020
K.32, -0.046812,-0.096837,0.47020
!
! **CREATE VOLUMES**
V. 1,2,3,4,5,6,7,8
V. 9,10,11,12
V. 13,14,15,16
V. 17,18,19,20
V. 21,22,23,24
V. 25,26,27,28
V. 29,30,31,32
!
! **DEFINE ELEMENT TYPES**
! **element type block and mortar**
ET.1,SOLID185
MPTEMP........
MPTEMP,1,295.37
MPDATA,EX.1,2.2e+10
MPDATA,PRXY.1,0.1
MPTEMP........
MPTEMP,1,295.37
MPDATA,EX.2,1.15e+9
MPDATA,PRXY.2,0.1
!
TB.CREEP,2,1,7,11
TBTEMP,295.37
TBDATA,4.76E-007,0.17,-0.723676415,-572.4462374,9.6736E-012,1,
TBDATA,0,....
! C***element attributes #1: block
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM_ Y,VOLU
VSEL, , , ,P51X
CM_ Y1,VOLU
CMSEL,S_ Y
!*
CMSEL,S_ Y1
VATT, 1, , 1, 0
CMSEL,S_ Y
CMDELE_ Y
CMDELE_ Y1
!
! C***element attributes #2: nonlinear mortar
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM_ Y,VOLU
VSEL, , , ,P51X
CM_ Y1,VOLU
CMSEL,S_ Y
!*
CMSEL,S_ Y1
VATT, 2, , 1, 0
CMSEL,S_ Y
CMDELE_ Y
CMDELE_ Y1
!
! C***MESH ELEMENT TYPES
FLST,5,4,4,ORDE,4
FITEM,5,5
FITEM,5,21
FITEM,5,37
FITEM,5,53
CM_ Y,LINE
LSEL, , , ,P51X
CM_ Y1,LINE
CMSEL,S_ Y
!*
LSEIZ,1, 10, , ,1
!*
FLST,5,3,4,ORDE,3
FITEM,5,13
FITEM,5,29
FITEM,5,45
CM_ Y,LINE
LSEL, , , ,P51X
CM_ Y1,LINE
CMSEL,.Y
!*
LESIZE,.Y1,.18,.1
!*
FLST,5,3,6,ORDE,3
FITEM,5,2
FITEM,5,4
FITEM,5,6
CM_.Y,VOLU
VSEL,.51X
CM_.Y1,VOLU
CHKM,,'VOLU'
CMSEL,.S_.Y
!*
VSWEEP,.Y1
!*
CMDELE,.Y
CMDELE,.Y1
CMDELE,.Y2
!*
! VPLOT
FLST,5,4,6,ORDE,4
FITEM,5,1
FITEM,5,3
FITEM,5,5
FITEM,5,7
CM_.Y,VOLU
VSEL,.51X
CM_.Y1,VOLU
CHKM,,'VOLU'
CMSEL,.S_.Y
VSWEEP,.Y1
CMDELE,.Y
CMDELE,.Y1
CMDELE,.Y2
FINISH
/SOL
FLST,2,1,5,ORDE,1
FITEM,2,1
/GO
DA,51X,ALL,0
FLST,2,1,5,ORDE,1
FITEM,2,36
FLST,2,1,5,ORDE,1
FITEM,2,36
/GO
SFA,51X,1,PRES,2130480
/SOLU
RATE,OFF
TIME,1.0E-8
SOLVE