IMPROVEMENT OF SOIL MODELING IN A TIRE-SOIL INTERACTION USING
FINITE ELEMENT ANALYSIS AND SMOOTH PARTICLE HYDRODYNAMICS

A Thesis in
Mechanical Engineering

by
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Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

May 2010
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ABSTRACT

In recent years, the advancement of computerized modeling has allowed for the creation of extensive pneumatic tire models. These models have been used to determine many tire properties and tire-road interaction parameters which are either prohibitively expensive or unavailable with physical models. More recently, computerized modeling has been used to explore tire-soil interactions. The new parameters created by these interactions were defined for these models, but accurate soil constitutive equations were lacking. With the previous models, the soil was simulated using Finite Element Analysis (FEA) with soil material models requiring calibration and validation. Furthermore, the meshless modeling method of Smooth Particle Hydrodynamics (SPH) may be a viable approach to more accurately simulating large soil deformations and complex tire-soil interactions.

For this thesis, a rigid tire model is used to perform an extensive sensitivity study on the previously used FEA soft soil (dense sand) in order to determine the importance of mesh size, soil plot size, and edge constraints. Then, parameters for SPH particles are determined for either complete or partial replacement of FEA elements in the soil model. Rolling resistance tests are conducted with a rigid tire model for different SPH and FEA/SPH soil models and compared to the previously determined best FEA soil model. Replacement of FEA elements with SPH particles is found to be the key variable as using a deeper amount of SPH particles increases rolling resistance while increasing the SPH particle density has little effect on rolling resistance. These results are then replicated using a pneumatic tire model.

For further validation, pressure-sinkage tests are conducted with the FEA and SPH soils to explore the differences in the two soil modeling methods. Also, shear-displacement tests are conducted with the SPH soil—a test which cannot easily be performed with an FEA soil model. These shear tests show that the SPH soil behaves more like a clay in initial shearing and more like a sand by exhibiting increased shearing due to vertical loading. Furthermore, both the pressure-sinkage and shear-displacement tests still indicate that a larger particle density is unnecessary.
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ACKNOWLEDGEMENTS

The author would like to express his appreciation to Volvo 3P engineers, Mukesh Trivedi, Fredrik Öijer, and Inge Johansson for their continuous financial and technical support through the course of this thesis research. The author would also like to thank the Applied Research Laboratory of the Pennsylvania State University and Dr. Richard Stern, director of the E&F Graduate Assistant Program. Special thanks go to Dr. Moustafa El-Gindy, the advisor and chair of the committee for this thesis who has supplied constant support and direction for this research. Finally, the author would like to express his gratitude to his graduate committee: Dr. Kevin Koudela, Dr. Ashok D. Belegundu, and Dr. Karen Thole.
CHAPTER 1
INTRODUCTION

This chapter is an introduction to the work completed for this thesis and consists of four sections including motivation, a literature survey, objectives and scope, and a statement of the work.

1.1 MOTIVATION

Computerized modeling has greatly changed both the design process and the way in which experimental information is gathered. Before extensive computer usage, many physical models needed to be created and, in some cases, destroyed for prototyping and experimental purposes. The continuous setup, running, and information gathering of these processes could quickly become very expensive. With the advent of computerized modeling, many more virtual experiments could be run in less time and at less cost than experiments on physical models. As computers became more powerful, the models which could be run became more detailed and more accurate.

In previous work with Mack Trucks/Volvo 3P, a Finite Element Analysis (FEA) truck tire model was created by Seokyong Chae (2006) as partial requirement for a Ph.D. in materials. This tire has been used to obtain rigid-ring model parameters to be used within a complete semi-trailer truck model created by Mack Trucks/Volvo 3P. Furthermore, as partial requirement for an M.S. in mechanical engineering, an off-road rigid ring model was created by Jeffery L. Slade (2009). Now that the off-road rigid ring model has been created, new potential resources exist for soil modeling. Previously, the soil had been modeled with FEA elements with soil material models requiring calibration and validation. Furthermore, the meshless modeling method of Smooth Particle Hydrodynamics (SPH) may be a viable approach to more accurately simulating large soil deformations and complex tire-soil interactions.
1.2 LITERATURE SURVEY

This literature survey presents relevant work done in the fields of terramechanics and smooth particle hydrodynamics. The literature is introduced in order to offer necessary background information and to examine the current state of research in these fields.

1.2.1 Terramechanics

Creating mathematical predictions for soil deformation is a complicated issue. Soil is inherently composed of a random combination of anisotropic, non-homogeneous particles which causes it to act differently from well understood elastic models. For this reason, many have attempted over the years to create equations and formulations to predict how soil will react under different situations. Some of the first to explore this field were Bekker (1956, 1960 and 1969) and Janosi and Hanamoto (1961).

Both Bekker and Janosi et al. measured soil parameters using available test instruments to create their relations. Bekker used this information along with the Mohr-Coulomb soil failure criterion in order to predict pressure distributions, rolling resistances, and traction under a tire. On the other hand, Janosi and Hanamoto used stress-strain relationships in the case of uniform pressure distribution under the tracks of a tracked vehicle to create equations for predicting tractive forces. Bekker's equations were intended to predict interactions for forces normal to the soil while Janosi and Hanamoto created formulations for predicting shearing of the soil.

Meanwhile, Osman (1964) aimed to verify the cohesion and angle of shearing resistance parameters obtained from available testing methods. These experimental methods included the translational shear box, the triaxial test, the N.I.A.E shear box, the bevameter, the shear vane, the weighted sand-coated annulus, and the friction trolley method. The translational shear box consists of two halves free to slide relative to one another. The box is filled with soil and a constant strain is applied while the transmitted shear force is measured. The triaxial test is performed on a cylinder of soil by compressing the ends while containing the sides. This creates a non-hydrostatic stress state in which shear stresses can be measured. The N.I.A.E shear box is a cylindrical shear box which measures the torque required to shear the
enclosed soil sample. The bevameter is a device which can measure soil values from load sinkage tests performed with a plate on top of the soil and from annular shear stress deformation tests performed with a finned plate, as shown in Figure 1.1, being twisted within the soil. The shear vane is another torque failure device consisting of four blades mounted to a thin circular shaft which is pushed into the soil. The weighted sand-coated annulus is a surface torque shear test in which an annulus, or ring, has sand bonded to it with a special adhesive. The friction trolley method is a test in which the force required to overcome the friction between a normally loaded sand-coated slider and the surface of the sand are recorded. By testing dry sand, wet sand, and clay, Osman was able to conclude that these methods can accurately obtain soil properties when performed properly.

In 1973, Wismer and Luth used tire parameters and the cone index, which is a way of measuring the strength of soil, to create equations for predicting tractive performance of pneumatic tires. Then Yong et al. (1975) compared the data obtained from various soil measurement devices to the terramechanics properties they intended to simulate and found that the shearing slip involved in terramechanics cannot be determined by the cone penetrometer (which is used to determine the cone index and is shown in Figure 1.2), shear-vane, sinkage plate, shear plate, or shear annulus.
In 1978, Yong et al. used experimental drawbar pull tests on soil to validate results predicted analytically and were able to show good correlations. In 1990, Wittig and Alcock created a single-wheel tester for measuring traction conditions of soil by measuring the maximum transferable torque at known wheel loads. They determined that using bulk density or soil water content was able to more accurately predict tractive performance than the equations developed by Wismer and Luth using cone index. In 1991, Okello found that the bevameter yields better results for determining soil properties than the previously favored cone penetrometer which had been used largely due to U.S. military preference (Knight et al. 1961). Okello's findings reaffirmed the work conducted by Wittig and Alcock as well as the previous findings of Golob (1981).

While the majority of soil and tire testing had been performed for discrete, quasi-static states, Grahn (1991) conducted tests to further understand the dynamic nature of driving a wheel over soil. As shown in Figure 1.3, for a penetration plate, the penetration velocity is equal across the entire face of the plate. Alternately, for a wheel, the penetration velocity is highest towards the front of the wheel, which is moving down and forward due to rotational and translational velocities, and reduces to zero directly underneath the wheel. Due to this observation, a study was conducted on the relationship between penetration velocity and pressure-sinkage. Circular plates, rectangular plates, and cones were subjected to varying penetration velocities. The results confirmed that, under a constant load, there is less sinkage at a higher penetration velocity. Also, due to an inverse relationship between sinkage and rolling resistance, the rolling resistance decreases with increasing driving speed. Therefore, tires moving through soil at a higher rate of speed will tend to glide across the surface much in the same way as boats or skiers at higher speeds glide across the surface of water. Furthermore, using these results, Grahn was able to create an equation based on Bekker's pressure-sinkage relationship which is able to predict sinkage, rolling resistance, and drawbar pull of a rigid wheel at a given driving speed.
As research progressed, different techniques were employed to create better soil models. In 1998, Masad et al. improved a critical state soil mechanics framework in order to model clays. The critical state model had originally been developed to deal with isotropic materials. The new model created by Masad took into account the anisotropic nature of soil by modeling the influence of the microstructure and macrostructure of the clay as well as the properties of the clay itself. Masad found that the strain-softening exhibited by some soils is caused by the collapse of the soil structure and was able to build this relationship into their new model.

In 2003, Al-Shayea et al. developed a model to simulate soils which exhibit a postpeak strain-softening behavior such as dense sand and stiff clay. These soils act both as a brittle material, experiencing a reduction in stress once a certain strain is reached, and an elasto-plastic material. This model simulated the stress-strain behavior by combining plasticity, determined by either the Drucker-Prager model or as a function of damage strain, with damage mechanics, found by the elasto-damage formulation. Al-Shayea found that very good approximations could be obtained with this model using data from a triaxial test of the soil to parameterize the model.
In 2004, Fervers used the Finite Element Method (FEM) to investigate the interaction of tires on soft soil. The aim of Fervers' research was not to create formulations to describe the tire-soil interaction. Instead, Fervers wanted to explore the possibility of creating a two-dimensional (2D) FEM reproduction of the basic components of a tire. The 2D FEM tire model was proven to be accurate on even and uneven rigid roads and was then applied to soils. Some of Fervers' results from running the tire at high and low pressures on both a wet loose loam with high cohesion and a dry sand with low cohesion can be seen in Figure 1.4 and Figure 1.5.

![Figure 1.4: Soil compaction on loam and sand at high tire pressure (Fervers, 2004)](image1)

![Figure 1.5: Soil compaction on loam and sand at low tire pressure (Fervers, 2004)](image2)
In 2007, Coetzee et al. used three meshless modeling methods—the discrete element method (DEM) and both polar and nonpolar continuums of the material-point method (MPM)—to simulate excavator bucket filling. Meshless methods were used instead of the standard FEM due to the large displacements and distortions of the mesh which present difficulties in the bucket filling process. Coetzee found that all three methods predicted drag force filling trends correctly, but the nonpolar MPM was the most accurate followed by polar MPM and then DEM. Also, the DEM model did not accurately predict the material flow while both MPM methods did.

In 2008 and 2009, respectively, Hambleton and Drescher jointly modeled wheel-induced rutting in soils for the cases of indentation and rolling. ABAQUS was used for the FEA and the soil was simulated as an elastic-plastic solid while the tire was approximated as a rigid right-cylinder. In the area of indentation, the differences between two- and three-dimensional simulations were studied and it was found that the three-dimensional effects of indentation are minor for clays and significant for sands. Also, it was shown that the theoretical results satisfactorily agreed qualitatively with experimental data. The rolling tests were conducted with three-dimensional (3D) models with the wheel being towed (non-driving). The results showed that the rutting process of a rolling wheel is steady and, furthermore, the formulation of an analytic model for accurately predicting sinkage under steady-state conditions is feasible.

### 1.2.2 Smooth Particle Hydrodynamics

Smooth Particle Hydrodynamics (SPH) is a meshless modeling method. Schlatter (1999) traces the origin of SPH usage to the study of galaxy formation. More recently, SPH has been used for fluid dynamics, hypervelocity impacts, soft body impacts, and soil flow analysis.

Traditional meshed modeling (i.e. Finite Element Analysis) works by dividing up the simulation region using a grid. This method works well for modeling solids with small deformation, but the elements in the grid can only interact with their immediate neighbors.
SPH, on the other hand, fills the simulation region with discrete particles which, as shown in Figure 1.6, can interact with all neighbors within a certain distance. This distance is known as the smoothing length and can either be set to a constant value or varied based on real-time particle density.

![Diagram showing particle interactions](image)

**Figure 1.6: Particle $i$ interacts with other particles out to a distance of $2h$ (Schlatter, 1999)**

In order to maintain relevance to a tire-soil interaction, SPH papers pertaining to fluid dynamics have been omitted in favor of those dealing with material strength models.

In 1997, Groenenboom used PAM-SHOCK to model 2D and 3D hypervelocity impacts with SPH. The computed shapes of craters, ejecta trajectories, and debris clouds were shown to be in very good agreement with experimental data. Also, it was found that using a variable smoothing length for the SPH solver yields better results. Furthermore, Groenenboom created a model, as shown in Figure 1.7, consisting of a combination of SPH particles and FEA elements. By using the tied coupling method within PAM-SHOCK to link the SPH particles to the FEA elements, nearly identical ejecta and crater shapes were observed with a deviation of less than 0.1% in crater depth between the two models.
Also in 1997, Clegg et al. used AUTODYN-2D to perform penetrator impacts on multi-layered soil and concrete targets. SPH was chosen over FEA due to the ability of SPH to more naturally model fracture and fragmentation. Clegg compared SPH/Lagrange, Euler/Lagrange, and Lagrange/Lagrange models, as shown in Figure 1.8, and found that the SPH/Lagrange simulation yields a much better representation for the failed regions. But, while SPH has a level of accuracy similar to that of Eulerian and Lagrangian techniques, it still tends to under-predict average peak deceleration by up to 30%.
In 1999, Faraud et al. performed SPH simulations of debris impacts using PAM-SHOCK 3D and AUTODYN 2D. The research group took the point-of-view of an end-user interested in using the software to supplement research as opposed to that of a software developer or an SPH algorithm expert. It was found that, within PAM-SHOCK, an FEA model requires less computing time than an equivalent SPH model. Coupling of SPH particle and FEA elements was used to reduce the necessary CPU time. Faraud concluded that, while both programs had some problems with low velocity and hypervelocity impacts, the results were very promising for PAM-SHOCK being prototype software and AUTODYN being a beta version.

In 2004, McCarthy et al. used PAM-SHOCK to model the impact of the fluid-like behavior of a bird strike (SPH) with the leading edge of an aircraft wing (FEA) due to the SPH’s ability of variable connectivity which allows for severe distortions. Stills from a video using a gelatin bird substitute as well as those from the aforementioned SPH/FEA model are shown in Figure 1.9. McCarthy concluded that SPH was very effective for modeling a bird strike as the solution predicted realistic load transfer to the airplane wing, matched the deformed shape of the SPH bird to the gelatin substitute, and did not produce instability problems.
Shortly thereafter, Johnson and Holzapfel (2006) published a paper about modeling soft body impacts of gelatin (for synthetic bird) and ice (for hailstone) on aircraft structures. A quarter model of an SPH ice projectile impacting a load cell is shown in Figure 1.10. Johnson and Holzapfel noted the difficulty of measuring the SPH impactor properties under relevant dynamic load conditions and stated that a comparison of geometrical flow characteristics and pressure or force pulses were used to calibrate the SPH parameters. The results showed the SPH impactor model methodology to be very promising for simulating soft body impacts.

Figure 1.9: Stills from (a) a video of a gelatin bird impacting an aircraft wing and (b) an SPH bird model impacting an FEA aircraft wing (McCarthy et al., 2004)

Figure 1.10: SPH model of ice projectile during impact on load cell (Johnson et al., 2006)
In that same year, Maeda et al. (2006) published a paper on seepage failure of ground using SPH to jointly model the three phases of solid (soil), liquid (water), and gas (air). Figure 1.11 shows an SPH model of a typical demonstration test in geomechanics used to demonstrate seepage failure. Maeda found that the simulation results fit the experimental results and concluded that SPH can be used to qualitatively reproduce seepage failure.

Figure 1.11: Seepage failure around sheet pile modeled with SPH (Maeda et al., 2006)

Also in 2006, Dong et al. used SPH within AUTODYN to simulate rock cutter performance. SPH was chosen for this research because of its ability to simulate rock crushing, fragmentation, and chipping which are generally not possible with FEA methods. The results of these simulations were used to study the rock-tool interaction.

In 2007, Bui et al. published a paper on the usage of SPH for modeling the interaction of soil and water. Dry soil was modeled by a single phase while saturated soil was modeled by separate water and soil phases. Bui found that SPH was able to model the extremely large deformation and the failure of soil and concluded that, while numerical results have not been quantitatively compared with experimental data, the calculations are stable and the results appear acceptable.

Bui et al. then published another paper in 2008 on using SPH to model large deformation and failure flows of soil. Bui points out that, when applied to solids, SPH particles mimic the behavior of atoms. When compressed, atoms repel each other; when stretched, atoms attract each other. SPH particles act similarly, but under tension, some clumping of particles can
occur. The performance of non-cohesive SPH soil was validated by comparing the deformation of stacked aluminum rods to a 2D SPH model. The results of these two tests can be seen in Figure 1.12. Bui concluded that, when using a Drucker-Prager model for elastic-plastic and cohesive soils, SPH can be used but it encounters some tensile instability problems. Through various means, such as the tension cracking treatment and the artificial stress method, tensile instability can be overcome. Once corrected, the SPH method showed good agreement with both experimental and FEM results. Also, SPH can predict shear bands during failure which is generally hard to do with FEM.

Figure 1.12: (Top) Experimental result using aluminum bars and (Bottom) SPH simulation result (Bui et al., 2008)
1.3 SCOPE AND OBJECTIVES

This thesis concentrates on the structural modeling of the soil and the tire-soil interaction. As such, soil material properties are not extensively considered. The main objective of this thesis is to recommend a method of modeling the soil for a tire-soil interaction. This is done through performing a sensitivity study on the meshing and boundary size of the currently used FEA soil model and determining the feasibility and any practicalities of using SPH to model the soil.

1.4 STATEMENT OF THE WORK

Chapter 2 introduces the previously-created pneumatic FEA tire and the rigid tires derived from it which are used for the soil-tire interaction models. Chapter 3 describes the soil modeling in terms of soil properties, FEA modeling, and SPH modeling. Chapter 4 discusses the various vertical displacement and rolling resistance tire-soil interaction models. Chapter 5 details the follow-up testing including pressure-sinkage and shear-displacement tests used to validate the SPH soil model. Chapter 6 contains the conclusions drawn from this thesis and suggests some potential future work which could be done in the area of tire-soil modeling.
CHAPTER 2

TIRE MODELS

While the main focus of this thesis is on the modeling of the soil, due to the desire to model tire-soil interactions, tire models are necessary. For the most accurate modeling of the tire-soil interaction, a pneumatic tire model is used; the first section of this chapter describes this pneumatic tire model. However, the pneumatic tire significantly increases the amount of time necessary to calculate the results. For this reason, two rigid tire models—as described in the second section of this chapter—were used for preliminary tire-soil testing.

2.1 PNEUMATIC TIRE MODEL

When simulating a full truck model, a system comprised of multiple different bodies linked together is commonly used. Within these models, a simplified rigid ring tire model is frequently chosen. This rigid ring model is able to give accurate results while reducing the computational load on the computer. In order to accurately represent a real pneumatic tire, several in-plane and out-of-plane parameters need to be determined for the rigid ring model. Due to the high expense of physical tire tests, the parameters for the rigid ring model are predicted by performing virtual tests using an FEA truck tire model (Chae, 2006).

The preliminary FEA truck tire model was created by Seokyong Chae as partial requirement for a Ph.D. in materials. This radial-ply, three-dimensional, nonlinear FEA truck tire was created using PAM-CRASH and was modeled after the Goodyear G357, 295/75R22.5 G. The masses and weights of the tire and rim are as shown in Table 2.1 while the dimensions of the tire are shown in Figure 2.1.
Table 2.1: Tire and rim weights

<table>
<thead>
<tr>
<th></th>
<th>Mass (kg)</th>
<th>Weight (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tire</td>
<td>51</td>
<td>112</td>
</tr>
<tr>
<td>Rim</td>
<td>32</td>
<td>70</td>
</tr>
</tbody>
</table>

Figure 2.1: Three groove truck tire basic dimensions

The creation of this FEA truck tire model has been a multi-phase process with Mack Trucks/Volvo 3P. The truck tire model from Phase I has been modified and improved in Phase II to more precisely predict the vertical loading and displacement behavior. This modified tire shows very good correlation to the data measured by Goodyear. For these validation tests, the tire is inflated to 110 psi (0.759 MPa), placed on a flat surface, and then loaded vertically at the center of the rim from 0 to 12,000 lbs (53,378 N). The load-displacement curves for the data from Goodyear and the Phase I and Phase II FEA tires are shown in Figure 2.2 while Figure 2.3 shows the improved tire contact area and better correlation with the data from Goodyear.
Figure 2.2: Static stiffness curve

Figure 2.3: Contact area versus vertical load
The FEA truck tire model used in the new simulations described herein is similar to the modified Phase II FEA truck tire. However, the vertical force-vs.-displacement values were rechecked by loading the tire from 3,000 lbs (13.34 kN) to 9,000 lbs (40.03 kN). The flat-ground loading case data from these tests is shown in Table 2.2 and Figure 2.4 while Table 2.3 shows the vertical stiffness (Fz/z) and the tangential stiffness as calculated by the slope of the curve at the given vertical loads. Furthermore, for all of the tire-soil interactions, the pneumatic tire is loaded at 6,000 lbs (26.69 kN) and inflated to 85 psi (unless otherwise noted).

**Table 2.2: Vertical force vs. vertical displacement**

<table>
<thead>
<tr>
<th>Fz (kN)</th>
<th>13.34</th>
<th>26.69</th>
<th>40.03</th>
</tr>
</thead>
<tbody>
<tr>
<td>z (mm)</td>
<td>15.9</td>
<td>29.6</td>
<td>42</td>
</tr>
</tbody>
</table>

**Figure 2.4: Vertical Force vs. Vertical Displacement**

\[ F(z) = 0.0043z^2 + 0.7731z \]

**Table 2.3: Tire vertical stiffness**

<table>
<thead>
<tr>
<th>Tire Loading</th>
<th>Vertical Stiffness (Fz/z) (kN/m)</th>
<th>Tangent Vertical Stiffness (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.34kN (3,000 lbs)</td>
<td>839.0</td>
<td>909.8</td>
</tr>
<tr>
<td>26.69 kN (6,000 lbs)</td>
<td>901.7</td>
<td>1027.7</td>
</tr>
<tr>
<td>40.03 kN (9,000 lbs)</td>
<td>953.1</td>
<td>1134.3</td>
</tr>
</tbody>
</table>
2.2 RIGID TIRE MODELS

Two main tire-soil interactions were studied for this thesis: vertical displacement and rolling resistance. For each of these interactions a different rigid tire model was used. The following sub-sections further define the rigid tires used for each interaction.

2.2.1 Vertical Displacement Rigid Tire Model

The tire model used for the vertical displacement soil tests was the abovementioned pneumatic 3-groove tire with the following modifications: the tread was removed in order to eliminate many solid elements; the tire was completely defined as a rigid body to reduce time-consuming compliance calculations; and the width was increased by 20% to simulate the wider footprint of a pneumatic tire. The dimensions of this modified rigid tire model are as shown in Figure 2.5.

![Figure 2.5: Rigid tire basic dimensions](image)
2.2.2 Rolling Resistance Rigid Tire Model

Before starting the rolling resistance virtual tests with the towed rigid tire, a preliminary test was run with the pneumatic tire. During this test, it was found that the pneumatic tire does not deform in the soft soil nearly as much as it does on rigid road. Therefore, for the rolling resistance tests, the rigid tire model was created by simply removing the tread from the FEA tire and defining it as a rigid body. The dimensions of this tire are shown in Figure 2.6.

Figure 2.6: Towed test rigid tire basic dimensions
CHAPTER 3
SOIL MODELING

Modeling soil is an inherently complicated issue. Most soil is composed of a non-homogenous mixture of particles causing it to act differently from well understood elastic-plastic materials. Even as such, standards have been set for measuring soil properties and different soils have been characterized as best as possible. For this thesis, material properties for soils are assumed to be complete and correct. Therefore, the first section of this chapter briefly covers different soil types and the properties used to model the chosen soil of “dense sand.” Then, the second and third sections of this chapter discusses the basic ways in which FEA and SPH are used to model the soil.

3.1 SOIL PROPERTIES

According to The Idaho Association of Soil Conservation Districts, soil is typically classified based on the relative proportions of silt, sand, and clay (Idaho OnePlan, 2010). The different soil types which result from the various composition ratios are shown by the triangle in Figure 3.1 (retrieved from the Idaho OnePlan website). For this thesis, the soil type being modeled is a “dense sand” which doesn’t appear in Figure 3.1, but is most closely analogous to the “sandy loam.”

Due to the low emphasis on soil property measurement for this thesis, the soil properties used were the mean values of ranges given by U.S. Department of Transportation and Federal Highway Administration publications as reported in previous soil-modeling work performed within this research lab (Slade, 2009). The properties selected were intended for a simplified linear soil model which takes into account stiffness, yield stress, and density, but did not include parameters for moisture or cohesion. The soil properties used—and their corresponding values—are shown in Table 3.1 as well as properties and values for some other soil types to serve as reference for comparison.
Figure 3.1: Soil composition ratios (Idaho OnePlan, 2010)

Table 3.1: Material properties for various soil types with the relevant soil highlighted

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Elastic Modulus, E (MPa)</th>
<th>Bulk Modulus, K (MPa)</th>
<th>Shear Modulus, G (MPa)</th>
<th>Yield Stress, Y (MPa)</th>
<th>Density, $\rho$ (ton/mm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense sand/sandy loam</td>
<td>22</td>
<td>15</td>
<td>9</td>
<td>0.016</td>
<td>1.60E-09</td>
</tr>
<tr>
<td>Loose sand</td>
<td>17</td>
<td>11</td>
<td>7</td>
<td>0.004</td>
<td>1.44E-09</td>
</tr>
<tr>
<td>Silty sand</td>
<td>45</td>
<td>30</td>
<td>18</td>
<td>0.02</td>
<td>1.68E-09</td>
</tr>
<tr>
<td>Sand and gravel</td>
<td>121</td>
<td>80</td>
<td>48</td>
<td>0.024</td>
<td>1.92E-09</td>
</tr>
</tbody>
</table>
3.2 FEA SOIL MODELING

The FEA soil was modeled as an elastic-plastic solid using the PAM-CRASH material type 1. The properties necessary for this material type are those listed in Table 3.1. The filled PAM-CRASH material card is shown in Figure 3.2. Values in Figure 3.2 not also in Table 3.1 are either user-defined identification information or unchanged default values.

![Figure 3.2: FEA material card for PAM-CRASH material type 1](image)

The original FEA soil was discretized using a meshing strategy loosely based on a soil model from J. Y. Wong's *Theory of Ground Vehicles* as shown in Figure 3.3 (2008). A non-uniform mesh is used in order to decrease the number of elements needed which decreases the amount of time necessary to run the simulations. This soil model was validated with some simple quantitative and qualitative measures (Slade, 2009), but it still required a sensitivity study as detailed later in this thesis to either validate the results or further improve the accuracy of the predictions.

![Figure 3.3: Stresses in FEA sand (Wong, 2009)](image)
3.3 SPH SOIL MODELING

The soil for the SPH model is composed of a finite collection of particles. These particles are created from an FEA mesh as shown in Figure 3.4.

![Figure 3.4: (a) FEA mesh, (b) creation of an SPH particle at the center of each FEA element, (c) initial SPH particle orientation](image)

The SPH soil was modeled as a hydrodynamic elastic-plastic material using the PAM-SHOCK material type 7. As detailed by McCarthy (2004), this SPH material type uses an equation of state (EOS) to govern its pressure-volume relationship. The full EOS is shown in Equation 1.

\[
p = p_0 + C_1 \mu + C_2 \mu^2 + C_3 \mu^3 + (C_4 + C_5 \mu + C_6 \mu^2)E_i
\]

with \( \mu = (\rho/\rho_0) - 1 \) (with \( \rho \) being the current material density and \( \rho_0 \) the initial material density), \( C_i \) the yet-determined material constants, and \( E_i \) the internal energy.

The simplest form of this EOS comes from setting all \( C_i \) except for \( C_1 \) to zero which results in a dilatational elastic material with bulk modulus of \( C_1 \). With this EOS format, the SPH soil uses the same material properties from Table 3.1 that the FEA soil uses. The filled PAM-SHOCK material card is shown in Figure 3.5. As with the FEA soil model, values in Figure 3.5 not also in Table 3.1 are either user-defined identification information or unchanged default values. However, the SPH soil requires more definition than just the material properties.
Figure 3.5: SPH material card for PAM-SHOCK material type 7

While material properties are enough to fully define the FEA soil, the SPH soil also requires part properties which further define the interactions of the individual SPH particles. These properties include the particle smoothing length to radius ratio (RATIO), the minimum (HMIN) and maximum (HMAX) smoothing lengths, the anti-crossing force parameter (ETA), and artificial viscosity variables based on the linear (ALPHA) and quadratic (BETA) terms of velocity. These properties and the values used for the soil simulations are shown in Table 3.2.

The RATIO, HMIN, and HMAX properties are all used to determine how far each SPH particle will search for neighbors. While the RATIO parameter is adjusted to affect how the particles react to each other, HMIN and HMAX are set to be approximately one order of magnitude less and greater, respectively, than the initial smoothing length. This is done because, as shown by previous research mentioned in Section 0, allowing the SPH solver to adjust the smoothing length based on instantaneous particle density yields the best results. Within PAM-SHOCK, the RATIO value must be set between 1.21 and 2.1. Due to the initial RATIO defined by PAM-SHOCK and the particle density, HMIN was set to 1 and HMAX was set to 100.
Furthermore, the ETA property is used to determine how the SPH particles react to one another and the ALPHA and BETA properties are used to constrain the movement of the individual particles. The basis for some of these values were found within other reports (Faraud, 1999; Bui 2007, 2008) but the specific values were determined through trial and error. The ETA property was chosen to be 0.1, but others reported using values as low as 0.01 and a value of zero didn’t greatly affect simulation results while resulting in reductions in computational time. Throughout virtual testing, it was found necessary to adjust the ALPHA value between zero and three to obtain the desired results while changing BETA had no discernable effect.

Table 3.2: SPH soil part properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle smoothing length to radius ratio</td>
<td>RATIO</td>
<td>1.3 to 2.1*</td>
</tr>
<tr>
<td>Minimum smoothing length</td>
<td>HMIN</td>
<td>1</td>
</tr>
<tr>
<td>Maximum smoothing length</td>
<td>HMAX</td>
<td>100</td>
</tr>
<tr>
<td>Anti-crossing force parameter</td>
<td>ETA</td>
<td>0.1</td>
</tr>
<tr>
<td>Linear artificial viscosity</td>
<td>ALPHA</td>
<td>0 to 3*</td>
</tr>
<tr>
<td>Quadratic artificial viscosity</td>
<td>BETA</td>
<td>0</td>
</tr>
</tbody>
</table>

* Adjusting these values affects the settling depth of the tire and are used to make the SPH soil approximate the FEA soil in that regard.
CHAPTER 4
TIRE-SOIL INTERACTIONS

This chapter describes the various tire-soil interaction simulations. The first set of simulations—vertical displacement—are used to perform a sensitivity study on the FEA soil and to verify the ability to use SPH to model the soil. The second set of simulations—rolling resistance—are used to compare the FEA soil to the SPH soil and to compare the rigid tire to the pneumatic tire.

4.1 VERTICAL DISPLACEMENT TESTS

For the vertical displacement virtual tests, the rigid tire detailed in Section 2.2.1 is loaded to a force of 26.7 kN (6000 lbs) slowly and steadily over the course of 2.2 seconds. The settling depth and the amount of CPU time required for each model are then analyzed quantitatively while the stresses in the soil are looked at qualitatively.

4.1.1 FEA Soil Models

This section details the sensitivity study of the FEA soil model.

4.1.1.1 Original Soil

The original 3D FEA soil model is 600 mm deep, 800 mm wide, and 1200 mm long. This soil model was created using symmetrical elements when viewed from above and non-symmetrical elements when viewed from the front or side. Figure 4.1 shows some basic dimensions of the elements and the dimensions of the soil plot while Figure 4.2 shows an isometric view of this original mesh. The outer edges and bottom of the soil (as shown in orange) are fixed in the X-, Y-, and Z-directions in order to simulate a fixed plot of soil. When the tire is loaded on this soil plot, significant edge effects can be seen along the top surface of the soil as shown in Figure 4.5.
Figure 4.1: Original soil mesh shown from (a) the top and (b) the front

Figure 4.2: Isometric view of the original mesh soil plot
4.1.1.2 Double Width Soil

The first solution for limiting the edge effects was to double the width of the soil plot to 1600 mm. An isometric view of the double width soil plot can be seen in Figure 4.3 and cutaways of the normal and double width soil showing the von Mises stress are presented in Figure 4.4 and Figure 4.5.

As shown in Figure 4.5, the edge effects significantly alter the curvature of the soil surface for the normal width soil plot. These surface stresses greatly reduce the depth at which the stresses penetrate the soil and end up reducing the settling depth of the tire. For the normal width soil model, the tire settles at a depth of 100 mm. For the double width soil model, the tire settles at 113.5 mm. This is an increase of 13.5%.
Figure 4.4: Von Mises stress on (a) normal width and (b) double width soil plots

Figure 4.5: Von Mises stress on (a) normal width and (b) double width soil plots with the soil curvature at the surface and the stress depth highlighted in red
4.1.1.3 Double Depth Soil

Next, the effect of the depth of the soil was evaluated by doubling the depth of both the normal width and double width soil plots to 1200 mm. Isometric views of these new soil plots can be seen in Figure 4.6 and von Mises stress distributions are presented in Figure 4.7 and Figure 4.8.

Figure 4.6: Isometric view of double depth soil plots

Comparing Figure 4.8 to Figure 4.5, the stress distributions at the surface and within the soil are very similar between the normal and double depth soil plots. Also, the settling depth is not greatly affected by doubling the depth of the soil. The settling depth of the tire increases from 100 mm to 102 mm for the normal width soil plot and from 113.5 mm to 116 mm for the double width soil plot.
Figure 4.7: Von Mises stress on double depth (a) normal and (b) double width soil plots

Figure 4.8: Von Mises stress on double depth (a) normal width and (b) double width soil plots with the soil curvature at the surface and the stress depth highlighted in red
4.1.1.4 Uniform Mesh

After investigating the effects of changing the size of the soil plot, changes to the actual mesh of the soil were studied. When creating meshes, better results generally occur when elements with uniform dimensions are implemented. In the case of the soil, the hexagonal elements should be cube shaped. Uniform meshes of 25 mm and 12.5 mm, as shown in Figure 4.9 and Figure 4.10, were used for comparison. Anything significantly larger than 25 mm (e.g. 30 mm) has problems with the tire-soil contact while anything smaller than 12.5 mm results in an unreasonably large computational time. The von Mises stress for these soil plots are presented in Figure 4.11 and Figure 4.12.

Figure 4.9: Uniform 25 mm mesh shown from (a) the top and (b) the front
Comparing Figure 4.12 to Figure 4.5, the stress distributions at the surface and within the soil are very similar between the original mesh and both the uniform 25 mm and 12.5 mm meshes. Also, the settling depth is only marginally affected by the increased mesh density. The settling depth of the tire increases from 100 mm to 102 mm for the 25 mm uniform mesh and to 103 mm for the 12.5 mm mesh.
Figure 4.11: Von Mises stress on uniform meshed (a) 25 mm and (b) 12.5 mm soil plots

Figure 4.12: Von Mises stress on uniform meshed (a) 25 mm and (b) 12.5 mm soil plots with the soil curvature at the surface and the stress depth highlighted in red
4.1.1.5 Free Edges

Due to the significant edge effects observed, the next adjustment made was to allow the edges of the soil to be free to move in the Z-direction. By doing this, the model more closely reflects a finite amount of soil in a box in which the outer edges of the soil can slide along the inner edges of the box. A representation of the modeling of the free edges is shown in Figure 4.13. A comparison of the von Mises stress, surface curvature, surface stress concentrations, and stress depth for fixed edge and free edge soil plots are shown in Figure 4.14 and Figure 4.15.

![Figure 4.13: Representation of edges free to move in the Z-direction](image-url)
Figure 4.14: Von Mises stress on original mesh (a) fixed and (b) free edge soil plots

Figure 4.15: Von Mises stress on original mesh with (a) fixed edges and (b) free edges with the soil curvature at the surface and the stress depth highlighted in red
The difference between fixed edges and free edges is very clearly shown in Figure 4.15. The abnormal stress concentrations at the surface are relieved, the stress depth penetrates deeper, and the tire settles to a depth of 114 mm (as opposed to 100 mm with fixed edges). Furthermore, comparing the free edge normal width to the fixed edge double width, as shown in Figure 4.16, similarities between the soil surface curvature, stress concentrations throughout the soil, and the stress depth can be seen. Furthermore, the settling depth of the tire is almost the same for the fixed edge double width soil as it is for the free edge normal width soil.

![Figure 4.16: Von Mises stress on original mesh with (a) free edge normal width and (b) fixed edge double width with the soil curvature at the surface and the stress depth highlighted in red](image)

Due to these results, free edge models were run for all of the previous cases as well as a 25 mm uniform mesh with both double width, double depth, and combined double width and double depth soil plots. The data for all of these free edge simulations can be seen in Table 4.1.
4.1.1.6 FEA Vertical Displacement Results

The FEA static deflection simulations yielded interesting results. The most important of which are:

- Edge effects at the surface due to fixed edges results in decreased tire settling depth and can be remedied by:
  - Doubling the width of the soil plot, or
  - Allowing the edges of the soil to move freely in the Z-direction.
- Edge effects at the surface due to fixed edges (generally) increase CPU runtime.
- Doubling the soil depth has no significant effects on tire settling depth.
- Doubling the soil width when the edges are free to move in the Z-direction does not significantly increase tire settling depth.
- Decreasing the uniform mesh size below 25 mm does not significantly affect tire settling depth.
- Increasing the uniform mesh size above 25 mm creates problems with the tire-soil contact.

The tire settling depth and CPU runtime results from all of the vertical displacement simulations are shown in Table 4.1.

Table 4.1: Settling depth and CPU runtime for all FEA models

<table>
<thead>
<tr>
<th>Soil Plot Type</th>
<th>Tire Settling Depth (mm)</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed Edges</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original mesh</td>
<td>100</td>
<td>2.671 E 3</td>
</tr>
<tr>
<td>Original mesh, double width</td>
<td>114</td>
<td>5.252 E 3</td>
</tr>
<tr>
<td>Original mesh, double depth</td>
<td>102</td>
<td>5.094 E 3</td>
</tr>
<tr>
<td>Original mesh, double width and depth</td>
<td>116</td>
<td>1.028 E 4</td>
</tr>
<tr>
<td>Uniform 25 mm mesh</td>
<td>102</td>
<td>4.384 E 3</td>
</tr>
<tr>
<td>Uniform 12.5 mm mesh</td>
<td>103</td>
<td>*</td>
</tr>
<tr>
<td><strong>Free Edges</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original mesh</td>
<td>114</td>
<td>2.664 E 3</td>
</tr>
<tr>
<td>Original mesh, double width</td>
<td>119</td>
<td>4.871 E 3</td>
</tr>
<tr>
<td>Original mesh, double depth</td>
<td>116</td>
<td>4.783 E 3</td>
</tr>
<tr>
<td>Original mesh, double width and depth</td>
<td>122</td>
<td>8.626 E 3</td>
</tr>
<tr>
<td>Uniform 25 mm mesh</td>
<td>117</td>
<td>4.423 E 3</td>
</tr>
<tr>
<td>Uniform 12.5 mm mesh</td>
<td>116</td>
<td>5.658 E 4</td>
</tr>
<tr>
<td>Uniform 25 mm double width</td>
<td>122</td>
<td>9.616 E 3</td>
</tr>
<tr>
<td>Uniform 25 mm double width and depth</td>
<td>126</td>
<td>1.395 E 4</td>
</tr>
</tbody>
</table>

*After settling, simulation terminates prematurely. Runtime data cannot be obtained.
4.1.2 SPH Soil Models

This section verifies the ability of SPH particles to both fully and partially replace FEA elements in the soil model.

4.1.2.1 SPH Soil Model

For this model, the normal width and depth 25 mm uniform model was converted from FEA elements to SPH particles. Then, a box made from shell elements was constructed around the outside of the SPH particles. The SPH soil model with and without the surrounding box can be seen in Figure 4.17 and a comparison of SPH and FEA deformations is shown in Figure 4.18.

Figure 4.17: SPH soil model shown (a) with and (b) without surrounding box
4.1.2.2 Combined SPH and FEA Soil Model

Like the all SPH model, the normal width and depth 25 mm uniform model was used as the basis. But, for this model, just the top three layers of FEA elements were converted to SPH particles with a single FEA element being left around the edges in order to contain the SPH particles. The combined SPH/FEA soil model with and without the FEA elements can be seen in Figure 4.19 and a comparison of the combined SPH/FEA and just FEA deformations is shown in Figure 4.20.
Figure 4.19: Combined SPH/FEA model (a) with and (b) without FEA elements

Figure 4.20: Deformation of (a) combined SPH/FEA and (b) FEA soil plots
4.1.2.3 SPH Vertical Displacement Results

The SPH models have proven that SPH particles can be used to simulate soil and obtain vertical deflections similar to that of the FEA soil. Due to the slow loading of the tire, changing the artificial viscosity parameter ALPHA did not greatly affect the settling depth and just the smoothing length to radius ratio, RATIO, was used to adjust the settling depth. Both the SPH only soil and the FEA/SPH soil have been calibrated with settling depths close to the free edge FEA soil plot with the uniform 25 mm mesh and for settling depths close to that of the double wide and double deep uniform 25 mm soil plot. The settling depths for the SPH models are presented in Table 4.2 along with the CPU runtimes which show replacing FEA with SPH results in a model which takes longer to run.

<table>
<thead>
<tr>
<th>Soil Plot Type</th>
<th>Tire Settling Depth (mm)</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Edge FEA</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Original mesh</td>
<td>114</td>
<td>2.664 E 3</td>
</tr>
<tr>
<td>Uniform 25 mm mesh</td>
<td>117</td>
<td>4.423 E 3</td>
</tr>
<tr>
<td>Uniform 25 mm double width and depth</td>
<td>126</td>
<td>1.395 E 4</td>
</tr>
<tr>
<td>SPH</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 1.4</td>
<td>119</td>
<td>1.704 E 4</td>
</tr>
<tr>
<td>RATIO 1.66</td>
<td>126</td>
<td>1.802 E 4</td>
</tr>
<tr>
<td>SPH/FEA</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 2.0</td>
<td>116</td>
<td>7.056 E 3</td>
</tr>
<tr>
<td>RATIO 2.1</td>
<td>127</td>
<td>7.740 E 3</td>
</tr>
</tbody>
</table>
4.2 ROLLING RESISTANCE TESTS

For the rolling resistance virtual tests, the rigid tire detailed in Section 2.2.2 is loaded to a force of 26.7 kN (6000 lbs) over the course of 0.001 seconds and simultaneously brought up to a towed, unlocked (i.e. free to roll) speed of 25 km/hr over the course of 0.1 seconds with the tire reaching the end of the soil plot in approximately one second. The rolling resistance coefficient of the tire—calculated by dividing the force necessary to tow the tire by the vertical force acting on the tire—and the CPU runtimes are then compared for the different models.

4.2.1 FEA Soil

The FEA soil used as a baseline for comparison of the SPH and FEA/SPH soil models is the 25 mm uniform mesh with outer edges free to move in the Z-direction. As determined previously, this FEA soil offers the best results while also acting as a good parallel to the SPH soil by having elements distributed in the same manner and density as the SPH particles. Two views of the initial state of this soil model are shown in Figure 4.21 while Figure 4.22 shows an isometric view of the tire and soil approximately halfway through the simulation.

![Figure 4.21: Uniform 25 mm mesh shown from (a) the top and (b) the front](image)
The soil plot shown in Figure 4.22 is 800 mm wide and 600 mm deep, but the length has been increased to 7200 mm. The outer edges of the soil (as shown in orange in Figure 4.22) are fixed in the X- and Y- directions, but are free to move in the Z-direction as if the soil were placed in a box. Unlike the vertical displacement simulations, for the rolling resistance simulations, the tire is loaded vertically relatively quickly and then pulled through the soil. In this loading scenario, the tire bottoms out and then rises to a steady-state position, much as a water skier would as he goes from standing still, submerged in the water, to being pulled at a given velocity, skimming along the top of the water. A cutaway of the FEA soil model at various time-steps in the test is shown in Figure 4.23.
For the all FEA soil model, the tire reaches a steady-state vertical displacement of approximately 127 mm below the starting position, as shown in Figure 4.24. The rolling resistance coefficient levels off at approximately 0.48 at this position, as shown in Figure 4.25. This steady-state vertical displacement will be used as the calibration metric for the SPH and FEA/SPH models and the rolling resistance coefficient will be used as the benchmark for comparison of these new models.
Figure 4.24: Vertical displacement for the FEA soil model

Figure 4.25: Rolling resistance coefficient for the FEA soil model
4.2.2 SPH Soil

Part properties for the SPH soil were previously determined, but the vertical loading was done very slowly and consistently maintained states closer to quasi-static. The more rapid loading and higher velocity impacts caused at the front of the tire while rolling create much more dynamic states. However, these velocities are still far below sonic speeds. Therefore, to obtain the correct settling depth, the linear artificial viscosity, ALPHA, has been changed in some instances, but the quadratic artificial viscosity, BETA, was set to a value of zero.

The baseline SPH model was once again created from the 25mm uniform FEA soil. The effect of the particle density was evaluated by creating a SPH model with approximately twice the number of particles. Also, in order to test the effect of the linear artificial viscosity, ALPHA (denoted by "a" in the figures’ data series labels), on the rolling resistance without changing the settling depth, parameters were chosen for both the minimum suitable ALPHA with its corresponding smoothing length ratio, RATIO (denoted by "Ratio" in the figures’ data series labels), as well as for the maximum RATIO with its corresponding (higher) ALPHA. An isometric view of the towed tire in SPH is shown in Figure 4.26 while Figure 4.27 shows a cutaway of the model at various time-steps in the simulation.

Figure 4.26: Towed tire test on the SPH soil, showing plastic strain, with a rigid tire
As shown in Figure 4.28, all of the SPH soil models were tailored to have a vertical displacement very close to that of the FEA soil model. But, despite this similar vertical displacement, the SPH soil models all gave a rolling resistance coefficient greater than that of the FEA.

The rolling resistance coefficient, as shown in Figure 4.29, levels off at about 0.55 for the SPH models as compared to approximately 0.48 for the FEA model. This is an increase of about 15%. Also visible in Figure 4.29 is the relative instability of the SPH soil case using a RATIO of 1.6 and an ALPHA value equal to 2.
Figure 4.28: Vertical displacement for the SPH soil models

Figure 4.29: Rolling resistance coefficient for the SPH soil models
4.2.3 Combined FEA/SPH Soil

Due to the relatively long CPU runtime caused by using only SPH particles for the soil, combined FEA/SPH soil was explored again. The combined FEA/SPH model presented previously used a contact defined between the FEA elements and the SPH particles to govern their relationship. However, the faster loading of the tire in the rolling resistance virtual tests causes the SPH particles to separate from the FEA elements if they are linked only with a contact. This problem was fixed by using a tied link within PAM to affix the outermost SPH particles along the bottom and sides to their closest FEA elements. This method results in the same vertical displacement as using the contact method, but by using a tied link, the soil doesn't separate and a computationally time-consuming contact definition is removed from the model.

Four different FEA/SPH soil model configurations were examined (explored in more detail within this section) each with a pair of SPH part properties (as with the SPH only soil models). An isometric view of the tire running in combined FEA/SPH soil is shown in Figure 4.30 while Figure 4.31 shows a cutaway of the model at various time-steps in the test. As with the FEA model, the edges, as shown in orange in Figure 4.30, are fixed in the X- and Y-directions, but are free to move in the Z-direction.
Figure 4.30: Towed tire test on the combined FEA/SPH soil, showing plastic strain, with a rigid tire

Figure 4.31: Cutaway of the combined FEA/SPH soil shown at the beginning, middle, and end of the simulation with the trajectory of the center point shown in red
4.2.3.1 Original density and 1/8 depth FEA/SPH model

For this model, the top three layers of FEA elements of the 25mm uniform mesh soil model were converted to SPH particles, excluding a one-element-thick barrier around the outside of the soil plot used to contain the SPH particles. A comparison between the FEA model and the FEA/SPH model is shown in Figure 4.32.

![Figure 4.32: Rigid tire with (a) all FEA soil and (b) original density and 1/8 depth FEA/SPH soil](image)

As for the all SPH soil model, and as shown in Figure 4.33, the SPH soil part parameters have been tailored to make the FEA/SPH models achieve a steady-state vertical displacement similar to that of the FEA model.

The rolling resistance coefficient, as shown in Figure 4.34, levels off at two different values for the two different soil part parameters. The soil with a RATIO of 1.41 and an ALPHA of zero settles at approximately the same level as the FEA model while soil with a RATIO of 2.1 and an ALPHA of 2.15 displays an increase of about 4% by settling at roughly 0.50.
Figure 4.33: Vertical displacement for the original density and 1/8 depth FEA/SPH soil models

Figure 4.34: Rolling resistance coefficient for the original density and 1/8 depth FEA/SPH soil models
4.2.3.2 Original density and 1/4 depth FEA/SPH model

For this model, the top six layers of FEA elements of the 25mm uniform mesh soil model were converted to SPH particles, excluding a one-element-thick barrier around the outside of the soil plot used to contain the SPH particles. A comparison between the FEA model and the FEA/SPH model is shown in Figure 4.35.

Figure 4.35: Rigid tire with (a) all FEA soil and (b) original density and 1/4 depth FEA/SPH soil

Figure 4.36 shows how the SPH soil part parameters have again been tailored to make the FEA/SPH models achieve a steady-state vertical displacement similar to that of the FEA model.

The rolling resistance coefficient, as shown in Figure 4.37, levels off at approximately 0.51 for both sets of SPH soil part parameters. This is an increase of about 6% over the FEA model.
Figure 4.36: Vertical displacement for the original density and 1/4 depth FEA/SPH soil models.

Figure 4.37: Rolling resistance coefficient for the original density and 1/4 depth FEA/SPH soil models.
4.2.3.3 Original density and 1/2 depth FEA/SPH model

For this model, the top 12 layers of FEA elements of the 25mm uniform mesh soil model were converted to SPH particles, excluding a one-element-thick barrier around the outside of the soil plot used to contain the SPH particles. A comparison between the FEA model and the FEA/SPH model is shown in Figure 4.38.

As shown in Figure 4.39, the SPH soil part parameters have been tailored to make the FEA/SPH models achieve a steady-state vertical displacement similar to that of the FEA model.

The rolling resistance coefficient, as shown in Figure 4.40, levels off at approximately 0.53 for both sets of SPH soil part parameters. This is an increase of about 10% over the FEA model.
Figure 4.39: Vertical displacement for the original density and 1/2 depth FEA/SPH soil models

Figure 4.40: Rolling resistance coefficient for the original density and 1/2 depth FEA/SPH soil models
4.2.3.4 Octuple density and 1/8 depth FEA/SPH model

For this model, the top three layers of FEA elements of the 25mm uniform mesh soil model, excluding a one-element-thick barrier around the outside of the soil plot used to contain the SPH particles, were each split into eight equal parts before being converted to SPH particles. A comparison between the FEA model and the FEA/SPH model is shown in Figure 4.41.

![Figure 4.41: Rigid tire with (a) all FEA soil and (b) octuple density and 1/8 depth FEA/SPH soil](image)

As before, and as shown in Figure 4.42, the SPH soil part parameters have been tailored to make the FEA/SPH models achieve a steady-state vertical displacement similar to that of the FEA model.

The rolling resistance coefficient, as shown in Figure 4.42, levels off at two different values for the two different soil part parameters. The soil with a RATIO of 2.1 and an ALPHA of 1 settles at approximately the same level as the FEA model while soil with a RATIO of 1.6 and an ALPHA of zero displays an increase of just 2% by settling at roughly 0.49.
Figure 4.42: Vertical displacement for the octuple density and original depth FEA/SPH soil models

Figure 4.43: Rolling resistance coefficient for the octuple density and original depth FEA/SPH soil models
4.2.4 Rigid Tire Results

By studying the results from the rolling resistance simulations, some observations were made:

- Adding SPH particles increases the CPU runtime, even when it is just a one-to-one replacement of existing FEA elements.
- While most of the models containing SPH particles resulted in an increase in the rolling resistance coefficient, some merely remained the same as the FEA model.
- Increasing the SPH particle density had almost no effect on the rolling resistance coefficient.
- Increasing the SPH particle depth increased the rolling resistance coefficient.
- Increasing particle smoothing length ratio, RATIO, and the linear artificial viscosity, ALPHA:
  - Sometimes increased and sometimes decreased the rolling resistance coefficient, although the effects were small
  - Always increased the CPU runtime

All of the rolling resistance coefficient and CPU runtime results can be seen in Table 4.3.

Table 4.3: Rolling resistance coefficient and CPU runtime for the rigid tire

<table>
<thead>
<tr>
<th>Soil Plot Type</th>
<th>Rolling Resistance Coefficient</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Uniform 25 mm mesh</td>
<td>0.48</td>
<td>1.918 E 4</td>
</tr>
<tr>
<td>SPH - Original</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 1.6 - ALPHA 2</td>
<td>0.55</td>
<td>7.794 E 4</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 2.4</td>
<td>0.55</td>
<td>1.473 E 5</td>
</tr>
<tr>
<td>SPH - Double Density</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 1.6 - ALPHA 1.5</td>
<td>0.55</td>
<td>1.559 E 5</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 2.1</td>
<td>0.54</td>
<td>3.274 E 5</td>
</tr>
<tr>
<td>SPH/FEA - 1/8 Depth</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 1.41 - ALPHA 0</td>
<td>0.48</td>
<td>2.442 E 4</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 2.15</td>
<td>0.50</td>
<td>3.012 E 4</td>
</tr>
<tr>
<td>FEA/SPH - 1/4 Depth</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 1.3 - ALPHA 0</td>
<td>0.51</td>
<td>2.317 E 4</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 1.5</td>
<td>0.51</td>
<td>3.407 E 4</td>
</tr>
<tr>
<td>FEA/SPH - 1/2 Depth</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 1.6 - ALPHA 1.1</td>
<td>0.53</td>
<td>3.705 E 4</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 1.6</td>
<td>0.53</td>
<td>5.849 E 4</td>
</tr>
<tr>
<td>FEA/SPH - 1/8 Depth, 8x Density</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 1.6 - ALPHA 0</td>
<td>0.49</td>
<td>7.450 E 4</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 1</td>
<td>0.48</td>
<td>1.529 E 5</td>
</tr>
</tbody>
</table>
4.2.5 Pneumatic Tire Tests

While a rigid tire model is good for tuning the soil, a pneumatic tire model should give more accurate results. After determining the best soil models with the rigid tire, the pneumatic tire was run on four different soil models: all FEA, all SPH, and FEA/SPH with 1/8 depth SPH of both the original density and eight times the density. A side-view of the tire running on the four different soil configurations is shown in Figure 4.44. In order to emphasize the deflection of the pneumatic tire, a red circle has been drawn around the tire in each image.

![Figure 4.44: Pneumatic tire at steady state on various soils with a circular outline shown in red](image-url)
Due to the findings with the rigid tire (namely, that differences in SPH part parameters don’t greatly affect rolling resistance), all of the SPH models use a smoothing length to radius ratio, RATIO, of 2.1 along with the corresponding linear artificial viscosity, ALPHA, which causes the tire to reach a steady-state vertical displacement similar to that of the FEA soil model.

For the FEA soil model, the tire reaches a steady-state vertical displacement of approximately 112 mm below the starting position at which the rolling resistance coefficient levels off at approximately 0.43. This steady-state vertical displacement was once again used as the target for the FEA/SPH and SPH models, as shown in Figure 4.45, and the rolling resistance coefficient on the FEA soil is used as the benchmark for comparing the FEA/SPH and SPH rolling resistance coefficients shown in Figure 4.46.
As expected, the all SPH soil had a rolling resistance coefficient higher than the all FEA soil. Leveling off at approximately 0.50, the SPH soil showed an increase over the FEA soil by approximately 16%. However, unexpectedly, the FEA/SPH model with a higher density of SPH particles showed a rolling resistance coefficient of about 0.46 while the normal density FEA/SPH model leveled off at 0.44. While this is only a small change, the rigid, treadless tire showed practically no change. Regardless, the overall trend with the rigid tire is observed with the pneumatic tire, including CPU runtime, as shown in Table 4.4.

Table 4.4: Rolling resistance coefficient and CPU runtime for the pneumatic tire

<table>
<thead>
<tr>
<th>Soil Plot Type</th>
<th>Rolling Resistance Coefficient</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Uniform 25 mm mesh</td>
<td>0.43</td>
<td>5.126E-5</td>
</tr>
<tr>
<td>SPH - Original</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 1.1</td>
<td>0.50</td>
<td>4.167E-6</td>
</tr>
<tr>
<td>FEA/SPH - 1/8 Depth</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 2.15</td>
<td>0.44</td>
<td>7.479E-5</td>
</tr>
<tr>
<td>FEA/SPH - 1/8 Depth, 8x Density</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 3</td>
<td>0.46</td>
<td>3.440E-6</td>
</tr>
</tbody>
</table>
CHAPTER 5

COMPARISON AND VALIDATION

After obtaining results from the tire-soil interaction virtual tests some comparison and validation work needed to be conducted in order to draw conclusions. In this chapter, the pneumatic tire is further examined, follow-up simulations are performed to confirm the difference in results between the FEA and SPH soil models, and finally some simpler soil virtual tests are conducted on the FEA and SPH soil models.

5.1 PNEUMATIC TIRE PRESSURE

In order to validate the results of the pneumatic tire, rolling resistance simulations were performed on the 25mm uniform free-edge FEA soil at four different inflation pressures: original pressure of 85 psi, half pressure of 42.5 psi, quarter pressure of 21.25 psi, and double pressure of 170 psi. The results are presented in Figure 5.1 while a side-view of the tire running at the four different pressures is shown in Figure 5.2. In order to emphasize the deflection of the pneumatic tire, a red circle has been drawn around the tire in each image.

![Figure 5.1: Pneumatic tire rolling resistance coefficient at various inflation pressures](image-url)
The rolling resistance coefficient results from these tests shows that the pneumatic tire follows the expected trend: lower rolling resistance at lower inflation pressures and higher rolling resistance at higher inflation pressures. (Note: this is the opposite of what is expected for a tire rolling on a rigid surface.)
5.2 RIGID TIRE ROLLING RESISTANCE FOLLOW-UP TESTS

Due to the results from the rigid tire rolling resistance virtual tests, some follow-up simulations were performed. First, the friction coefficient for the contact between the tire and the soil for the FEA soil model was adjusted to see if an increased rolling resistance coefficient, as was shown in some of the SPH tests, could be obtained with the FEA model. Then, the length of the combined FEA/SPH model which settled significantly faster than the FEA only model (FEA/SPH, original density and 1/8 depth, RATIO 2.1, ALPHA 2.15) was shortened in order to observe the effect on CPU runtime.

5.2.1 Friction Coefficient Tests

For the majority of the SPH and FEA/SPH soil simulations, the rolling resistance coefficient was higher than that of the FEA soil. Assuming this higher rolling resistance coefficient is correct, the FEA soil tests still run faster than tests containing SPH particles. Therefore, it would be beneficial to be able to replicate the higher rolling resistance coefficient with the FEA soil.

The rolling resistance coefficient is calculated by dividing the force acting against the tire in its direction of travel by the force from the vertical load. The vertical load is a constant, so in an attempt to increase the rolling resistance coefficient, the friction coefficient for the contact between the tire and the soil was increased. Values for the friction coefficient where varied from the original 0.8 up to 1.6. This range was used for the friction coefficient because a value of 0.6 introduced contact issues and a value of 1.8 caused extra instabilities. The vertical displacements for these test are presented in Figure 5.3 while the rolling resistance coefficients are shown in Figure 5.4.
Figure 5.3: Vertical displacement for the increased friction coefficient tests

Figure 5.4: Rolling resistance coefficient for the increased friction coefficient tests
5.2.2 Shorter Soil Test

During the rolling resistance tests, it was observed that the original density and 1/8 depth combined FEA/SPH soil with SPH part parameters of RATIO 2.1 and ALPHA 2.15 reached steady state significantly faster than the FEA only model. Therefore, as shown in Figure 5.5, the tire can still easily reach steady state on 75% the length of soil. Also, due to the shorter soil length, a simulation time of 0.8 seconds was used instead of the original full one second. The combination of both the smaller amount of soil being modeled and the shorter simulation time results in a shorter CPU runtime than the full length FEA model.

Figure 5.5: Cutaways of (top) the original length FEA model and (bottom) the shortened FEA/SPH model with the trajectories of the center points shown in red
5.2.3 Follow-Up Results

The friction coefficient virtual tests show that increasing the friction between the tire and the soil only marginally increases the rolling resistance coefficient of the tire. However, as shown in Figure 5.3, increasing the friction coefficient causes the tire to settle faster. Therefore, increasing the friction between the tire and the soil may allow for a shorter soil plot to be used while maintaining the same results.

Along those lines, the shortened FEA/SPH model effectively obtained results similar to that of the normal length FEA model, but was able to do so in a shorter amount of time. That shows that, under the right circumstances, SPH particles can be used more efficiently than FEA elements. The rolling resistance coefficient and CPU runtime results from these follow-up simulations are presented in Table 5.1.

Table 5.1: Rolling resistance coefficient and CPU runtime for follow-up tests

<table>
<thead>
<tr>
<th>Soil Plot Type</th>
<th>Rolling Resistance Coefficient</th>
<th>CPU Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA - Uniform 25 mm Mesh</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Friction - 0.8 (Original)</td>
<td>0.48</td>
<td>1.918E+4</td>
</tr>
<tr>
<td>Friction - 1.0</td>
<td>0.48</td>
<td>1.524E+4</td>
</tr>
<tr>
<td>Friction - 1.2</td>
<td>0.48</td>
<td>1.660E+4</td>
</tr>
<tr>
<td>Friction - 1.4</td>
<td>0.49</td>
<td>1.663E+4</td>
</tr>
<tr>
<td>Friction - 1.6</td>
<td>0.49</td>
<td>1.877E+4</td>
</tr>
<tr>
<td>FEA/SPH - Shortened</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RATIO 2.1 - ALPHA 2.15</td>
<td>0.50</td>
<td>1.806E+4</td>
</tr>
</tbody>
</table>
5.3 SIMPLIFIED SOIL TESTS

Due to the large differences between the FEA and SPH soil models observed in the rolling resistance models, a further understanding of the SPH soil was desired. For this reason, simplified pressure-sinkage and shear-displacement virtual tests were chosen for comparing the FEA and SPH models.

5.3.1 Pressure-Sinkage Tests

The pressure-sinkage simulation consists of applying a known pressure to the soil—usually via a plate, as shown in Figure 5.6—and measuring the amount of sinkage (or vertical displacement) of the plate.

![Figure 5.6: Pressure plate (a) before and (b) after the application of the pressure force](image)

For the pressure-sinkage tests, a 300 mm diameter plate is loaded on four main soil model geometries: the original FEA soil with non-uniform elements; the original FEA soil with the top surface area increased by a factor of four and the depth increased by a factor of two; the FEA soil with uniform elements; and the SPH soil. The normal-sized soil plots have the dimensions of 800 mm in width and length and 600 mm in depth. These four soil models are presented in Figure 5.7. Each of the non-uniformly meshed FEA models were run with fixed edges and the two smaller FEA models were run with free edges for a total of four FEA
models. Three SPH models were run including one with higher values for the smoothing length ratio (RATIO) and linear artificial viscosity (ALPHA) and one using approximately double the density of SPH particles. Using these seven soil models, pressures from 5 to 35 kPa were applied to the plate in 5 kPa increments. The results from these simulations are shown in Figure 5.8.

![Figure 5.7: Four soil model geometries used for the pressure sinkage tests](image)

(a) FEA: non-uniform  
(b) FEA: non-uniform, 4x surface area, 2x depth  
(c) FEA: uniform  
(d) SPH

Figure 5.7: Four soil model geometries used for the pressure sinkage tests
As shown by Figure 5.8, when the part parameters for the SPH soil (i.e. Ratio and Alpha) are left constant, there is much more sinkage on the SPH soil than on the FEA soil at an equal pressure. By comparing these results to data from J.Y. Wong’s *Theory of Ground Vehicles, 4th Edition* (shown in Figure 5.9 and Table 5.2), it would appear that the FEA model is merely representing a different type of soil than the SPH model.

**Table 5.2: Terrain values: $n$**
(Wong, 2008)

<table>
<thead>
<tr>
<th>Terrain</th>
<th>$n$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>0.1 – 0.2</td>
</tr>
<tr>
<td>Clayey soil</td>
<td>0.5 – 0.7</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.2 – 1.1</td>
</tr>
<tr>
<td>Sand</td>
<td>0.8 – 1.1</td>
</tr>
<tr>
<td>Snow</td>
<td>1.4 – 1.6</td>
</tr>
</tbody>
</table>
However, as shown in Figure 5.10, looking at cross-sections of the plate loaded on FEA soil showing plastic strain and SPH soil showing displacement magnitudes of the particles tells a different story. For the FEA model, the pressure plate is supported largely due to tensile forces created by the stretching of the surface elements. Conversely, for the SPH model, the pressure plate is supported by the compressive forces created by compacting the particles directly underneath the plate.

![Figure 5.10: Cross-sections of the pressure plate loaded on (a) FEA soil showing plastic strain and (b) SPH soil showing displacement magnitudes of the particles](image)

### 5.3.2 Shear-Displacement Tests

For the shear-displacement virtual tests, initially both a rectangular shear plate and a shear ring, as shown in Figure 5.11, were used. However, due to the plowing caused by the shear plate (as shown in Figure 5.12) and the inherent difficulties of finding the soil shear strength with the shear ring, a shear box was used instead. For the shear box, a volume of soil is placed into a box which is split into top and bottom halves. The soil is then loaded vertically with a known pressure and the two halves of the box are slid with respect to each other. The force necessary to slide the box is measured and, from this, a shear force in the soil can be calculated.
The shear box created for these tests has a cross sectional area of 0.15 m$^2$ and is moved a distance of 200 mm over the course of 1 second after vertically loading the soil at 5, 10, 20, 40, and 60 kPa. The shear box at the beginning and end of the simulation is shown in Figure 5.13 with Figure 5.14 and Figure 5.15 showing how the SPH soil deforms inside the shear box. Due to penetration issues—shown in Figure 5.16—shear results could not be obtained with the FEA soil.
Figure 5.13: Shear box (a) before and (b) after displacement

Figure 5.14: Cutaway of the shear box showing the SPH soil (a) before and (b) after displacement with deformation

Figure 5.15: Side view of the shear box showing the SPH soil (a) before and (b) after displacement with deformation

Figure 5.16: Side view of shear box showing the FEA soil (a) before and (b) after displacement with deformation and penetration
The three SPH soils from the pressure-sinkage tests in Section 5.3.1 were used for the shear-displacement tests. The peak of the sliding force curve—shown for the 5 kPa vertical load case for the three different soils in Figure 5.17—is divided by the cross-sectional area of the box to find the maximum shear force (known as the shear strength) of the soil. The shear strength is then plotted against the vertical load for all cases and soils as shown in Figure 5.18 and linear trendlines are added. From these trendlines, two terrain values can be found: \( c \) from the y-intercept and \( \phi \) from the slope. The values for \( c \) and \( \phi \) for the SPH soils from the shear-displacement tests are presented in Table 5.3 while data from J.Y. Wong’s *Theory of Ground Vehicles, 4th Edition* is presented in Table 5.4. By comparing these data, it appears that the SPH soil approximates the shear characteristics of a clay with a high moisture content for the \( c \) value and the characteristics of a sand or a clay with low moisture content for the \( \phi \) value.
Table 5.3: Shear-displacement SPH terrain values

<table>
<thead>
<tr>
<th>SPH Soil Parameters</th>
<th>$c$ (kPa)</th>
<th>$\Phi$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio: 1.6 – Alpha: 2</td>
<td>21</td>
<td>28</td>
</tr>
<tr>
<td>Ratio: 1.6 – Alpha: 2 – 2x Density</td>
<td>22</td>
<td>27</td>
</tr>
<tr>
<td>Ratio: 2.1 – Alpha: 2.4</td>
<td>23</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 5.4: Terrain values: $c$ and $\phi$ (Wong, 2008)

<table>
<thead>
<tr>
<th>Terrain</th>
<th>Moisture content (%)</th>
<th>$c$ (kPa)</th>
<th>$\Phi$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>0</td>
<td>1.04</td>
<td>28</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>23</td>
<td>9.65</td>
<td>35</td>
</tr>
<tr>
<td>Lean clay</td>
<td>32</td>
<td>13.79</td>
<td>11</td>
</tr>
<tr>
<td>Heavy clay</td>
<td>40</td>
<td>20.69</td>
<td>6</td>
</tr>
<tr>
<td>-</td>
<td>22</td>
<td>68.95</td>
<td>20</td>
</tr>
<tr>
<td>-</td>
<td>25</td>
<td>68.95</td>
<td>34</td>
</tr>
</tbody>
</table>
CHAPTER 6
CONCLUSIONS AND FUTURE WORK

6.1 CONCLUSIONS

A rigid tire model was used to perform an extensive sensitivity study on the FEA soft soil to determine the importance of mesh size and soil plot size for vertical loading. It was found that the original soil model had significant edge effects at the surface of the soil. These edge effects can be relieved by either doubling the width of the soil or simply allowing the edges of the soil to be free to move in the Z-direction resulting in an increased settling depth of the tire. Furthermore, doubling the depth of the soil did not greatly affect the settling depth of the tire. This rigid tire was then used to show that SPH soil could be used to partially or fully replace the FEA soil model for vertical displacement simulations.

Using a narrower rigid tire model, the rolling resistance coefficient on the FEA soil was compared to the coefficient calculated for various SPH and FEA/SPH soil models. Even though all of the models containing SPH particles were tailored to have a steady-state vertical deflection similar to that of the FEA model, the SPH models almost always resulted in a larger rolling resistance coefficient and never resulted in a lower coefficient. Also, it was shown that SPH part parameters and SPH particle density did not greatly affect the rolling resistance and that SPH particle depth had the greatest effect. These trends were then confirmed by using a pneumatic tire model.

Finally, through simplified soil virtual tests, the FEA soil was further compared to the SPH soil. Pressure-sinkage simulations showed that the SPH soil models a different type of soil than the FEA soil and also helped to highlight some of the inadequacies of using an FEA soil for large deformations. The SPH soil was further analyzed through the use of a shear box simulation. These shear virtual tests showed that the SPH soil behaves more like a clay in initial shearing and more like a sand by exhibiting increased shearing due to vertical loading. Furthermore, both the pressure-sinkage and shear-displacement tests showed a larger particle density to be unnecessary.
6.2 RECOMMENDATIONS FOR FUTURE WORK

The SPH soil model used in this thesis was largely a “proof-of-concept” model. It was shown that SPH can be used for a material strength model, but the SPH soil still needs to be further refined.

The SPH soil model parameters were defined mainly through comparison to the FEA model which has been shown to have many limitations. Instead, the SPH soil model should be parameterized through simple virtual testing such as the pressure-sinkage test and the shear-displacement test in order to find the SPH parameters which yield the desired results. Furthermore, different material types for the SPH should be examined as the hydrodynamic elastic-plastic model (PAM-SHOCK material type 7) used created a highly viscous material as shown by the shear box virtual testing. The Murnaghan Equation of State solid (PAM-SHOCK material type 28) yields good qualitative shear results as shown in Figure 6.1, but the correct parameters for this material type are unknown and current models are unable to support any load for pressure-sinkage tests.

Once the optimal material type and material parameters are determined, the rolling resistance simulations should be repeated along with other in-plane and out-of-plane virtual testing such as steering and slip tests. These results can be compared to the FEA soil for reference but should not be compared for validation purposes.

Ideally the soil being modeled will be a specific soil which can be physically tested. In that case, the soil should be tested both for validation of parameters from the pressure-sinkage and shear-displacement tests and also for validation of the physics of the tire-soil interactions.
REFERENCES


