FORCE- AND POWER VELOCITY RELATIONSHIPS
IN A MULTI JOINT MOVEMENT

A Thesis in
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by
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Abstract

Force-velocity characteristics in multi-joint movements, specifically the vertical jump, have been relatively unexplored in the literature. There were five main goals in this study: 1) to accurately define the force-velocity relationships for a multi-joint movement and compare them to Hill’s classic force-velocity curve, 2) to compare several options for presenting force-velocity curves in a multi-joint movement, 3) to accurately define the power-load and power-velocity relationships in a multi-joint movement for the ranges of force and velocity that were obtainable, 4) since the entire theoretical power-velocity curve was not obtainable because of physical limitations, to determine whether the data in this experiment lies in the ascending or descending portion of the theoretical power-velocity curve, and 5) to determine the load and velocity at which maximum power was produced.

Ten well-trained subjects were asked to perform maximum effort, noncountermovement vertical jumps with a range (80% of bodyweight unloading to 125% of bodyweight additional loading) of external loads applied. Each subject performed 28-34 trials with two trials at each condition. The instant of the maximum levels of the various velocity measures was used as the time to measure all of the other variables and the trial with the highest maximum center of mass velocity was selected for analysis.

Relationships were identified as ‘Hill-like’ if they were descending, had upward concavity, and $0 < \alpha/F_o < 1$. All of the variables studied in this investigation (maximum velocity of the center of mass, maximum knee angular velocity, maximum leg extension velocity, ground reaction force, and knee moment) were plotted against load. None of these relationships compared well with Hill’s curve. Various logical combinations of these variables were compared to each other and to Hill’s curve. Only ground reaction force vs. maximum center of mass velocity (video), vs. maximum knee angular velocity, and vs. maximum leg extension velocity were Hill-like. The best fit was the ground reaction force vs. maximum center of mass velocity (video) relationship, mathematically fit with Hill’s curve.
Power, calculated by multiplying ground reaction force and maximum center of mass velocity, varied as expected with maximum center of mass velocity and was on the descending part of the theoretical power-velocity curve. Maximum power corresponded to approximately 37-61% of the maximum squat lift of the subjects and 56% of maximum velocity. This was higher than predicted from theoretical models, but was similar to weightlifting studies.

This study successfully determined the force-velocity and power-velocity relationships for a multi-joint movement. Theoretical reasons why there was limited agreement with Hill’s curve were discussed. No other study known by the author covers as wide a range of forces and velocities in vertical jumping.
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-Andrew Hardyk, October, 2000-
“It is odd how one’s brain fails to work properly when pet theories are involved.”

- A.V. Hill, 1970 -
Chapter 1   Introduction
Everyday experience shows that as heavy objects are lifted, the speed of lifting is slower than when lighter objects are lifted. Everyone intuitively knows about the relationship between force and velocity. Force-velocity and power-velocity relationships of skeletal muscle are popular topics in the fields of Biomechanics and Physiology. Research began in earnest in the late 1920’s with studies completed by Hill (1922), Gasser and Hill (1924), Levin and Wyman (1927), Fenn and Marsh (1935), Hill (1938), and Katz (1939) and has continued into today’s current scientific literature. Study in this field can be divided into four main areas: 1) classic studies, 2) single fiber/muscle preparations in vitro, 3) single-joint studies in vivo, and 4) multi-joint studies. A. V. Hill is perhaps the most famous researcher in this field. He and his team were among the first to clearly define the force-velocity relationship in muscle. Using muscle removed from amphibians, Hill determined that as the resistance against the muscle increased, the contraction velocity decreased in a predictable, non-linear, concave upward fashion. The now famous ‘Hill’s force-velocity equation’ was written to describe the form of the displaced rectangular parabolic curve:

\[(V+b)(F+a) = (F_o + a)b = \text{constant}, \tag{1}\]

where \(F\) is the external force on the muscle, \(V\) is the velocity of shortening, \(F_o\) is the isometric tension, and \(a\) and \(b\) are constants. Nearly all scientists who have investigated the force-velocity characteristics of muscle compared their results to this classic curve.

Researchers studying single fibers and muscles typically removed them from the body of the animal and tested them using sophisticated equipment, which measured the force produced by the muscle and the shortening velocity throughout a single muscular action while keeping the velocity or the force constant. The muscles were maximally stimulated either chemically or electrically, and the muscles were tested over a number of trials against different loads. Research in this area has been diverse, and the effects of many factors on the shape of the force-velocity curve have been studied. Topics have included: fiber type (Baratta, et al., 1995; Bottinelli, et al., 1991), type of stimulation (DeHaan, 1998; Heckman, et al., 1992), fatigue (Ameredes, et al., 1992; Curtin and Edman, 1994; Lannergren and Westerblad, 1989), and temperature (Assmussen, et al.,
Animal muscles were the primary models in this area of investigation. Nearly all researchers in this area have compared their results to the Hill force-velocity curve and, with a few exceptions, reported similar force-velocity curves.

Single-joint investigations have allowed for the study of human muscles and involved one joint while the rest of the body was kept stationary during a maximum effort. The force-velocity characteristics of the muscles or the torque-angular velocity characteristics at the joint of interest were studied by either keeping the velocity of the motion or the force against which the subject must move constant. The measurements were taken at the instant of a maximum force, torque, or velocity, or at a specific joint angle. There were two main joints investigated in the literature: the elbow (Dern, et al., 1947; Wilkie, 1950; Cavagna, et al., 1968; Van Leemputte, et al., 1987; Martin, et al., 1995), and the knee (Thorstensson, et al., 1976; Perrine and Edgerton, 1978; Johansson, et al., 1987; Marshall et al., 1990; Ikegawa, et al., 1995; Seger and Throstensson, 2000), but the wrist (Chow and Darling, 1999), the finger (Cook and McDonagh, 1996), the hip (Hawkins and Smeulders, 1999), and the ankle (Bobbert, et al., 1990) have recently been studied as well. In general, most of these studies claimed that the resulting force-velocity, or torque-angular velocity, curves compared favorably to Hill’s force-velocity curves.

Force-velocity characteristics in multi-joint movements have been relatively unexplored compared to the single fiber/muscle and single-joint relationships. Many authors have investigated the force-velocity relationship in cycling (e.g. Baron, et al., 1999; Seck et al., 1995; Hautier, et al., 1996). Other models for study have included: horizontal arm pulls (Grieve and van der Linden, 1986), rowing ergometer (Hartmann, et al., 1993), bench throws (Newton, et al., 1997), weighted baseball bats (Bahill and Karnavas, 1991), and weight lifting (Thomas, et al., 1996). The vertical jump has been studied by several scientists (Tsarouchas and Klissouras, 1981; Bosco and Komi, 1979; Viitasalo, 1985; Jaric, et al., 1986; Bobbert, et al., 1986). Changes in force and velocity were produced by requiring the subjects to jump with additional loads, or by attaching the subjects to a calibrated spring via a harness and pulley system to unload them.
(Tsarouchas and Klissouras, 1981). Once again, the results were favorably compared to previous work by Hill (1938) and other force-velocity investigators. However, there is a gap in the force-velocity literature for the vertical jump because several of the studies mentioned above had questionable methods or did not actually attempt to mathematically fit Hill’s curve to their data, and none of them had very wide ranges of force and velocity.

Power-velocity relationships in muscle have not been discussed in the literature to the same extent as force-velocity relationships. Several authors reported power-velocity curves in single-joint investigations (e.g. Johansson, et al., 1987; Perrine and Edgerton, 1978; Gregor, et al., 1979; Fugel-Meyer, et al., 1982) using isokinetic dynamometry. The main finding in these studies was that power increased with joint velocity, but the curve began to level off at high velocities. Power-velocity curves seem to be rare in the literature for multiple joint movements with the exception of studies involving cycle ergometry (e.g. Baron, et al., 1999; Seck et al., 1995; Hautier, et al., 1996), but authors often mention peak or average power (e.g. Cavagna, et al., 1971; Harman, et al., 1990; Bartosiewicz, et al., 1990). Power was often reported in the vertical jumping literature as a measure of athletic ability or as a predictor for vertical jumping ability (e.g. Aragon-Vargas and Gross, 1997; Dowling and Vamos, 1993; Thomas, et al., 1996) and in reference to individual muscle power (e.g. Voigt, et al., 1995; Jacobs, et al., 1996; Bobbert, et al., 1986; Pandy and Zajac, 1991) and joint power (e.g. Prilutsky and Zatsiorsky, 1994; Jacobs, et al., 1996; van Soest, et al., 1993; Gregoire, et al., 1984).

A similarity between all of the methods described above was that they were all parametric, that is, a single instant in time was chosen during each trial for study and all of the single points from each trial were combined to form one curve. A parameter of the motor task was changed from trial to trial (e.g. the external load). However, there are several significant differences between single fiber/muscle and single-/multi-joint force-velocity studies. The velocity of shortening in the first class of experiments is normally relatively constant for most of the shortening range. This means that it is not as important at which point in the contraction history the velocity or force is measured. As long as the velocity or force is not measured during the initial acceleration or the final deceleration during the movement, the measurement will be accurate. Single-joint testing
has the intrinsic complication of non-constant force and velocity profiles due to anatomical constraints. Multi-joint movement has the added complication that several muscles are involved with the movement in addition to the fact that there are multiple joints, all moving in an unknown coordination pattern. Therefore, the options when deciding at which point to measure or analyze force and velocity are virtually limitless and complicated to interpret.

The subjects in this study were asked to perform maximum-effort, noncountermovement vertical jumps against loads that varied from trial to trial. The subjects were loaded with a barbell and weights. The subjects were unloaded with stretched elastic bands. The force-velocity and power-velocity relationships were studied using force plate and video analyses and used the instant of maximum velocity as the time at which the parameters were measured.

The theoretical shape of the force-velocity curve in an investigation such as this is a curve that resembles the Hill curve, i.e. a descending, concave upward curve. As the resistance is increased, the velocity of movement should decrease. An important measurement in the study of Hill-like curves is the parameter $a/F_o$. Typically this parameter is between zero and one in the literature. For the purposes of this study, Hill-like curves were defined as follows:

1) Descending,
2) Concave upward,
3) Ratio of constants $a/F_o$ between zero and one.

All curves that did not fit into this definition were rejected as Hill-like.

The theoretical power-velocity curve should look like an ascending-descending parabolic shape with its peak somewhere in the middle of the velocity range. The entire theoretical force and velocity ranges (zero velocity to very high velocity and maximum load to zero load) was impossible to cover in this study because the amount of load that would be needed would become dangerous to lift, and the unloading force would pull the subject from the floor before a jump could be attempted. Therefore, the curves that were produced were only a portion of the entire theoretical curve.
The main goals of this investigation were to:

1) Determine the general shape of the various force-velocity and velocity-load curves for a noncountermovement vertical jump (see #2)

2) Compare several velocity and force combinations:
   a) Maximum velocity of the center of mass vs. the corresponding ground reaction force and external load.
   b) Maximum angular velocity of the knee joint vs. the corresponding ground reaction force, corresponding knee moment and external load.
   c) Maximum linear velocity of leg extension as measured by the change in linear distance between the ankle and hip joints vs. the corresponding ground reaction force, corresponding knee moment, and external load.

3) Determine the general shape of the various power-velocity and power-load curves:
   a) Power, calculated by multiplying maximum center of mass velocity and the corresponding ground reaction force.
   b) Power, calculated by multiplying maximum leg extension velocity and corresponding ground reaction force.
   c) Power, calculated by multiplying maximum knee angular velocity and the corresponding knee moment.

4) Since the power-velocity/force/load curves in this study did not cover the complete theoretical range possible, determine whether this experiment lies in the ascending or descending portion of the theoretical curves.

5) Determine the load and velocity at which maximum power is produced (if possible, since maximum power may not be included in the range of data).
The hypotheses for the results of the tests that meet the above goals are:

1) The shape of all of the force-velocity curves will be similar in shape to the Hill model for muscle force-velocity characteristics, i.e. they will be Hill-like as strictly defined above.

2) All of the combinations for force-velocity relationships will be comparable, i.e. they will have similar correlation coefficients.

3) The shape of all the power-velocity curves will be similar to the theoretical power-velocity curve; i.e. it will be a parabolic ascending-descending curve peaking between the maximum and minimum velocity.

4) The power-force and -load curves will be in the ascending portion of the theoretical curve and the power-velocity curves will be in the descending portion.

5) The peak power will occur at approximately 33% of the maximum load and velocity (as determined by the self-reported maximum squat lift of the subjects and the maximum velocity predicted by the data).
Chapter 2  Review of Literature
The force-velocity relationship in muscle has been a topic of intense research for a number of years. This review covers the main types of force-velocity studies. It is intended to be comprehensive of the field, but no claim is made that all force-velocity studies are included. The main focus of the review is parametric studies in the concentric range of force production.

2.1 Force-Velocity Relationships

There are two main types of force-velocity studies in the literature. The first is single-trial studies and the second is multi-trial studies. The main focus of the review is on the multi-trial studies since this project is in that area. However, it is important to understand the difference between the two to avoid confusion.

2.1.1 Single-Trial Force-Velocity Experiments

The idea behind this type of research is simple; the researcher measures the force and the velocity concurrently and continuously during a trial. This can be accomplished using a number of experimental models such as a single muscle fiber or whole muscle preparation in vitro, or during a controlled movement using some sort of an animal model.

There is a relatively small amount of research that uses this approach for the specific purpose of measuring force-velocity relationships. van Ingen Schenau, et al. (1985) used video and force plate data with an inverse dynamics approach to study the torque-angular velocity characteristics of the ankle joint during a vertical jump. Komi, et al. (1990) used a buckle-type force transducer temporarily implanted directly into the achilles tendon of human subjects to measure force and used video data and muscle modeling to calculate muscle shortening velocity during walking, running, and jumping. The results from both of these studies were similar and were manifested in a force-velocity relationship that was somewhat circular in shape. The results of these studies are typical of other single-trial studies.

The point of mentioning these types of studies is that the force-velocity relationship during a single trial is different than when multiple trials are presented in a parametric way and does not strictly follow the famous force-velocity curves that are
commonly found in the literature. One of the common explanations for the differences in these types of studies is that during *in vivo* testing, none of the variables (activation levels, force, velocity, etc.) are held constant, whereas in parametric force-velocity experiments, some part of the experiment is controlled. In other words the two experiment types are different and have different results. Another possible reason for the differences in the results is that *in vivo* testing includes all of the energy-storing connective tissue, but in parametric tests, this tissue is often removed (Komi, et al., 1990).

2.1.2 Multi-Trial Force-Velocity Experiments

The bulk of the force-velocity literature uses a multiple trial approach to study the force-velocity relationship in muscle. In general, this involves a procedure where the force and velocity are measured at a specific position or specific instant in the movement during a single trial. One of the parameters of the experiment, i.e. external load, velocity of shortening, etc., is then changed and the experiment is repeated. The single force-velocity point from each trial is then plotted on one graph to produce a ‘parametric’ relationship for force and velocity (Zatsiorsky, 1995). This type of experiment has been performed on a variety of muscle models including isolated animal and human muscle fibers, whole muscle preparations *in vitro* and *in situ* for both animals and humans, and human *in vivo* single-joint and multi-joint models. All of these models will be discussed in detail.

Parametric force-velocity experiments typically attempt to control (keep ‘constant’) either the force that the fiber/muscle/subject is required to move against or the velocity at which the fiber/muscle/subject will move during the trial. The other variable is then measured at some time during the movement. Holding the force constant has been attempted by subjecting the fiber/muscle/subject to a constant external load such as a hanging weight or a barbell. Another method uses special equipment, called an isotonic dynamometer, which has been designed to maintain constant resistance levels during a movement. Holding velocity constant is usually accomplished using an isokinetic dynamometer. This is a machine designed to absorb all of the energy produced in the
movement that would normally cause acceleration, thereby keeping the velocity at a constant level.

In reality, none of these techniques perfectly maintains constant force or velocity. In every experiment, the fiber/muscle/subject starts in a stationary position with no force applied and then accelerates to a ‘constant’ state. This situation can be troublesome because the acceleration period at low loads and/or high velocities can sometimes take up the entire time of the movement. To overcome this problem, many experiments use the so called ‘quick release’ method where the fiber/muscle/subject is allowed to build up maximum isometric force at the beginning position of the movement and is then suddenly released to the desired force or velocity level. Another potential problem is the inertia of the apparatus and/or the fiber/muscle/subject itself. When the movement is produced in a plane where gravity has an effect (e.g. seated leg extension), the actual forces/moments measured will include the intrinsic force/moment produced against the load (isotonic) or at the controlled velocity (isokinetic) and the force needed to lift and/or accelerate the mass of the limb/apparatus. This problem can be corrected for mathematically or experimentally. Correction for inertial properties is important when comparing the results for different protocols, i.e. comparing isolated muscle preparations in vitro to isotonic elbow flexions in vivo (Wilkie, 1950).

Since the force-velocity relationship is built up from multiple trials and only one moment in time from each trial is used, the researcher must choose a time in the movement to record the data. There are an infinite number of possibilities for this choice and the literature reflects this variability. Some protocols have the advantage of having a significant period of constant force and velocity during the movement. In these cases, the specific time that the variables are measured is not critical as long as the measurements occur during the constant periods. Most protocols do not have this luxury. Even if force or velocity is held constant as the independent variable, the dependent variable is often not constant over the range of movement, especially in experiments involving one or more joints in human subjects. Even if the external load is held constant, the researcher may be interested in the actual force and velocity produced, neither of which may be constant throughout the range of movement (e.g. multi-joint movement against a constant
external load). Therefore, in these cases, the dependent variable(s) must be measured either continuously, at a specific position in the movement, or at a specific moment in time (i.e. time when maximum velocity or force occurs). The choices of time made for measuring the force and velocity can have an effect on the interpretation of the results and may have ramifications on how comparable individual studies may be.

The muscle activation level in these experiments is typically assumed to be maximal. This means that in the in vitro fiber/muscle preparations the muscle is maximally stimulated either chemically or electrically and in the in vivo human trials maximum effort is produced. Recently there has been some effort to determine the force-velocity relationship of submaximal efforts (Chow and Darling, 1999; Hawkins and Smeulders, 1998, 1999).

Regardless of the protocol, the results of nearly all of these types of experiments have been very similar. As velocity of movement increased, the force that was produced in a maximal contraction decreased in a non-linear way. This has become known as the famous force-velocity relationship (often called Hill’s curve, after A. V. Hill, see below) and it seems to hold up under a wide variety of parametric protocols.

Researchers have also studied the force-velocity relationship in the eccentric range of the force-velocity curve, i.e. when the force is so high that the muscle is extended instead of shortened (Katz, 1939; Granzier, et al., 1989; Dudley, et al., 1990; Kues and Mayhue, 1996; Seger and Thorstensson, 2000; Jorgensen, 1976; Amidiris, et al., 1995). They found that Hill’s curve does not adequately describe the force-velocity relationship during forced extension. The force increases very quickly flattens to a constant value with increasing extension velocity. This area of research remains relatively undefined because the force-velocity relationship has not been explored to a great extent and because the properties for lengthening muscle are not as consistent or easy to measure compared to shortening muscle (Epstein and Herzog, 1998).

2.1.3 Types of Multi-Trial Force-Velocity Experiments

As mentioned above, there are four main models for parametric force-velocity experiments: single muscle fiber preparations in vitro, single muscle preparations in vitro/in situ, single-joint in vivo, and multi-joint in vivo. Each type will be discussed
individually. Before that, a brief history of parametric force-velocity relationships will be discussed in the form of a review of some of the classic early studies.

2.1.3.1 Classical Force-Velocity Studies

In 1922, A. V. Hill used a flywheel, a series of pulleys, and a hand held tachometer to control and record the velocity when calculating the work done by the elbow flexors during a simple, single-joint movement at the elbow. Maximum force pulls on a string wrapped around wheels of progressively smaller radii for each trial connected to the flywheel produced a range of inertial resistances and thus, a range of average angular velocities of movement of the forearm. The results indicated that as the inertia increased, the velocity of the movement increased, and the work done by the muscles decreased somewhat linearly. Since the movement was carried out over the same distance in each trial, the same relationship follows for the force, i.e. as velocity of movement increased, the force decreased. This relationship was attributed to viscous properties of the muscle and was modeled as a spring in a viscous damping fluid.

Gasser & Hill (1924) attempted to confirm the linear work (force)-velocity relationship using isolated frog sartorious muscles. They used a quick-release method with a tension lever to calculate the average velocity and work done by the muscles against various resistances. They found that the work again decreased as the contraction velocity increased, but the resulting work-velocity curve was not linear but hyperbola-like. The viscous spring model did not predict this result.

Levin and Wyman (1927) continued this line of research by developing an isokinetic measuring device that measured the tension on dogfish muscles during constant velocity contractions. They found similar, non-linear work-velocity curves as Gasser and Hill (1924) and fit the curve using an exponential model based on a viscous spring in series with another spring. This model explained the data well. Fenn and Marsh (1935) corroborated this work with experiments using frog and cat muscles and an isotonic lever system and for the first time produced the familiar force-velocity curve.

Using frog and toad sartorius muscles, Hill (1938) used the measurement of heat liberated during muscular contractions to predict the force-velocity characteristics of muscle. He used an isotonic lever system and maximally electrically stimulated the
muscles. Thermodynamic sensors were used to measure the amount of energy expended by the muscle in the form of excess heat during the contraction. He found that the heat energy produced by the muscle (in excess of maintenance heat) was proportional to the velocity:

\[ \Delta E = (a + F)V, \]  

(1)

where \( V \) is the shortening velocity, \( F \) is the suspended weight (i.e. force produced by the muscle), and \( a \) is a constant. He also found that the liberated thermal energy was proportional to the difference between the maximum isometric force produced by the muscle and the muscle force produced during a specific trial:

\[ \Delta E = b(F_o - F), \]  

(2)

where \( F \) is the shortening velocity, \( F_o \) is the maximum isometric force, and \( b \) is a constant. Combining these equations led to a general equation describing the force-velocity relationship of the muscle:

\[ (F+a)V = b(F_o - F), \]  

(3)

This equation can be rewritten as:

\[ (V+b)(F+a) = (F_o+a)b = constant. \]  

(4)

These equations determine the relationship between the muscular force and speed of the contraction and are in the form of a displaced rectangular hyperbola. Another useful form of the curve is:

\[ (V/b+1)(F/F_o+a/F_o) = (1+a/F_o) = constant, \]  

(5)

where \( a/F_o = V/b \) are parameters that describe the amount of curvature of the hyperbola.

The force-velocity curve and the values for the constants were confirmed with purely mechanical methods with no heat measurements by measuring the force and velocity directly using the same isotonic lever (Hill, 1938; Katz, 1939). The hyperbolic model also successfully explained the data of Fenn and Marsh (1935), Levin and Wyman (1927), and Gasser and Hill (1924).

The results of these studies were the foundation for the famous classic force-velocity curve for individual muscle, often called Hill’s curve as mentioned above (Figure 2.1). Hill’s force-velocity equation, which describes the shape of the force-velocity curve, is a phenomenological model that has limited correlation to actual muscle
structures (Hill, 1970). Possibly because of that fact, various forms of Hill’s curve have been applied to a variety of force-velocity and torque-angular velocity experimental results from single fiber experiments to multi-joint activities. Hill’s curve has stood up to a huge amount of scrutiny and is generally accepted as one of the most robust descriptors of muscle mechanics. Nearly every author who has written about muscle mechanics has compared their results to Hill’s basic equations for force and velocity relationships of muscle or has used his model in some form or another to predict muscle properties. The studies in this section are summarized in Table 2.1.

2.1.3.2 Isolated Fiber/Single Muscle Force-Velocity Studies

There have been a large number of force-velocity experiments that use a single muscle fiber, a small group of muscle fibers, or an entire muscle from animals or humans. Most of these experiments used a device in which the muscle/fiber is clamped on both ends, the muscle was maximally stimulated either chemically or electrically, and the device controlled either force or velocity at a desired level as the independent variable which was changed from trial to trial. The dependent variable, velocity or force, was then measured either during a constant period, at a specific time during the movement, or at specific fiber/muscle length. The quick release method (described above) was nearly always used in these experiments.

Table 2.2 shows some examples of recent studies and their specific techniques for the single-fiber model. Table 2.3 shows examples of the single-muscle model. Nearly all of the studies found that Hill’s force-velocity equations adequately explained the force-velocity relationship. Exceptions include Granzier, et al. (1989) who used a linear fit for their curve, and Lou and Sun (1993) and Edman, et al. (1988) who found that a bi-phasic hyperbolic curve fit their data best because the Hill curve overestimated maximum velocity and isometric force.
**Table 2.1. Classic Force-Velocity Studies**

<table>
<thead>
<tr>
<th>Author</th>
<th>Animal</th>
<th>Experiment Type</th>
<th>QR</th>
<th>V measured?</th>
<th>F Measured?</th>
<th>P calculated?</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dern et al ‘47</td>
<td>Human</td>
<td>Elbow flexors, isotonic lever</td>
<td>N</td>
<td>@ 90 deg</td>
<td>Const T</td>
<td>No</td>
<td>Hyperbolic equation fits T-ang V data</td>
</tr>
<tr>
<td>Fenn &amp; Marsh ‘35</td>
<td>Frog, cat</td>
<td>Single muscle, isotonic lever</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td>F-V non-linear, exponential curve fit</td>
</tr>
<tr>
<td>Gasser &amp; Hill ‘24</td>
<td>Frog/sartorius</td>
<td>Single muscle, tension lever</td>
<td>Y</td>
<td>Average</td>
<td>Work calculated</td>
<td>No</td>
<td>Non-linear work (force) – velocity relationship</td>
</tr>
<tr>
<td>Hill ‘22</td>
<td>Human</td>
<td>Elbow flexors, inertia wheel w/ different gear ratio</td>
<td>N</td>
<td>Const V</td>
<td>Work calculated</td>
<td>No</td>
<td>Linear work (force) – velocity relationship</td>
</tr>
<tr>
<td>Hill ‘38</td>
<td>Frog/sartorius</td>
<td>Single muscle, isotonic lever, thermodynamics</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td>Classic hyperbolic F-V curve, constants derived using thermo, confirmed w/ F-V measurements</td>
</tr>
<tr>
<td>Hill ‘39</td>
<td>Human</td>
<td>Elbow flexors, ’22 inertia wheel data</td>
<td>N</td>
<td>Const V</td>
<td>Work calculated</td>
<td>No</td>
<td>Hyperbolic model fits data fairly well</td>
</tr>
<tr>
<td>Katz ‘39</td>
<td>Frog, tortoise</td>
<td>Single muscle, isotonic lever</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td>Used hyperbolic model to fit F-V curves</td>
</tr>
<tr>
<td>Levin &amp; Wyman ‘27</td>
<td>Dogfish, tortoise, crab</td>
<td>Single muscle, isokinetic</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Work calculated</td>
<td>No</td>
<td>Non-linear work (F) – V curve, exponential curve fit</td>
</tr>
<tr>
<td>Wilkie ‘50</td>
<td>Human</td>
<td>Elbow flexors, isotonic lever, inertia correction</td>
<td>N</td>
<td>@ 80 deg</td>
<td>Const F</td>
<td>No</td>
<td>Hyperbolic equation fits F-V curve measured at hand</td>
</tr>
</tbody>
</table>
Table 2.2. Single Fiber Force-Velocity Studies

<table>
<thead>
<tr>
<th>Author</th>
<th>Animal</th>
<th>Experiment Type</th>
<th>QR</th>
<th>V measured?</th>
<th>F measured?</th>
<th>P calculated?</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bangart et al ’97</td>
<td>Rat/soleus</td>
<td>Effect of non-use &amp; Ca++, chemical stim</td>
<td>Y</td>
<td>Max V</td>
<td>Isotonic</td>
<td>Max P</td>
<td>Parabolic Eq.</td>
</tr>
<tr>
<td>Bottinelli et al ’91</td>
<td>Rat/soleus</td>
<td>Chemical stim, fiber type</td>
<td>Y</td>
<td>@ 30ms</td>
<td>Isotonic</td>
<td>FxV</td>
<td></td>
</tr>
<tr>
<td>Bottinelli et al ’95</td>
<td>Rat/plantaris</td>
<td>Chemical stim, light vs heavy myosin chains</td>
<td>Y</td>
<td>@20ms</td>
<td>Isotonic</td>
<td>FxV</td>
<td></td>
</tr>
<tr>
<td>Bottinelli et al ’96</td>
<td>Human</td>
<td>Chemical stim, fiber type, temperature</td>
<td>Y</td>
<td>@20ms</td>
<td>Isotonic</td>
<td>FxV</td>
<td></td>
</tr>
<tr>
<td>Curtin &amp; Edman ’94</td>
<td>Frog/ant tib</td>
<td>Electrical stim, fatigue</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Granzier et al ’94</td>
<td>Frog</td>
<td>F-V &amp; length, conc &amp; eccent</td>
<td>Y</td>
<td>@ 3 diff L</td>
<td>Isotonic</td>
<td>FxV</td>
<td></td>
</tr>
<tr>
<td>Lannergren &amp; Westerblad ’89</td>
<td>Xenopus</td>
<td>Electrical stim, fatigue, effect of caffeine &amp; K+</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Lou &amp; Sun ’93</td>
<td>Frog/semi-</td>
<td>Chemical stim, high load interest</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Malmqvist &amp; Arner ’96</td>
<td>Guinea pig/taenia coli</td>
<td>Chemical stim, effect of Ca++ &amp; okadaic acid</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td>Increased pCa++ &amp; increased okadaic acid increased F-V</td>
</tr>
<tr>
<td>McDonald et al ’94</td>
<td>Rat/soleus</td>
<td>Chemical stim, effect of non-use</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>FxV</td>
<td>Increased time of non-use decreased F-V curve</td>
</tr>
<tr>
<td>Sobol &amp; Nasledov ’94</td>
<td>Lamprey</td>
<td>Electrical stim, temperature effects</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td>Increased temp decreased curvature &amp; increased max V</td>
</tr>
<tr>
<td>Widrick et al ‘98</td>
<td>Human/soleus</td>
<td>Chemical stim, effect of bed rest</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>From Hill’s F-V equation</td>
<td>Bed rest decreased F-V curve</td>
</tr>
<tr>
<td>Author</td>
<td>Animal</td>
<td>Experiment Type</td>
<td>QR</td>
<td>V measured?</td>
<td>F measured?</td>
<td>P calculated?</td>
<td>Result</td>
</tr>
<tr>
<td>------------------------</td>
<td>-------------------------</td>
<td>--------------------------------------</td>
<td>----</td>
<td>-------------</td>
<td>-------------</td>
<td>---------------</td>
<td>-------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Ameredes et al ‘92</td>
<td>Dog/gastroc</td>
<td>In situ, electrical stim, fatigue</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td>Fatigue decreased F-V curve</td>
</tr>
<tr>
<td>Askew &amp; Marsh ‘98</td>
<td>Mouse/soleus</td>
<td>Electrical stim, cyclical</td>
<td>Y</td>
<td>Max V</td>
<td>Isotonic</td>
<td>FxV</td>
<td>Inc cycle freq dec power</td>
</tr>
<tr>
<td>Assmussen et al ‘94</td>
<td>Rabbit/eye</td>
<td>In vitro, elect stim, temperature</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Extrapolated</td>
<td>No</td>
<td>Increase temp, increase F-V</td>
</tr>
<tr>
<td>Baratta et al ‘95</td>
<td>Cat/9 hind</td>
<td>Electrical stim, fiber type</td>
<td>N</td>
<td>Max V</td>
<td>Susp Load</td>
<td>No</td>
<td>Increase FT fiber composition, increase F-V</td>
</tr>
<tr>
<td>Baratta et al ‘96</td>
<td>Cat/9 muscles</td>
<td>Electrical stim, fiber type</td>
<td>N</td>
<td>Max V</td>
<td>Hanging</td>
<td>No</td>
<td>Increased ST decreases curvature in F-V</td>
</tr>
<tr>
<td>Beckers-Bleukx &amp;</td>
<td>Rat/soleus &amp; EDL</td>
<td>Electrical stim, fiber type</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Const F</td>
<td>Max P, equation</td>
<td>Increase FT fiber composition, increase F-V</td>
</tr>
<tr>
<td>Marechal ‘89</td>
<td>Rat/soleus</td>
<td>Electrical stim, compare to single fibers</td>
<td>Y</td>
<td>isokinetic</td>
<td>Const F</td>
<td>No</td>
<td>Differences explained by fiber heterogeneity in muscle</td>
</tr>
<tr>
<td>Colomo et al 2000</td>
<td>Frog</td>
<td>Electrical &amp; chemical stim</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Const F</td>
<td>No</td>
<td>Electrical &amp; Chemical same</td>
</tr>
<tr>
<td>de Haan ‘88</td>
<td>Rat/soleus, EDL, gastroc</td>
<td>In-situ, electrical nerve stim, twitch vs tetanus</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Const F</td>
<td>FxV</td>
<td>No difference twitch vs tetanus</td>
</tr>
<tr>
<td>de Haan ‘98</td>
<td>Rat/gastroc</td>
<td>In-situ, electrical nerve stim, varied stim freq</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Const F</td>
<td>FxV</td>
<td>Decreased freq decreased F-V curve</td>
</tr>
<tr>
<td>Edman et al ‘88</td>
<td>Frog/ant tib</td>
<td>Electrical stim</td>
<td>Y</td>
<td>@ specific L</td>
<td>Isotonic</td>
<td>No</td>
<td>Different from Hill @ high F, two hyperbolic function fit</td>
</tr>
<tr>
<td>Hatcher &amp; Luff ‘85</td>
<td>Kitten/ FDL, soleus</td>
<td>Developing kitten, fiber type</td>
<td>Y</td>
<td>Max V</td>
<td>Isotonic</td>
<td>No</td>
<td>@ 2 days FDL &amp; soleus different in max V</td>
</tr>
<tr>
<td>Heckman et al ‘92</td>
<td>Cat/gastroc</td>
<td>Stimulation frequency effect</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Const F</td>
<td>No</td>
<td>F-V curve less steep with increased stim freq</td>
</tr>
<tr>
<td>Marechal &amp; Beckers-Bleukx ‘93</td>
<td>Mouse, 2 types/ soleus</td>
<td>Electrical stim, fiber type</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Extrapolated</td>
<td>No</td>
<td>Fiber type can be predicted w/ muscle parameters</td>
</tr>
<tr>
<td>Ratanunga &amp; Thomas ‘90</td>
<td>Rat/soleus, EDL, peroneus longus</td>
<td>In situ, electrical nerve stim, muscle fiber type</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>No</td>
<td>Fiber type explains differences in F-V curves</td>
</tr>
<tr>
<td>Swoap et al ‘97</td>
<td>Rat/soleus, plantaris</td>
<td>Electrical stim, in situ, optimal V using normal F-V &amp; cyclical</td>
<td>Y</td>
<td>Const V</td>
<td>Isotonic</td>
<td>P=FxV, work loops</td>
<td>Optimal V was the same using both methods</td>
</tr>
</tbody>
</table>
Recent research has concentrated on determining the effects of certain variables on the shape of the force-velocity curve. An example is to correlate the muscle fiber type with the shape of the curve. Several authors (Bottinelli, et al., 1996; Bottinelli, et al., 1991; Beckers-Bleukx and Marechal, 1989; Marechal and Beckers-Bleukx, 1993; Ratanunga and Thomas, 1990) found that the curvature decreases (i.e. the curve becomes more linear) when fast twitch fibers are tested compared to slow twitch fibers. However, Baratta, et al. (1995) and Baratta, et al. (1996) found the opposite: increases in slow twitch fibers decreased the curvature of the force-velocity curve. This difference was attributed to the fact that the other experiments used a quick release method that builds up elastic energy in the muscle that leads to increases in velocity upon release, and the authors of the latter studies used hanging weights with no quick release.

Many researchers have addressed the effects of various factors on the force-velocity curve. Bottinelli, et al. (1996), Sobol and Nasledov (1994), and Assmussen et al. (1992) found that the curvature of the force-velocity curve decreases with increasing temperature. Curtis and Edman (1994) and Ameredes et al. (1992) found that increased fatigue increased the curvature, but Lannergren and Westerblad (1989) determined that caffeine aids the speed of recovery to normal. The effect of non-use of the muscles was studied by several authors (Bangart, et al., 1997; McDonald, et al., 1994; Widrick, et al., 1998) and they determined that non-use increased the curvature of the force-velocity relationship. Other studies focused on stimulation frequency (Askew and Marsh, 1998; de Haan, 1998; Heckman, et al., 1992) and found that curvature decreased with increasing frequency. Cavagna, et al. (1968) determined that a prestretch shifted the entire work-velocity curve upward. Colomo, et al. (2000) compared electrical and chemical stimulation protocols and found that maximal stimulation in both cases produced similar results. de Haan (1988a) determined that there is no difference in the force-velocity curve when the electrical stimulation is a single maximum twitch or in tetanus, but de Haan (1998b) and Heckman, et al. (1992) showed a decrease in the curvature and the slope respectively with decreases in stimulation frequency.

When single-muscle results were compared to single-fiber results, some differences were observed. Several authors have observed decreases in the high force
region of the force-velocity curve when compared to single fiber preparations (Edman, et al., 1988). This was attributed to the elastic properties of the connective tissue of the muscle. Another study (Claflin and Faulkner, 1989) indicated that differences in the predicted maximum velocity were due to different muscle fiber types in the whole muscle, compared to the single muscle fiber.

2.1.3.3 Single-Joint Force-Velocity Studies

This class of force-velocity experiments requires that one joint only is moving during the experiment. Usually torque at a joint is measured or computed along with angular velocity of the joint at a given time. The continuous measurement of torque and angular velocity throughout the range of joint angles is typical for the experiments, but the data for the analyses are sometimes collected at the time of maximum angular velocity or maximum torque. Some studies simply measure the torque and angular velocity at a predetermined joint angle. The elbow and the knee joints are the most commonly studied in human subjects (see Table 2.4), but other joints such as the finger (Cook & McDonagh, 1996), thumb (DeRuiter, et al., 1999), wrist (Chow and Darling, 1999), ankle (Bobbert et al., 1990; Fugel-Meyer, et al., 1982), and hip (Hawkins and Smeulders, 1999) have also been investigated.

There is an important difference between single-joint studies and the previously discussed isolated fiber/muscle studies. In the isolated fiber/muscle studies, the velocity and force are produced by and measured from the muscle directly. In single-joint studies, the torque and angular velocity of the apparatus are measured directly and the torque and angular velocity at the joint are assumed to be the same (Gulch, 1994). The movement at the joint is certainly caused by muscular movement, but in order to ascertain the properties of the muscles themselves in vivo, models of the joint configuration, insertion angles, muscle moment arms, etc., must be used. In other words, just because the dynamometer is isotonic/isokinetic does not mean that the movement of the muscle is isotonic/isokinetic. However, it may be predictable and a joint model that takes into account the moment arm and length of the muscle as the joint rotates may allow the computation of the mechanics of the muscle itself (Cabri, 1991). For example, Marshall,
et al. (1990) used x-rays of the knee in conjunction with isokinetic knee extensions to create a model to calculate muscle forces and velocities.

Despite the above differences, force-velocity and torque-angular velocity plots for single-joint movements tended to have a very similar shape as the single fiber/muscle force-velocity plots. Numerous authors have reported Hill-like force-velocity and torque-angular velocity curves in the literature. Exceptions include Kues and Mayhue (1996) and Fugel-Meyer, et al. (1982) who used a linear fit to describe their data, and Wakayama, et al. (1995) who used Fenn’s exponential equation to fit their data. In addition, the torque-angular velocity curve was found to deviate from the force-velocity curve predicted by Hill’s force-velocity equation and by isolated fiber/muscle data in the high-torque range of the curve. At the lower angular velocities, the torque was not as high as expected. This was postulated to be due to a central nervous system inhibition designed to protect the muscles at high forces (Perrine and Edgerton, 1978; Kojima, 1991).

The most common approach by far to studying the force-velocity relationship for the single-joint model has been isokinetic knee extension and flexion (Table 2.4). The subject was strapped into the dynamometer in a seated position and the moving part of the apparatus was strapped to the ankle. The center of rotation of the knee was aligned with the rotation point of the apparatus. The subject then performed a series of maximum effort full range of movement extensions or flexions at a predetermined set of angular velocities. The torque was measured either by measuring the force exerted on the dynamometer at the ankle and multiplying this by the length of the rotating arm of the apparatus (tibia length) or directly at the rotation point of the dynamometer with sensors. A similar approach was used for isokinetic studies of other joints.

Other researchers attempted to control the external force that the muscle had to overcome in some fashion. Hanging weights (Wilkie, 1950; Kojima, 1991), elastic resistance (Hawkins and Smeulders, 1998; 1999), a constant force spring (DeKoning, et al., 1985) and changing the moment of inertia of the dynamometer (Tihyani, et al., 1982) are all examples of this type protocol.
<table>
<thead>
<tr>
<th>Author</th>
<th>Animal</th>
<th>Experiment type</th>
<th>QR</th>
<th>V measured?</th>
<th>F measured?</th>
<th>P calculated?</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amidiris et al ’95</td>
<td>Human</td>
<td>Knee extension, concentric &amp; eccentric, trained vs sedentary</td>
<td>N</td>
<td>Isokinetic</td>
<td>@ 65 deg</td>
<td>No</td>
<td>T-angV higher in trained, greater antag activ in eccen in sed, no curve fit</td>
</tr>
<tr>
<td>Bobbert et al ’90</td>
<td>Human</td>
<td>Ankle plantar flexion, comparison of T-angV data and model calculations for muscles</td>
<td>N</td>
<td>Isokinetic</td>
<td>Max T</td>
<td>Max T x corr ang V</td>
<td>Model and reality did not agree</td>
</tr>
<tr>
<td>Cavagna et al ’68</td>
<td>Human</td>
<td>Elbow flexion, isokinetic dynamometer, prestretch</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Work calculated</td>
<td>No</td>
<td>Prestretch increased W-V curve</td>
</tr>
<tr>
<td>Chow &amp; Darling ’99</td>
<td>Human</td>
<td>Wrist flexor, max vs submax effort</td>
<td>Y</td>
<td>@ 5 deg from release</td>
<td>@ 5 deg from release</td>
<td>No</td>
<td>Decreased activation – decreased F-V</td>
</tr>
<tr>
<td>Cook &amp; McDonagh ’96</td>
<td>Human</td>
<td>Finger abduction, electrical stim, vary stim freq &amp; level</td>
<td>Y</td>
<td>Isokinetic</td>
<td>Const</td>
<td>No</td>
<td>Hill curves – changes with activation and frequency</td>
</tr>
<tr>
<td>De Koning et al ’85</td>
<td>Human</td>
<td>Elbow flexion, trained vs untrained, const F spring</td>
<td>N</td>
<td>Ang V @ 110 deg</td>
<td>T @ 110 deg</td>
<td>No</td>
<td>Training increased F-V curve</td>
</tr>
<tr>
<td>DeRuiter et al ’99</td>
<td>Human/thumb</td>
<td>Electrical stim, fatigue</td>
<td>Y</td>
<td>Isokinetic</td>
<td>@ 51 deg</td>
<td>FxV</td>
<td>Fatigue reduced F-V curve</td>
</tr>
<tr>
<td>Dowling et al ’95</td>
<td>Human</td>
<td>Elbow flexion, isokinetic fatigue</td>
<td>N</td>
<td>Isokinetic</td>
<td>Every 2 deg</td>
<td>No</td>
<td>Hill-like surfaces</td>
</tr>
<tr>
<td>Dudley &amp; Djamil ’85</td>
<td>Human</td>
<td>Knee extension, strength vs endurance training</td>
<td>N</td>
<td>Isokinetic</td>
<td>T @ 0.52 rad bel horiz</td>
<td>No</td>
<td>Strength trained athletes increased @ training speeds</td>
</tr>
<tr>
<td>Dudley et al ’90</td>
<td>Human</td>
<td>Knee extension, max voluntary vs electrical stim, conc &amp; eccen</td>
<td>N</td>
<td>Isokinetic</td>
<td>T @ 45 deg below horiz</td>
<td>No</td>
<td>Stim conc F-V lower than max effort, opposite for eccen</td>
</tr>
<tr>
<td>Froese &amp; Houston ’85</td>
<td>Human/vastus lat</td>
<td>Knee extension, %FT fiber, FT fiber area</td>
<td>N</td>
<td>Isokinetic</td>
<td>T @ 30 deg below horiz</td>
<td>No</td>
<td>%FT fiber &amp; area corr w/ T @ 30 deg, but not w/ max T</td>
</tr>
<tr>
<td>Fugel-Meyer et al ’82</td>
<td>Human</td>
<td>Ankle plantar flexion</td>
<td>N</td>
<td>Isokinetic</td>
<td>Max T</td>
<td>Avg P form work calc</td>
<td>Linear T-Ang V</td>
</tr>
<tr>
<td>Gregor et al ’79</td>
<td>Human</td>
<td>Knee extension, fiber type</td>
<td>N</td>
<td>Isokinetic</td>
<td>@ 30 deg</td>
<td>T x ang V</td>
<td>FT F-V higher than ST</td>
</tr>
<tr>
<td>Hawkins &amp; Smeulders ’98</td>
<td>Human</td>
<td>Knee flexion/extension, muscle activation level, rubber band resistance</td>
<td>Y</td>
<td>Max before 100 ms</td>
<td>@ max V before 100ms</td>
<td>No</td>
<td>F-V increased with effort level, modified Hill model predicts F-V</td>
</tr>
<tr>
<td>Hawkins &amp; Smeulders ’99</td>
<td>Human</td>
<td>Hip extension, effort level, elastic resistance</td>
<td>Y</td>
<td>Max V before 100ms</td>
<td>@ Max V</td>
<td>No</td>
<td>F-V decreased w/ effort level</td>
</tr>
<tr>
<td>Author</td>
<td>Animal</td>
<td>Experiment type</td>
<td>QR</td>
<td>V measured?</td>
<td>F measured?</td>
<td>P calculated?</td>
<td>Result</td>
</tr>
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<td>------------------------</td>
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<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>Ikegawa et al ‘95</td>
<td>Human</td>
<td>Elbow &amp; knee extension &amp; flexion, muscle F-V calculation</td>
<td>N</td>
<td>Isokinetic</td>
<td>Max T</td>
<td>No</td>
<td>Hill like F-V, no curve fit</td>
</tr>
<tr>
<td>Johansson et al ‘87</td>
<td>Human</td>
<td>Knee extension, sprinters vs marathon runners</td>
<td>N</td>
<td>Isokinetic</td>
<td>Max T</td>
<td>T x ang V &amp; avg P=Work/t</td>
<td>Hill-like curves, no curve fit, sprinters higher F-V than marathoners</td>
</tr>
<tr>
<td>Jorgensen ‘76</td>
<td>Human</td>
<td>Elbow flexors &amp; extensors, concen &amp; eccen, train/untrain</td>
<td>Y</td>
<td>Isokinetic</td>
<td>@ 90 deg</td>
<td>No</td>
<td>Hill-like curves, plotted by eye, trained F-V higher</td>
</tr>
<tr>
<td>Jorgensen &amp; Bankov ‘71</td>
<td>Human</td>
<td>Elbow flexion, supinated vs pronated forearm</td>
<td>Y</td>
<td>Isokinetic</td>
<td>@ 90 deg</td>
<td>No</td>
<td>Hill-like F-V, no curve fit, sup F-V higher than pron F-V</td>
</tr>
<tr>
<td>Kojima ‘91</td>
<td>Human</td>
<td>Elbow flexion, F-ang V x-ferred to muscle by m model</td>
<td>N</td>
<td>Ang V @ 90 deg</td>
<td>@ 90 deg, weights</td>
<td>No</td>
<td>Hill’s F-V equation fit well, possible problem @ high F</td>
</tr>
<tr>
<td>Komi ‘73</td>
<td>Human</td>
<td>Elbow flexion &amp; extension, muscle F-V calculation</td>
<td>N</td>
<td>Isokinetic</td>
<td>Average F @ midpoint</td>
<td>No</td>
<td>Hill-like curve, no curve fit</td>
</tr>
<tr>
<td>Kues &amp; Mayhue ‘96</td>
<td>Human</td>
<td>Knee extension, electrical stim @ 30% max isometric F</td>
<td>N</td>
<td>Isokinetic</td>
<td>@ Max T</td>
<td>No</td>
<td>F-V linear, both concentric &amp; eccentric, different slopes</td>
</tr>
<tr>
<td>Marshall et al ‘90</td>
<td>Human</td>
<td>Knee extension, x-ray model, converted to muscle F-V</td>
<td>N</td>
<td>Isokinetic</td>
<td>@ various knee angles</td>
<td>No</td>
<td>Muscle max F occurs before max T in single trial</td>
</tr>
<tr>
<td>Martin et al ‘95</td>
<td>Human</td>
<td>Elbow flexion, eccentric training effects</td>
<td>N</td>
<td>Isokinetic</td>
<td>@ Max T</td>
<td>No</td>
<td>Eccentric training increased Max V &amp; flattened F-V curve</td>
</tr>
<tr>
<td>Perrine &amp; Edgerton ‘78</td>
<td>Human</td>
<td>Knee extension</td>
<td>N</td>
<td>Isokinetic</td>
<td>@ 30 deg</td>
<td>T x ang V</td>
<td>Hill-like, lower force @ low velocity than Hill curve</td>
</tr>
<tr>
<td>Prietto &amp; Caiozzo ‘89</td>
<td>Human</td>
<td>Knee extension/flexion, force ratio</td>
<td>N</td>
<td>Isokinetic</td>
<td>@ specific angle</td>
<td>No</td>
<td>Knee flex &amp; ext similar at low V, differ @ high V</td>
</tr>
<tr>
<td>Seger &amp; Thorstensson 2000</td>
<td>Human</td>
<td>Knee extension, max voluntary vs electrical stim, conc &amp; eccen</td>
<td>N</td>
<td>Isokinetic</td>
<td>Max T</td>
<td>No</td>
<td>Stim F-V higher in eccentric</td>
</tr>
<tr>
<td>Thorstensson et al ‘76</td>
<td>Human</td>
<td>Knee extension</td>
<td>N</td>
<td>Isokinetic</td>
<td>Various angles</td>
<td>No</td>
<td>Hill-like curves, no curve fit</td>
</tr>
<tr>
<td>Tihanyi et al ‘82</td>
<td>Human</td>
<td>Knee extension, change MOI of dynamometer, fiber type</td>
<td>N</td>
<td>Max ang V</td>
<td>@ max ang V</td>
<td>T x ang V</td>
<td>Fiber type &amp; x-sect area combo predicted inc P for FT</td>
</tr>
<tr>
<td>Van Leemputte et al ‘87</td>
<td>Human</td>
<td>Elbow flexion</td>
<td>N</td>
<td>Isokinetic</td>
<td>@ various angles</td>
<td>No</td>
<td>Hill-like, standard start pos improved results</td>
</tr>
<tr>
<td>Wakayama et al ‘95</td>
<td>Human</td>
<td>Knee extension &amp; flexion, sprinters vs distance runners</td>
<td>N</td>
<td>Isokinetic</td>
<td>Max T</td>
<td>No</td>
<td>T-ang V fit with Fenn’s eq, F-V inc for faster runners</td>
</tr>
</tbody>
</table>
The force-velocity or torque-angular velocity curve has been investigated in relation to a number of other single-joint variables. The force-velocity curve was shown to have less curvature in subjects with higher percentages of fast-twitch fibers (Froese and Houston, 1985; Tihanyi, et al., 1982; Gregor, et al., 1979), and similar results were found when comparing sprinters and distance runners (Johansson, et al., 1987; Wakayama, et al., 1995). Increased stimulation frequency (Cook and McDonagh, 1996), increased training level (Dudley and Djamil, 1985; DeKoning et al., 1985; Martin, et al., 1995; Jorgensen, 1976; Amidiris, et al., 1995), and prestretch (Cavagna, et al., 1968) shifted the force-velocity curve upward, while fatigue (DeRuiter et al., 1999) shifted it downward. Knee extension and flexion force-velocity curves had different slopes; however similar force was observed at low velocities (Prietto and Caiozzo, 1989).

2.1.3.4 Multi-Joint Force-Velocity Studies

Experiments of this type include movements that involve multiple joints. Examples of multi-joint systems are the arms/shoulder and the legs. Multiple muscles are involved in these movements making the analysis much more complex. Similar to the single joint protocol, multi-joint force-velocity studies require detailed joint models to calculate forces and velocities of actual muscles during a movement.

There are a number of possible methods to measure both force and velocity. A common protocol is to have the subject move against a constant external load or mass, similar to previous types of protocols. Other possible methods include measuring the force exerted on the load, the force at an endpoint of the system (e.g. ground reaction force in a vertical jump), the torque produced at a joint, the calculation of the actual muscle force, or average force or torque over the entire movement. Possible choices for measuring the velocity include: the velocity of the center of mass of the system, the velocity of an endpoint (e.g. velocity of the hand in an arm pull), the angular velocity of a particular joint, or average velocity of any of the previous velocity options over the entire movement.

A characteristic of multi-joint movements is that there often is no period of constant force or velocity during the movement. This complicates the choice of the time of measurement during the movement. Common choices were to measure both force and
velocity at the time of maximum velocity (Newton, et al., 1997) or angular velocity (Bosco and Komi, 1979), or at a specific position (Grieve and van der Linden, 1986) or joint angle (Bahill and Karnavas, 1991; Kunz, 1974; Jaric, et al., 1986). Another common technique was to average force and velocity over the entire movement (Viitasalo, 1985; Bosco and Komi, 1979; Newton, et al., 1997). There are virtually an infinite number of possibilities and the literature reflects this. The variability of these choices can affect the interpretation of the results of a study and can influence the comparability of the results of different protocols.

There are many examples of multi-joint force-velocity studies (Table 2.5). One of the most common was the use of a cycle ergometer or stationary bicycle. The hips, knees, and ankle were free to move in these studies but the feet were typically strapped to the pedals and the subject was required to remain seated at all times during a trial. There are two types of protocols in force-velocity testing on a cycle ergometer, single-trial (Baron, et al., 1999; Buttelli, et al., 1999, 1997, 1996; Capmal and Vandewalle, 1997; Seck, et al., 1995; Hautier, et al., 1996; Arsac, et al., 1995) and multi-trial (Baron, et al., 1999; Driss, et al., 1998; Jaskolska, et al., 1999; Linossier, et al., 1996; Vandewalle, et al., 1987). In the single trial protocol, the subject performed a single maximal sprint from zero revolutions per minute to the subject’s maximum speed against a constant resistance. The torque exerted on the crank and pedal frequency was measured once during each revolution of the pedals, or the torque and velocity was averaged for each revolution of the pedals. The torque and its corresponding rotation speed were then plotted parametrically. The multi-trial protocol required that the subject pedal at maximum speed against several braking forces, one braking force for each trial. The maximum pedaling speed during each trial was plotted against the braking force or the maximum torque. Both of these protocols produced a linear torque-angular velocity relationship with negative slope. In fact Seck, et al. (1995) found that the curves were nearly the same for both protocols. The results on cycle ergometry had a vague resemblance to Hill’s relationship in that as the pedaling speed increased, torque decreased, but since it was a linear relationship, a curve as complex as Hill’s was not needed.
<table>
<thead>
<tr>
<th>Author et al '95</th>
<th>Animal</th>
<th>Experiment type</th>
<th>QR</th>
<th>V measured?</th>
<th>F measured?</th>
<th>P calculated?</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arsac et al '95</td>
<td>Human</td>
<td>Cycle erg, single trial rest, effect of short term training</td>
<td>N</td>
<td>Avg ang V for one cycle</td>
<td>Avg F for one cycle</td>
<td>F x ang V</td>
<td>Max P increased with training but not optimal V</td>
</tr>
<tr>
<td>Bahill &amp; Karnavas '91</td>
<td>Human</td>
<td>Swinging baseball bat with different weights</td>
<td>N</td>
<td>Max V</td>
<td>Bat weight</td>
<td>No</td>
<td>Bat weight-velocity curve was Hill-like, some linear</td>
</tr>
<tr>
<td>Baron et al '99</td>
<td>Human</td>
<td>Cycling erg, Compare isokinetic multi-trial &amp; single trial F-V</td>
<td>N</td>
<td>Isokinetic &amp; Max ang V</td>
<td>Avg T &amp; Each Revol</td>
<td>Avg P &amp; MaxP=Txw</td>
<td>Same Peak P, Optimal V use isokinetic method</td>
</tr>
<tr>
<td>Bartosiewicz et al '90</td>
<td>Human</td>
<td>Vertical jump, vary level of countermovement</td>
<td>N</td>
<td>@ max P</td>
<td>@ max P</td>
<td>F x V</td>
<td>Optimal level of countermovement found for each subject</td>
</tr>
<tr>
<td>Bobbert et al '86</td>
<td>Human</td>
<td>Vertical jump, ankle power</td>
<td>N</td>
<td>Ang V calculated from video</td>
<td>T calculated from FP &amp; video</td>
<td>T x ang V</td>
<td>F-V not plotted but implied &amp; used for calculations</td>
</tr>
<tr>
<td>Bosco &amp; Komi '79</td>
<td>Human</td>
<td>Vertical jump with varied loads, non CM, CM, &amp; drop jumps</td>
<td>N</td>
<td>Max Knee ang V</td>
<td>Avg GRF</td>
<td>Avg P, calculated w/ F-t curve</td>
<td>F-ang V &amp; P-ang V similar to Hill’s curve, no curve fit</td>
</tr>
<tr>
<td>Bosco et al '95</td>
<td>Human</td>
<td>Horiz slide machine – load varies 35%-210% body mass</td>
<td>N</td>
<td>Avg V</td>
<td>Avg F</td>
<td>Avg P</td>
<td>Dynamometer works</td>
</tr>
<tr>
<td>Buttelli et al '96</td>
<td>Human</td>
<td>Cycle erg, one trial test</td>
<td>N</td>
<td>Avg ang V</td>
<td>Avg T</td>
<td>Max P, Avg P</td>
<td>Linear F-V curve</td>
</tr>
<tr>
<td>Buttelli et al '97</td>
<td>Human</td>
<td>Cycle erg, one trial test, aerobic fatigue</td>
<td>N</td>
<td>Avg ang V</td>
<td>Avg T</td>
<td>Max P</td>
<td>Harder exercise dec maxP&amp;V Easier exercise dec max P&amp;T</td>
</tr>
<tr>
<td>Buttelli et al '99</td>
<td>Human</td>
<td>Cycle erg, one trial test, anaerobic fatigue</td>
<td>N</td>
<td>Avg ang V</td>
<td>Avg T</td>
<td>Max P</td>
<td>Recovery in 1 minute to norm</td>
</tr>
<tr>
<td>Capmal &amp; Vandewalle '97</td>
<td>Human</td>
<td>Cycle erg, one trial test, w/ vs w/o toe clips</td>
<td>N</td>
<td>Avg ang V</td>
<td>Avg T</td>
<td>Max P</td>
<td>Same shape F-V curve, peaks decreased w/o toe clips</td>
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<tr>
<td>Driss et al '98</td>
<td>Human</td>
<td>Cycle &amp; arm crank ergs, multiple trial, vary braking force, vertical jump</td>
<td>N</td>
<td>Max V</td>
<td>Brake F</td>
<td>FxV</td>
<td>Linear F-V, VJ correlated to max P in arms &amp; legs</td>
</tr>
<tr>
<td>Grieve &amp; van der Linden '86</td>
<td>Human</td>
<td>Horiz arm pulls, viscous resistance</td>
<td>N</td>
<td>@ several arm positions</td>
<td>@ several arm position</td>
<td>FxV</td>
<td>F-V-Arm position surfaces</td>
</tr>
<tr>
<td>Hartmann et al '93</td>
<td>Human</td>
<td>Rowing erg, 10 consecutive reps</td>
<td>N</td>
<td>Max V each stroke</td>
<td>Max F each stroke</td>
<td>FxV</td>
<td>Nearly linear F-V, men higher than women</td>
</tr>
<tr>
<td>Hautier et al '96</td>
<td>Human</td>
<td>Cycle erg, single trial test, optimal velocity &amp; fiber type</td>
<td>N</td>
<td>Avg ang V</td>
<td>Avg T</td>
<td>T x ang V</td>
<td>Linear F-V, optimal V and FT fiber type correlated</td>
</tr>
<tr>
<td>Author</td>
<td>Animal</td>
<td>Experiment type</td>
<td>QR</td>
<td>V measured?</td>
<td>F measured?</td>
<td>P calculated?</td>
<td>Result</td>
</tr>
<tr>
<td>-------------------------</td>
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<td>-------------------------------------------------------------------------</td>
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<tr>
<td>Jaric et al ’86</td>
<td>Human/quadriceps</td>
<td>Vertical jump w/ additional loads, elite athletes vs normals, muscle F-V calculated w/ eq</td>
<td>N</td>
<td>@ 5 knee angles</td>
<td>@ 5 knee angles</td>
<td>FxV?</td>
<td>Hill curves fitted, elite athletes higher F-V</td>
</tr>
<tr>
<td>Jaskolska et al ’99</td>
<td>Human</td>
<td>Cycle erg vs treadmill</td>
<td>N</td>
<td>Max V</td>
<td>Braking F</td>
<td>FxV</td>
<td>Linear F-V, similar in both</td>
</tr>
<tr>
<td>Kunz ’74</td>
<td>Human</td>
<td>Throwing ball of different mass</td>
<td>N</td>
<td>Max V @ release</td>
<td>Ball mass</td>
<td>No</td>
<td>Release V decreased w/ increased ball mass</td>
</tr>
<tr>
<td>Linossier, et al ’96</td>
<td>Human</td>
<td>Cycle erg, optimal brake force</td>
<td>N</td>
<td>Max V</td>
<td>Braking F</td>
<td>BfxVmax</td>
<td>Optimal BF @ Max P</td>
</tr>
<tr>
<td>Newton et al ’97</td>
<td>Human</td>
<td>Bench throws w/ different loads</td>
<td>N</td>
<td>Max V, Avg V</td>
<td>Load &amp; force plate</td>
<td>Avg P, Max P</td>
<td>Similar to Hill’s curve, no curve fit, L-V nearly linear</td>
</tr>
<tr>
<td>Seck et al ’95</td>
<td>Human</td>
<td>Cycle erg, one trial test vs multi trial test</td>
<td>N</td>
<td>Max V &amp; continuous</td>
<td>Brake torque &amp; contin</td>
<td>P=BTxang V, P=Txang V</td>
<td>Similar F-V for both, Max P higher in single trial test</td>
</tr>
<tr>
<td>Thomas et al ’96</td>
<td>Human</td>
<td>Weight lifting</td>
<td>N</td>
<td>No</td>
<td>External load</td>
<td>Avg P from work</td>
<td>P-load curves similar to Hill-type, optimal load higher than expected</td>
</tr>
<tr>
<td>Tsarouchas &amp; Klissouras ’81</td>
<td>Human</td>
<td>Vertical jump, loaded and unloaded</td>
<td>N</td>
<td>Takeoff</td>
<td>External load</td>
<td>Load x V</td>
<td>Hill-like curves</td>
</tr>
<tr>
<td>Vandewalle et al ’87</td>
<td>Human</td>
<td>Cycle erg, multiple trials, vary braking force, vertical jump</td>
<td>N</td>
<td>Max V</td>
<td>Braking force</td>
<td>F x ang V</td>
<td>Max power correllated w/ vertical jump</td>
</tr>
<tr>
<td>Viitasalo ’85</td>
<td>Human</td>
<td>Vertical jump w/ additional loads</td>
<td>N</td>
<td>Avg knee ang V, jump height</td>
<td>Avg GRF, load</td>
<td>No</td>
<td>Avg knee ang V-avg GRF &amp; jump height-load curves were similar &amp; Hill-like, no fit</td>
</tr>
</tbody>
</table>
Bahill and Karnavas (1991) measured velocity as baseball players swung baseball bats of differing weights, and Kunz (1974) measured the speed of release as subjects threw balls of differing mass. Both authors found that the speed of the implements decreased with increasing weight or mass, and the velocity of the baseball bat decreased in a Hill-like manner that could be described with a hyperbolic curve fit. In a related study, Newton, et al. (1997) studied the load-velocity relationship in bench press throws. The results showed a nearly linear, but Hill-like relationship. Two additional upper body experiments (Grieve and van der Linden, 1986; Hartmann, et al., 1993) showed Hill-like force-velocity relationships for pulls against viscous resistance and for repetitive strokes on a rowing ergometer respectively. Both of these studies measured the force and velocity at the hand grip.

The final group of experiments under review for studying multi-joint force-velocity relationships used the vertical jump as the multi-joint model. Bosco and Komi (1979) compared the results of noncountermovement, countermovement and drop vertical jumps with different additional loads placed on the shoulders of the subjects with a barbell for each jump. The maximum angular velocity of the knee joint was plotted parametrically against the average ground reaction force. The resulting curves were found to be Hill-like but no curve was fit to the data. The ground reaction force was assumed to be a direct result of the action of the knee extensors, as was the knee angular velocity, similar to isokinetic leg extensions. Since no joint model was used, the validity of these results could be questioned. In a similar study, Viitasalo (1985) plotted average knee angular velocity vs. average ground reaction force during loaded vertical jumps, as well as the height of the vertical jump vs. the additional load. Both curves were found to be Hill-like, but no curves were fit to the data.

Jaric, et al. (1986) accomplished a vertical jump study where elite and untrained subjects jumped against additional resistance. A joint model was used to calculate the force and velocity produced in the quadriceps and measured at five joint angles. However, this model did not include the any moment arm calculations, it seemed to be merely a phenomenological conversion model. Hill’s curve was fitted to the resulting force-velocity data, with the force-velocity of the elite subjects above the untrained
subjects. Tihanyi, et al. (1987) fit loaded vertical jump force-velocity data with Hill’s curve, but did not report which specific forces or velocities were used. They determined that at least four data points are required to properly fit Hill’s curve as long as the range of forces used is at least 15 to 100% of the maximum isometric force.

Attempting to increase the range of the force-velocity curve in vertical jumping, Tsarouchas and Klissouras (1981) used loaded and unloaded vertical jumps. The extra load was provided with bags filled with sand on the shoulders, and the unloaded conditions were accomplished using a calibrated spring, harness, and pulley system. External load was plotted parametrically with takeoff velocity, producing Hill-like curves that were not mathematically fit to the data. However, they did not measure the actual force of unloading throughout the jump, only the average unloading force as calculated from the calibrated spring. This may have caused inaccuracies in the velocity calculations (the values for velocity were very high). They also reported the unusual result that maximum ground reaction force did not continue to decrease with increased unloading and did not increase above 150% of body weight loading. This may indicate a problem with the experimental protocol or the physical abilities of their subjects.

Bosco, et al. (1995) used a horizontal slide machine connected to a weight stack to validate a dynamometer for evaluating human performance using a jumping style protocol. The range of loads was 35% to 210% of body weight. Average force and velocity were reported and produced linear force-velocity relationships when combined parametrically. This method appears to have potential for use in future force-velocity research.

There appears to be a gap in the literature in the force-velocity relationship for vertical jumping. Many of the relevant variables such as ground reaction force, knee torque, maximum knee angular velocity, and maximum center of mass velocity have not been addressed sufficiently. In addition, some of the methods for measurement and analysis could be questioned. The force and velocity ranges in vertical jump studies have been very limited for the most part; therefore, the vertical jump model has not been sufficiently researched. This is a very commonly used movement in sport and science and deserves more attention.
2.2 Power-Velocity and Power-Force Relationships

The relationship between power and velocity and/or force is an interesting way to examine the force-velocity relationship from another point of view. Power has been associated with many positive attributes of athletics and the performance of explosive movements including the vertical jump (e.g. Dowling and Vamos, 1993). Therefore, it is natural that study into the production of power produced in muscle would be of interest to scientists and athletes. For the purposes of this review, all references to power are in the context of the force-velocity curve unless mentioned specifically in the text.

Power is most often calculated using the product of each force and velocity pair in the parametric force-velocity relationship. Each individual point can then be plotted against either force or velocity, depending on the interests of the individual study (see Tables 2.2-2.5). Average power was sometimes reported in the literature as the work produced in a movement divided by the time of the movement for each trial (Thomas, et al., 1996; Fugel-Meyer, et al., 1982; Johansson, et al., 1987; Bosco and Komi, 1979). Both methods usually result in a parabolic-shaped curve (Epstein and Herzog, 1998).

The choice of the specific force and velocity variables and the choice of the time where they are measured in the movement can make the calculation of power complex, especially in the case of multi-joint movements. For example, the corresponding ground reaction force in a vertical jump could be multiplied by the takeoff velocity or the maximum velocity; the maximum ground reaction force or the maximum knee torque could be multiplied by the corresponding angular velocity of the knee. Power calculations in single joint/fiber studies are much simpler since there are fewer choices for force and velocity.

Many authors were interested in the maximum power point of the curve. When the force-velocity relationship was linear, such as in cycle ergometer studies, the maximum power was easily calculated as the product of $0.5 \times$ maximum force and $0.5 \times$ maximum velocity, which equals $0.25 \times$ maximum force $\times$ maximum velocity (Buttelli, et al., 1999, 1997, 1996; Capmal and Vanderwalle, 1997). The velocity or force where parametric power was maximal was also a common source of interest (Swoap, et al., 1997; Baron, et al., 1999; Newton, et al., 1997; Arsac, et al., 1995; Thomas, et al., 1996).
The value of optimal velocity and force, i.e. the force and velocity where power was maximal, has been reported to be approximately $0.33 \times$ maximum force and $0.33 \times$ maximum velocity (Newton, et al., 1997; Kraemer and Newton, 1994). However, Thomas, et al. (1996) found that the force where maximum power occurred in weight lifting was somewhat higher.

Power was used in conjunction with the force-velocity relationship to analyze many of the same variables mentioned above for single fiber/muscle preparations, single-joint studies, and multi-joint studies. Examples include: fiber type, temperature, fatigue, non-use, electrical cycling frequency, comparison of protocol, training effects, and level of countermovement. Please see Tables 2.2-2.5 for specific references. There appears to be very little or no research in the area of power in the eccentric range of the force-velocity relationship.

2.3 Conclusion

The force-velocity relationship of muscle is well known and has received considerable attention in the literature. The concentric portion of the curve is the best defined because of the ease of measurement and consistency of the results. The eccentric portion of the curve is less defined because of a distinct lack of research in the area and the difficulty in producing consistent and reliable results.

Nearly all researchers either compared their results to or used the results of Hill and his hyperbolic equation for the force-velocity relationship. This is a descriptive equation that has little direct correlation to actual muscle properties or structure. However, the curve seemed to adequately describe the mechanics of muscle in many situations from single fiber studies to multi-joint studies. Power-velocity and power-force relationships are another common way to express and analyze muscle mechanics.

There appeared to be a gap in the literature for well-done multi-joint studies with a wide range of forces and velocities. This was the focus of this study.
Chapter 3 Methods
This chapter outlines the experiment, the experimental protocol, and the methods for analysis. A set of pilot experiments also was performed. A brief description of these experiments and their influence on the current study can be found in the Appendix.

3.1 Subjects

Ten well-trained athletic males between the ages of 19 and 29 selected from Penn State University athletic teams and graduate school served as subjects for this experiment. The experimental protocol was explained to all of the subjects and all of them signed an informed consent form reviewed and approved by the Penn State University Office of Regulatory Compliance. When possible, they were chosen for their ability in jumping- or power-type events because of the explosive nature of the experiment. This was determined mainly by their sport and/or event in their sport.

The subjects were identified only by a letter or a number (e.g. subject A, B, C… or subject 1, 2, 3…). The best performance of each subject in their sport was recorded as well as their height, weight, best vertical jump, and best lift in the squat exercise (Table 3.1). The squat parameter was needed to determine if the subject could easily perform a squat with a weight equal to their body weight. Sufficient experience and training was important for the safety of the subjects and to produce a wide range of experimental data. The athlete was not chosen if sufficient experience and ability were not demonstrated.
Table 3.1. Subject Data

<table>
<thead>
<tr>
<th>Subject</th>
<th>Age</th>
<th>Height</th>
<th>Weight</th>
<th>Best Squat</th>
<th>Sport/Event</th>
<th>Best Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>19</td>
<td>1.83m</td>
<td>798.5N</td>
<td>1446N</td>
<td>Athletics/Long Jump</td>
<td>7.91m</td>
</tr>
<tr>
<td>B-2</td>
<td>21</td>
<td>1.83m</td>
<td>867.5N</td>
<td>1401N</td>
<td>Athletics/110m Hurdles</td>
<td>13.94s</td>
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<tr>
<td>C-3</td>
<td>20</td>
<td>1.93m</td>
<td>892N</td>
<td>1224N</td>
<td>Athletics/110m Hurdles</td>
<td>14.33s</td>
</tr>
<tr>
<td>D-4</td>
<td>20</td>
<td>1.83m</td>
<td>862N</td>
<td>1847N</td>
<td>Athletics/Decathlon</td>
<td>7600 pts</td>
</tr>
<tr>
<td>F-5</td>
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<td>1.73m</td>
<td>781N</td>
<td>1780N</td>
<td>Athletics/100m</td>
<td>10.9s</td>
</tr>
<tr>
<td>G-6</td>
<td>21</td>
<td>1.83</td>
<td>970N</td>
<td>1780N</td>
<td>Athletics/Javelin</td>
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<tr>
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<td>1.88m</td>
<td>1001N</td>
<td>1780N</td>
<td>Athletics/Javelin</td>
<td>72.76m</td>
</tr>
<tr>
<td>I-8</td>
<td>21</td>
<td>1.83m</td>
<td>768N</td>
<td>1113N</td>
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<tr>
<td>J-9</td>
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<td>1.80m</td>
<td>870N</td>
<td>1335N</td>
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<td>2:12 min</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>200 Individual Medley</td>
<td></td>
</tr>
<tr>
<td>K-10</td>
<td>29</td>
<td>1.75m</td>
<td>920N</td>
<td>1914N</td>
<td>Olympic Weightlifting</td>
<td>Snatch: 1224N</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>Clean-Jerk: 1521N</td>
</tr>
<tr>
<td>Avg</td>
<td>22.2</td>
<td>1.82m</td>
<td>873N</td>
<td>1562N</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.2 Experiment

The experiment consisted of a series of maximum effort, non-countermovement vertical jumps with different amounts of resistance. The subjects jumped under three loading conditions: with body weight only, with extra weight in addition to body weight, and with assistance, which could be considered less than body weight.

Resistances greater than body weight were accomplished by adding load to the shoulders of the subjects with a barbell, similar to the squat exercise in weight lifting. The load was increased until the subjects had extreme difficulty jumping or until there was a safety concern due to lack of control or technique. The barbell that was used was a normal weightlifting barbell (45 lbs.) and the loads were normal iron weightlifting plates (5 - 45 lbs.). By changing the combination of the plates on the barbell, various loaded conditions could be accomplished (Figure 3.1 A).
Resistances less than body weight were accomplished by unloading the subject with stretched, filled elastic surgical tubing. The ends of the tubes were doubled over forming a loop and secured using nylon tie wrap for attachment purposes. The subjects wore a full body harness that was originally designed as a rescue harness and had various attachment points where cables could be connected (Figure 3.1 B). The elastic tubing was stretched from the harness between the shoulder blades to a point directly above the takeoff area via mountain climbing carabiners. The number of tubes was varied; more tubes caused more unloading. When stretched, the tubing produced a linear increase in force up to about 300% of its original length. The original length of the tubes was 4.5 feet and the maximum stretched length was approximately 12 feet, depending on the height of the subjects and the depth of the squats before the jumps. The amount of shortening during the ground contact time of the jumps was approximately two feet. This resulted in a vertical force on the subjects that decreased approximately 15-25% as the subjects jumped. Although the force was not constant, it was measured continuously.

Figure 3.1. Subject with a barbell in a loaded condition (A). Subject in harness in an unloaded condition (B).
Jumps performed with resistance equal to body weight were accomplished by simply performing non-countermovement vertical jumps. No additional weight was added or subtracted for these conditions.

A special A-frame structure was constructed to carry out this experiment. A wood crossbeam was supported by two wood A-shaped structures approximately 15 feet high. Two large eyebolts were mounted vertically in the crossbeam, one directly above the jumping area, and the other offset by approximately three feet. This configuration allowed for pulley attachment for the loaded trials and for the filled surgical tubes to be connected high above the subject for the unloaded trials without damaging the ceiling.

The body configuration of the subjects in the initial squatting position was kept as consistent as possible from condition to condition and from trial to trial but was not strictly controlled. The subjects were allowed to assume a comfortable starting position and were encouraged to keep it the same throughout the experiment. The arms of the subjects were not actively involved in the jumping motion. In the loaded trials, the subject grasped the barbell on either side of the shoulders. In the bodyweight and unloaded jumps, the subjects similarly grasped a light wooden rod that was placed on the shoulders in the same manner as the barbell in the loaded trials. The weight of the rod was included as part of the body weight.

The subjects were instructed to squat to a consistent, comfortable height before jumping, as indicated above. Since these were noncountermovement jumps, the subjects were required to remain motionless in the squatting position for one to two seconds before the initiation of the jump. From the squatted position, the subjects were required to move directly upward with no preparatory lowering of any part of the body. The force-time curve obtained from a force plate (see below) was examined after each attempt, and all jumps with countermovements were repeated or rejected for analysis. Typically, one or two trials were repeated during the experiment for each subject. Pilot work indicated that subjects were unable to remain flat-footed during the initial squat before the jumps in the unloaded conditions. The subjects were therefore instructed to raise their heels from the floor as they squatted in all of the conditions.
The safety of the subjects in the loaded trials was facilitated in several ways. A rack similar to a squat rack in weightlifting was placed beneath the subjects and the barbell to protect the subject in case they lost control of the weight. The barbell was connected to a pulley system that enabled the barbell to be “caught” at the maximum height of the jump. This system protected the subject from the high loads that would be experienced if the subject landed on the floor with the barbell, and provided additional safety if the subject lost control of the barbell. The barbell was padded with several soft towels that provided comfort to the subjects as they jumped with the bar and protected them in the case of accidental contact after the bar was “caught” at the peak of the jump.

Since most of the tasks in the experiment were novel to most people (people are not used to non-countermovement, loaded, and unloaded jumping), a practice session was held for each subject at least one day before the experiment. This session enabled them to gain experience with the experiment and the experimental protocol and minimized any learning effects that might have been present.

3.3 Experimental Protocol

The subjects completed a standardized warm-up and stretching routine before the start of the experiment. The subjects performed two trials at each experimental condition. This number was chosen to give the subjects two chances to give their best effort but at the same time minimize fatigue over the entire experiment. Each subject performed 14 to 17 conditions for a total of 28 to 34 trials, depending on the strength and body weight of the subject. The testing began with two trials in the bodyweight vertical jump condition. The test proceeded to the loaded jumping trials with the barbell. The weight was increased in approximately 45-pound increments for each condition until the subject could no longer jump or did not demonstrate good control during the jumps. The two bodyweight trials were then repeated and the test moved to the unloaded jumping trials. The number of bands was incrementally increased for each condition until the subject could not reach the full squatted position required because the force of the stretched bands exceeded the body weight of the subjects. The experiment was then terminated, resulting in approximately 80% unloading and 125% extra loading in relation
to body weight, depending on the abilities of the subject. There was a minimum of two minutes rest between each trial to reduce the effects of fatigue. The trial at each condition resulting in the highest peak velocity was selected for analysis.

In all of the trials, the subject was asked to perform jumps for maximum height with no countermovement. A reminder of both of these requirements was given to the subject before each trial, and verbal encouragement was provided to the subject during each trial. The subject was commanded to assume the squatting position. The data collection program was then started and the command to jump was given. The subject then jumped as high as possible. The two-minute rest period then followed during which the apparatus was adjusted for the next condition, if appropriate. The video cameras were paused during the rest periods.

The order of the trials was not randomized for several reasons. There was considerable changeover time between the loaded and unloaded trials. In addition, the difference between the unloaded jumps and the possible extreme loaded jumps was believed to be too much of a shock to the subjects and could have been unsafe. These considerations lead to a progressive loading and unloading scheme instead of a randomized order. This minimized the time of the experiment and maximized the safety of the subjects.

3.4 Data Collection

The subjects jumped from a force plate (Bertec 4060S, 40cm x 90cm, natural frequency = 1800Hz) that measured the vertical ground reaction force of the subjects during the jumps (Figure 3.2 A). In the unloaded trials, the force produced by the elastic surgical tubing was measured with an axial force transducer (PCB 221B04, range = +/- 4448kN, natural frequency = 70kHz) attached directly to the harness in series with the tubing. The transducer was small (3.1cm long x 1.6cm diameter) and light (31g) and did not affect the jumping motion (Figure 3.2 B).
The activity of six major muscles of the right leg (anterior tibialis, gastrocnemius (Figure 3.3 A), vastus lateralis, rectus femoris, biceps femoris (Figure 3.3 B), and gluteus maximus) was monitored via surface electromyography (EMG) during all of the jumps. The EMG signals were amplified using preamplifiers (x10) and an amplifier built by technicians at Rush Presbyterian Hospital, Chicago, IL. The self-adhesive electrodes (Medi-Trace ECL 1801) were placed on the surface of the shaved skin over the muscles as described in Basmajian and Blumenstein. The electrode wires were run upward along the leg of the subjects, under the shorts, and out above the waistband to the amplifier. The wires were secured with the shorts (described below) and with athletic tape if necessary (Figure 3.3 C).

All of the above data was collected at 600 Hz via an analog to digital (A/D) converter (National Instruments AT-MIO-64E-3) and a personal computer (Gateway 2000, 100MHz Pentium processor, 16 Mbytes RAM).

The motion was videotaped (Panasonic SVHS AG450) in the sagittal and frontal planes at 60 Hz to enable position data to be analyzed. To facilitate digitizing, six reflective markers (1.5 cm diameter) were placed on the left side of the subjects using double-sided tape. The locations of the markers were: 1) on the shoe of the subject at the heel and 2) at the fifth metatarsal-phalangeal joint, 3) the lateral maleolus, 4) the
approximate center of rotation of the knee, 5) the greater trochanter, and 6) the side of the neck in line with the jaw of the subject directly below the ear. In the loaded trials where the marker on the neck was obstructed from the view of the camera, the frontal view was used to determine the height of the neck marker in relation to the other markers that could be seen in both views. This precluded any sagittal movement analysis of the neck marker in all of the weighted trials. A one-meter scaling rod was used to compute the scaling factor for each of the camera views.

The subjects were provided with dark colored shorts and socks for contrast with the reflective markers in the video images. The subjects wore their normal training shoes. The shorts were made of Spandex to help minimize the motion of the EMG electrodes and to control the motion of the EMG wires (Figure 3.3 C).

### 3.5 Data Analysis

The positions of the reflective markers were digitized using the Peak Performance System software package. All other calculations were carried out using Microsoft Excel and Mathworks Matlab. A four-segment (feet, shanks, thighs, head-arms-torso (HAT)) model of the human body in the sagittal plane was used. The position data were interpolated by a factor of ten. This multiplied the number of data points by ten and effectively changed the frequency of the data collection from 60Hz to 600Hz (which
matches the rest of the data and makes data analysis easier). This interpolation did not increase the accuracy or change the values of any of the original calculated variables. The video data was filtered with a fifth order Butterworth filter. The filtering frequency was determined by filtering the data from one to 20Hz, computing the center of mass velocity curve (see below), and comparing the velocity curve with the corresponding center of mass velocity curve calculated with force plate data (see below). A least squares difference approach was used to choose the best filtering frequency.

The velocity and acceleration of each of the markers was computed by numerically differentiating the position data. In most of the loaded trials, the neck marker was obstructed from the view of the sagittal camera by the barbell plates. To construct the segment model the vertical position of the neck marker from the front view was combined with the vertical positions of the other markers in the sagittal view. This technique enabled vertical motion to be analyzed (which was most important for this study), but horizontal motion for the HAT segment could not be analyzed.

The positions of the centers of mass of each of the segments and of the whole body were computed using the body mass parameters of Zatsiorsky, et al. (1990). The position of the center of mass of the HAT was adjusted to take into account the position of the arms, which were raised to hold on to the barbell or wooden rod as described above. The vertical velocities and accelerations of these points and the center of mass were calculated by numerical differentiation of the position data.

In the bodyweight vertical jump and loaded jump trials, the governing equation is:

\[ GRF - W = ma, \]  

where \( GRF \) is the vertical ground reaction force measured from the force plate, \( W \) is the weight of the subject and the barbell combined, \( m \) is the mass of the subject and the barbell, and \( a \) is the acceleration of the center of mass of the system. In the unloaded trials, the governing equation is:

\[ GRF - W + F_b = ma, \]  

where \( GRF, W, m, \) and \( a \) are as above, and \( F_b \) is the force provided by the elastic bands. Using these equations, vertical ground reaction force was computed.
The vertical ground reaction force as measured from the force plate was also used to calculate the velocity of the center of mass. The above equations, when solved for acceleration and numerically integrated to estimate the area under the acceleration curve, allowed the vertical velocity of the center of mass to be computed at each instant of time. Vertical center of mass velocity and vertical ground reaction force were simply called center of mass velocity and ground reaction force in this paper.

The angular velocity of the knee joint was calculated by numerically differentiating the joint angle at each instant in time as determined from the video analysis. An additional measure of leg extension velocity was computed by computing the linear velocity between the hip and the ankle using the video data. This was a technique that has not been seen in the force-velocity literature and was called leg extension velocity in this paper.

The torque produced at the knee joint was computed using an inverse dynamics approach (Winter, 1990) combining the data from the force plate, video, and body segment parameter assumptions (Zatsiorsky, et al., 1990). The ground reaction force data and the movement data of the foot were combined to calculate the resulting torque at the ankle. This data was then used to calculate the torque produced at the knee.

Power was calculated in both the video method and the force plate method by multiplying the corresponding ground reaction force by the velocity of the center of mass at each instant in time. Power calculated in this way has sometimes been called ‘pseudo-power’ because, strictly speaking, the ground reaction force does not act directly on the center of mass, but it is quantitatively equal to the effective force that is acting at the center of mass. However, it is also a common practice in the literature to refer to pseudo-power as simply power; this paper will follow that practice as well. Power at the knee joint was calculated by multiplying the corresponding torque by the angular velocity at each moment in time. Another measure of power was attempted by multiplying leg extension velocity by the corresponding ground reaction force. This has not been seen before in the literature.

When necessary, the data from the force plate and the video analyses were synchronized using the instant of takeoff as a reference. This moment in time was easily
determined from the ground reaction force-time curve as the point where there is zero force. The moment of takeoff was also easily observed in the video analysis. The ratio of recording speeds between the video and the other equipment was 1/10 producing minimal error when the two sets of data were synchronized.

The data for each of the subjects was normalized so that the results from all of the subjects could be combined and compared. The external load was normalized by dividing the load at each condition by the body weight of the subject. Thus, the normalized load in the body weight conditions was zero and the rest of the normalized loads were fractions of body weight. Since the load was not constant in the unloaded conditions, the load during the static squat before the jump was used for these plots. All force, velocity, power, and time variables, regardless of the calculation procedure, were normalized by dividing by the average body weight values of each parameter. Thus, all of the normalized variables at the zero-load condition were equal to one.

Originally, EMG was to be analyzed in detail to determine the coordination patterns of the vertical jumps under the various loading conditions. This proved to be beyond the reasonable scope of the project, therefore only a few examples of EMG plots were created for several loading conditions. This was accomplished by rectifying the EMG signal and subjecting it to a low pass filter (same as above, 20 Hz), thereby producing an EMG-envelope showing the major activation patterns of each muscle.

3.6 Parameter Combinations

One of the purposes of this study was to determine the best way to present force-velocity and power-velocity relationships for multi-joint movements since there was an unlimited number of choices for combinations of variables. Therefore, several combinations were chosen to highlight combinations with high correlations and to eliminate some of the possible combinations with low correlations. All of the following combinations described were parametric relationships, i.e. one instant in time during each condition was chosen and the values for each experimental variable, i.e. force, velocity, or power, were determined at this instant; the individual points from each condition were then combined in one ‘parametric’ plot.
The efficacy of the methods and protocols was investigated by examining the center of mass velocity and ground reaction force-time relationships. The reactions of each of the key variables in the study were examined in relation to the external load, the only independent variable in this study. These key variables, maximum velocity of the center of mass, the corresponding ground reaction force, maximum knee angular velocity, maximum leg extension velocity, and the corresponding knee moment, were all plotted against the external load. Total movement time from the onset of the movement to takeoff, the time from initial movement to maximum center of mass velocity, and the time from the maximum center of mass velocity to takeoff were investigated as a function of load.

The instant when maximum velocity occurred during each jump was chosen as the moment in time to analyze the force-velocity and power-velocity relationships in this study. As discussed in the literature review, this selection is somewhat arbitrary. The time of maximum velocity was chosen because it was easily identifiable, and it was consistent with other force-velocity studies in the literature (see Tables 2.2-2.5). The maximum center of mass velocity also corresponds to an instant in time where the acceleration is zero. Analyzing force-velocity properties at this instant may make this multi-joint study more comparable to single joint and/or muscle/fiber studies in the literature that had constant velocity (zero acceleration) as a constraint. The instant of time for maximum velocity was different for each type of analysis for a given jump, depending on the velocity that was being measured. For example, the instant when maximum velocity of the center of mass occurred was not the same as when the maximum leg extension velocity or maximum knee angular velocity occurred.

The relationship between the maximum velocity of the center of mass of each jump from the force plate and video analyses as well as the maximum leg extension velocity and maximum knee angular velocity was investigated by plotting these terms against their corresponding ground reaction forces. Maximum ground reaction force was also studied briefly in relation to maximum center of mass velocity and to the center of mass velocity that corresponded to the instant of maximum ground reaction force. Knee moment was plotted as a function of maximum knee angular velocity and of maximum
leg extension velocity. Other than those involving leg extension velocity, these were all common variable combinations that have been investigated in the literature (see Tables 2.4 and 2.5).

Power was investigated in several ways in this study. Power, in a force-velocity context, is simply another way of presenting the data that may have additional meanings in alternate contexts, i.e. athletic events. The first group of power curves used power calculated by multiplying maximum center of mass velocity by its corresponding ground reaction force. The relationship between power and load, which potentially has relevance in weight training theory, was investigated by plotting the power corresponding to the instant of maximum center of mass velocity against the corresponding external load. The relationship between power and ground reaction force corresponding to the instant of maximum center of mass velocity was also studied. Power corresponding to the instant of maximum center of mass velocity and maximum center of mass velocity were plotted together, as were corresponding power and maximum knee angular velocity, and corresponding power and maximum leg extension velocity.

Knee angular velocity multiplied by maximum knee torque was used to calculate knee power in the second group of power curves. Corresponding knee power calculated in this way was plotted against maximum knee angular velocity and maximum leg extension velocity. The third method for calculating power has not been seen in the literature. This power measure was accomplished by multiplying the ground reaction force by the maximum leg extension velocity and was presented by plotting it against the maximum leg extension velocity and the external load.

3.7 Statistics

The force vs. velocity, force vs. load and velocity vs. load curves mentioned above were fitted with two models: a second order polynomial using a least-squared difference approach (except the ground reaction force vs. load relationship, which was fit with a first order curve) and a hyperbolic model (Hill’s curve) using Newton’s Method and the method of least squares to determine the constants. The power curves were fit with the polynomial only. The second order polynomial was chosen because it was
simple, seemed to describe the force-velocity data well, and in the case of the power curves, the theoretical model for power is a parabola which is a second order curve.

A measure of goodness of fit was determined for each relevant curve by performing an intra-class correlation (Fleiss, 1986) between the data and the fitted predictions. This correlation was a test of the relative homogeneity between the data and the fit. Intra-class correlation reflects systematic differences in the data sets that a more common measure such as Pearson’s product moment correlation would miss which makes the former a better measure of fit. These models were then compared to each other and the relevant literature. The polynomial and hyperbolic fits were not compared statistically because there is no statistical meaning to comparing the shape of the curves. The parabolic fit was either favorably or unfavorably compared to the Hill model with the criteria described previously.

Since data from both the force plate method and the video method were used in combination for some of the analyses, it was important to determine that the two methods produced similar results. To that end, the maximum velocity of the center of mass, corresponding ground reaction force, and load relationships were compared by correlating the curve fits from each variable combination using the intra-class correlation technique.

### 3.8 Delimitations of the Study

No matter how much effort is expended in the effort to control every aspect of a scientific experiment, there are always weaknesses in the study. This experiment was no exception. The main weakness of this study was the fact that body position could not be controlled throughout the entire movement. This led to many questions as the data was analyzed as reflected in the Discussion. However, this is an intrinsic problem found in nearly all multi-joint investigations of any type.

Another problem was the fact that the barbell weights prevented the sagittal camera from picking up the neck marker. This meant that no analysis of the hip joint was possible for the loaded conditions. There was a plan in place to digitize the end of the barbell and to transpose that data to the position of the neck, but that method proved to be
impractical if not impossible with the data available. The vertical position of the neck marker was measured from the front view and used as described earlier in this chapter.

The starting position of the subjects was not strictly controlled. The subjects were instructed to start in a thighs horizontal position. The feet were not able to be placed flat on the force plate in the unloaded conditions, so the subjects were instructed to begin with the heels off of the ground in all of the conditions. Differences in ankle position may have affected the initial length of the gastrocnemius, which could have affects at both the ankle and the knee since it is a two-joint muscle. There was also the possibility of a countermovement at the ankle, which may have been too small to be picked up by the force plate. These factors can adversely affect several of the calculations if there are large differences between trials and/or between subjects. Visual checks and verbal encouragement were used to assist the subjects in assuming their starting position, and the subjects were all required to attend a practice session before performing the experiments. It is believed that this is not a serious problem.

Another potential delimitation was the fact that the subject population was more diverse than originally hoped due to problems in the recruitment process. However, all of the subjects were good athletes and had similar strength training regimes. Most of the major differences between the subjects seemed to be greatly reduced with the simple normalization techniques described in this chapter.

The elastic bands did not provide a constant unloading force on the subjects throughout the entire movement. Using short bands stretched over a long distance minimized this problem but it was impossible to eliminate completely. Many solutions for unloading were discussed before the experiment, and this procedure was deemed to be the best overall (see Appendix for pilot studies on alternate solutions). The non-constant force issue was addressed by continuously measuring the force of the bands during the movement with a force transducer and including this measurement in the calculations. This was not done in other studies that used a similar protocol (Tsarouchas and Klissouras, 1981).

The length of time that the entire experimental protocol took to perform for each subject could have been a possible source of fatigue, which could preclude the subjects
from producing maximal efforts. This was addressed by giving the subjects at least two minutes rest between trials and by accomplishing the more difficult loaded trials first when the subjects were fresher. None of the subjects complained of fatigue when asked during the experiment.

Because of fatigue, mechanical, and safety issues the order of the trials was not randomized. Randomizing is always desirable in experimental procedures because it reduced the chances that the performance order itself influences the results. As mentioned in this chapter, the above issues outweighed the experimental concerns. There is no way to know for sure that the order of the trials did not influence the outcome of the experiment.

There is no way to determine that the subjects actually performed with maximum effort. They were reminded of this requirement before each trial and were verbally encouraged during the trials.

Most of the potential delimitations in this project were anticipated and compensated for in the experimental protocol or in the analysis. It is believed that the problems in this study are minimal and that the results seen in the next chapter are the true reflection of the biomechanics of the vertical jump during loaded and unloaded conditions.
Chapter 4  Results
The main purpose of this chapter is to present the results of the experiments described in the Methods. The first section verifies the effectiveness of the experimental apparatus and protocol and reveals some basic relationships between many of the variables to be used in the following sections and the loading level. The next section explores some of the possible variable combinations for force and velocity, and the consistency of the results between subjects will be discussed. After that, the two main measuring techniques for collecting force-velocity data will be compared. The fourth section will compare the results to Hill’s curve for force-velocity. Finally, power generated during the takeoffs will be discussed in relation to load, force and velocity. Unless stated otherwise, all of the plots include data for all of the subjects from all of the loading conditions.

4.1 Results of Individual Variables with Changes in Load

This section verified that the methods and protocol used in this experiment were effective in producing a wide range in the main variables by showing how these variables varied with changes in loading condition. These variables were: 1) center of mass velocity, 2) ground reaction force, 3) knee angular velocity, 4) leg extension velocity, and 5) knee torque. Load was varied from approximately 80% of bodyweight unloading to approximately 125% of bodyweight additional loading during the series of noncountermovement jumps. The desired outcome was to produce as wide of a range of forces, velocities, and torques as possible.

The loading had a predictable effect on the jumping performances of the subjects. Figure 4.1 (A) shows how the velocity of the center of mass varied for one subject with changes in external load. A general effect of increasing the load was to increase the total time of the movement, i.e. the time from the onset of movement to the instant when the feet lost contact with the ground (Figure 4.1 B). As shown by Figure 4.1 (C), the time from the onset of the movement to the achievement of maximum velocity increased as the external load increased. The time between the instant of maximum velocity and the instant when the feet lost contact with the ground increased slightly with increased load (Figure 4.1 D).
Figure 4.1. Typical COM velocity vs. time relationship for one subject for all loading conditions (A). Curves are aligned in time at the takeoff point. Normalized time vs. normalized load plots: total time (B), time from initial move to max COM velocity (C), time from max COM velocity to takeoff (D).
The maximum velocity of the center of mass decreased as the load increased in a nearly linear fashion. The most unloaded condition increased the maximum velocity by approximately 18% when compared to a normal, noncountermovement vertical jump, and the highest barbell weight decreased the maximum velocity by approximately 45% (Figure 4.2).

The ground reaction force-time curve typically had two peaks (Figure 4.3). The first peak was always smaller than the second was; therefore, the second peak was where the maximum force occurred. The time between these two peaks decreased with decreased load to nearly zero in the most unloaded conditions, resulting in a ground reaction force-time curve with only one peak.

Figure 4.2. Normalized maximum center of mass velocity vs. normalized external load relationship.

Figure 4.3. Typical ground reaction force vs. time relationship for one subject for all loading conditions. Curves are aligned in time at the takeoff point.

\[ V = 0.0657L^2 - 0.2307L + 1.0052 \]
The maximum rate of force production decreased with increased load (Figure 4.4 A); however, the time when the maximum occurred varied from trial to trial and subject to subject, i.e. sometimes it occurred near the beginning of the movement and sometimes it occurred later in the movement. Average rate of force production also decreased with increased load (Figure 4.4 B).

![Figure 4.4. Normalized maximum (A) and average (B) rate of force production for ground reaction forces as a function of normalized external load.](image)

The ground reaction force that corresponded with the instant of maximum velocity increased nearly linearly as the load increased. Maximum unloading of the subjects resulted in a decrease in ground reaction force of approximately 75% when compared to a normal jump, and maximum barbell loading increased the ground reaction force by approximately 125% (Figure 4.5).

![Figure 4.5. Ground reaction force corresponding to the maximum center of mass velocity vs. load relationship.](image)
The maximum knee angular velocity generally decreased as the load increased, but the data were scattered (R=0.4942) and a specific trend was difficult to see (Figure 4.6 A). When fitted with a second-degree polynomial, individual subjects produced relationships which were typically concave downward, but the shape of the curves were only generally similar, ranging from a nearly linear decreasing line (Figure 4.6 B) to an increasing-decreasing curved line (Figure 4.6 C).

Figure 4.6. Normalized maximum knee angular velocity vs. normalized load relationship (A). Maximum angular velocity vs. external load examples for two different subjects (B and C).
Leg extension velocity decreased nearly linearly with increasing load consistently from subject to subject. Normalizing and combining the data for all of the subjects confirmed this trend (Figure 4.7 A). Knee torque varied widely from subject to subject as load increased (R=0.0752). There was no discernable pattern in the data (Figure 4.7 B).

![Figure 4.7. Normalized maximum leg extension velocity (A) and normalized knee moment corresponding to the maximum knee angular velocity (B) vs. normalized load relationships.](image)

EMG was plotted for each muscle for two subjects in the maximally loaded condition, the bodyweight condition, and the maximally loaded condition (Figure 4.8, for example, and Appendix A). EMG will be discussed as needed.

![Figure 4.8. EMG for Subject A, most loaded condition.](image)
4.2 Parameter Combinations for Exploring Force-Velocity Relationships

As illustrated by the Literature Review, there are virtually an infinite number of combinations for exploring force-velocity relationships in human movements. This section presents the results for several of the possible combinations. Some combinations were analyzed for one or two subjects and rejected, but some were chosen for a more detailed analysis. The results presented in this section are for force plate analyses only (unless video data were required to calculate the variable). The results using the video data follows in Section 4.3. The combinations described here are: 1) maximum center of mass velocity and corresponding ground reaction force, 2) maximum ground reaction force and maximum center of mass velocity, 3) maximum ground reaction force and corresponding center of mass velocity, 4) maximum knee angular velocity and corresponding ground reaction force, 5) maximum leg extension velocity and corresponding ground reaction force, 6) maximum knee angular velocity and corresponding knee moment, and 7) maximum leg extension velocity and corresponding knee moment.

The ground reaction force that corresponded with the instant of maximum velocity of the center of mass was plotted against maximum velocity of the center of mass for each loading condition. Normalized ground reaction force decreased nearly linearly as velocity increased (Figure 4.9 C). However, there appeared to be two patterns in the individual subjects. A few subjects had a concave upward curve as illustrated by Figure 4.9 (A), but most had a concave downward curve (Figure 4.9 B).

Maximum ground reaction force was plotted against the maximum center of mass velocity and the corresponding center of mass velocity for each condition for one subject (Figure 4.10). Ground reaction force again decreased nearly linearly as velocity increased in both cases, indicating that the ground reaction force-center of mass velocity relationship is robust. The only difference between the plots is the range of data that is plotted because of how the ground reaction force and velocity curves line up in time. When maximum velocity is used as the independent variable, ground reaction force is less than when ground reaction force is the independent variable, in which case velocity is decreased. There does not appear to be any advantage to analyzing the data using
Figure 4.9. Ground reaction force corresponding to the maximum center of mass velocity vs. center of mass velocity relationships. Plots showing examples of two typical subjects (A and B). Normalized plot with all of the subjects combined (C).

Figure 4.10. Maximum ground reaction force vs. maximum center of mass velocity (A) and corresponding center of mass velocity (B) from the force plate analysis.
maximum ground reaction force as the independent variable in a multi-joint movement since maximum ground reaction force does not guarantee maximum force in the muscles at that instant. Plotting maximum ground reaction force with maximum velocity does not appear to have any real meaning since the two do not occur at the same instant in time. Therefore, both of these analysis options were rejected and not further analyzed.

The corresponding ground reaction force was plotted against the maximum knee angular velocity and the maximum leg extension velocity. The ground reaction force decreased with increasing angular velocity as expected, however, it did not seem to do so in a consistent way across the subjects (the data were very scattered). When the data were normalized and combined for all of the subjects, the data were still scattered (R=0.413), but the resulting best-fit polynomial indicated a concave upward curve (Figure 4.11 A). The ground reaction force decreased with increasing leg velocity very regularly for all of the subjects, which was confirmed when the data were normalized and combined with the best-fit polynomial indicating a nearly linear, but slightly concave upward, curve (Figure 4.11 B).

![Graph A](image1.png)

\[ F = 1.0856V^2 - 2.9353V + 2.7899 \]

\[ R = 0.41 \]

![Graph B](image2.png)

\[ F = 0.3443V^2 - 1.374V + 1.9971 \]

\[ R = 0.88 \]

**Figure 4.11.** Normalized ground reaction corresponding to maximum knee angular velocity vs. normalized maximum knee angular velocity relationship (A) and normalized ground reaction force corresponding to maximum leg extension velocity vs. normalized maximum leg extension velocity relationship (B).
The corresponding knee moment was plotted against maximum knee angular velocity and maximum leg extension velocity. As might be expected from the results of the moment-load analysis, these results were no better and showed virtually no trends (Figure 4.12). These parameter combinations were rejected for further analysis.

![Figure 4.12. Normalized knee moment corresponding to maximum knee angular velocity vs. normalized maximum knee angular velocity relationship (A) and normalized knee moment corresponding to maximum leg extension velocity vs. normalized maximum leg extension velocity relationship (B).](image)

This section presented the results of several variable combinations for force and velocity. Corresponding ground reaction force decreased with increasing maximum center of mass velocity, maximum knee angular velocity, and maximum leg extension velocity. Knee moment did not vary regularly with changes in maximum knee angular velocity or maximum leg extension velocity and these combinations were rejected from further analysis. Combinations involving maximum ground reaction force were shown to be either meaningless or redundant and were also rejected.

### 4.3 Force-Velocity Comparison Between Force Plate and Video Analysis

Two methods were used in this experiment to calculate or measure the ground reaction force and the center of mass velocity. The first used ground reaction force measured with the force plate to calculate the velocity of the center of mass. The second used video position data to calculate velocity and ground reaction force. Using video data can be problematic since differentiation is used in the calculation of velocity and
acceleration, which in turn is used to calculate ground reaction forces. Differentiation magnifies any noise present in the data, making the resulting curves increasingly noisy. Filtering can decrease this problem, but excessive filtering can cut off maximum points in the data that may be real data. Therefore the compromise solution described in the Methods was used, which should make the data from the two methods as similar as possible, unless there is an underlying difference in the two methods. The results from the two methods are compared in this section.

The ground reaction force corresponding to the maximum center of mass velocity varied very similarly in both the force plate and video methods across increasing loads (Figure 4.13). As the load increased, ground reaction force increased nearly linearly in both methods. The force determined by the force plate method was higher than the force determined by the video method for every subject. However, after combining and normalizing the data for all of the subjects, the two methods produced nearly identical results (R=0.95).

The maximum velocity of the center of mass decreased in a nearly linear fashion as load increased in both methods. The force plate velocity was higher for a given load than the video velocity for every subject, but at high velocities this difference was decreased. When the data from all of the subjects was combined and normalized, the two curves continued to be very similar (R=0.95), especially at high velocities (Figure 4.14 A).
The force-velocity relationship for the two methods were both nearly linear for all subjects, but the ground reaction force from the force plate method was nearly always greater than the video method at a given velocity level. When the data was normalized and combined for all subjects, the curves were nearly identical (R=0.95). The only difference between them was the slight difference in their shape tendencies. The force plate force-velocity curve tended to be concave downward, and the video force-velocity tended to be concave upward (Figure 4.14 B).

4.4 Force-Velocity Relationships and Hill’s Curve

As mentioned in the Literature Review, the author of nearly every force-velocity study undertaken compared the results to classic results of Hill, namely the famous Hill’s curve and/or Hill’s force-velocity equation. A curve can be considered Hill-like if it is descending and concave upward. This was described analytically by Hill by fitting the curve hyperbolically. At the time of Hill's initial experiments (1938), he placed special meaning on the resulting constants because they influenced the selection of the hyperbola as the equation to use to best describe the data. Since then, Hill and others have determined that the constants do not have special meaning and that Hill’s curve is just another method for fitting and describing the data (Hill, 1970; Epstein and Herzog, 1998;...
Phillips and Petrofsky, 1983). However, the general shape of the force-velocity curve itself seems to be generally consistent across many different experimental models and protocols, so Hill’s curve is still used today by many authors for comparison and curve fitting.

Hill’s force-velocity equation was fit to the eight sets of data in this experiment that varied in a predictable manner: ground reaction force and load plotted against center of mass velocity for the video method and the force plate method, knee angular velocity, and leg extension velocity (Figure 4.15 and Figure 4.16). The resulting values for the constants $a$, $b$, and $F_o$ varied widely from curve to curve, as did the ratio $a/F_o$. These results are summarized in Table 4.1. A striking feature of these results was the fact that the constant $a$ was much greater than $F_o$ for all but two of the curves ($a/F_o > 1$). This is unusual and is different from the classical Hill model.

<table>
<thead>
<tr>
<th>Combination</th>
<th>a</th>
<th>b</th>
<th>$F_o$</th>
<th>$a/F_o$</th>
<th>$R_H$</th>
<th>$R_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max V vs Load - Force Plate</td>
<td>31.12</td>
<td>6.89</td>
<td>4.42</td>
<td>7.04</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td>Max V vs Load – Video</td>
<td>43.25</td>
<td>13.16</td>
<td>3.06</td>
<td>14.13</td>
<td>0.90</td>
<td>0.92</td>
</tr>
<tr>
<td>Max Ang V vs. Load</td>
<td>49.77</td>
<td>9.57</td>
<td>4.57</td>
<td>10.89</td>
<td>0.43</td>
<td>0.49</td>
</tr>
<tr>
<td>Max Ext V vs. Load</td>
<td>41.19</td>
<td>16.04</td>
<td>2.44</td>
<td>16.88</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td>GRF vs Max V - Force Plate</td>
<td>26.41</td>
<td>8.29</td>
<td>4.25</td>
<td>6.21</td>
<td>0.85</td>
<td>0.87</td>
</tr>
<tr>
<td>GRF vs Max V – Video</td>
<td>5.72</td>
<td>1.78</td>
<td>4.56</td>
<td>1.25</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>GRF vs Max Ang V</td>
<td>0.02</td>
<td>0.24</td>
<td>4.98</td>
<td>0.004</td>
<td>0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>GRF vs Max Ext V</td>
<td>0.92</td>
<td>1.75</td>
<td>2.05</td>
<td>0.45</td>
<td>0.88</td>
<td>0.88</td>
</tr>
</tbody>
</table>

$R_H$ – Intra-class correlation coefficient for Hill’s fit  
$R_p$ – Intra-class correlation coefficient for polynomial fit
Figure 4.15. Hill fits for maximum center of mass velocity, force plate analysis (A), maximum center of mass velocity, video analysis (B), maximum knee angular velocity (C), and maximum leg extension velocity (D) vs. external load.
It was immediately apparent that the concavity of the Hill’s curve fits was different from the polynomial fits in most of these curves. The goodness of fit analysis indicated that the Hill fits were statistically worse than their polynomial counterparts by a small amount, except in the cases of ground reaction force vs. center of mass velocity (video) and vs. maximum angular velocity (Table 4.1).

Figure 4.16. Hill fits for corresponding ground reaction force vs. maximum velocity, force plate analysis (A); maximum velocity, video analysis (B), maximum knee angular velocity (C), and maximum leg extension velocity (D) vs. normalized external load.
4.5 Power-Velocity and Power-Force Relationships

Power is another way to represent force-velocity relationships. Power was calculated in three ways in this study: 1) corresponding ground reaction force × maximum center of mass velocity, 2) maximum knee angular velocity × corresponding knee moment, and 3) corresponding ground reaction force × leg extension velocity. Power (1) vs. load, power (1) vs. ground reaction force, power (1) vs. center of mass velocity, power (2) vs. maximum angular velocity, power (2) vs. maximum leg extension velocity, power (3) vs. load, and power (3) vs. maximum leg extension velocity relationships were all investigated. The results of these relationships will be reported in the order in which they were listed above.

The power that corresponded to the instant of maximum velocity in each condition increased in a concave downward parabolic fashion with increasing load (Figure 4.17 A). When maximally unloaded, the power was approximately 70% less than at the zero load condition. The maximum power was reached at approximately 85% of body weight of additional loading (185% total weight) and was approximately 20-30% higher than in a normal body weight jump. As the loading increased beyond 180%, the power began to decrease slightly. Force plate and video analyses produced similar curves, with the video data giving lower power values for a given load in almost every

![Figure 4.17. Normalized power corresponding to the maximum center of mass velocity vs. external load (A) and normalized ground reaction force corresponding to the maximum center of mass velocity (B) relationships for force plate and video analysis methods.](image)

65
case for all subjects. When the data were normalized and combined for all subjects, the
two curves were very similar (R=0.95), especially in the unloaded region. When power
was plotted against ground reaction force, nearly identical results to the power-load plot
were produced (Figure 4.17 B).

Power decreased with increased center of mass velocity in a concave-downward
parabolic fashion (Figure 4.18).
Power was highest in the low velocity
region, which corresponds to high
loads and ground reaction forces. The
lowest power was associated with the
high velocity region of the curve,
which corresponds to low loads and
ground reaction forces. Maximum
power occurred at approximately 60-
75% of the velocity of a normal body
weight jump. The force plate and
video methods produced similar
results (R=0.88).

Knee power calculated by multiplying maximum knee angular velocity by the
corresponding knee angular velocity was plotted against maximum knee angular velocity
(Figure 4.19 A) and maximum leg extension velocity (Figure 4.19 B). Both plots showed
virtually no relationship between the variables.

Power calculated by multiplying maximum leg extension velocity by the
corresponding ground reaction force decreased with increasing load (Figure 4.20 A) and
increased with increasing extension velocity (Figure 4.20 B). This trend is the opposite
of the curves highlighted above (Figure 4.7 and 4.18).

This experiment did not cover the entire theoretical range of forces and velocities
that make up a complete force-velocity curve because of the physical limitations of the
subjects as discussed previously. If this were possible, the power curve would be
expected to start at zero at zero load, force, or velocity, increase to a peak, then decrease
to zero again at maximum load, force, or velocity. Since this situation was not possible, it was interesting to determine where on the curve this data lay, i.e. on the ascending or descending portion of the power curve. The data above indicated that the power in this experiment lay in the ascending portion of the power-load and power-force curves and in the descending portion of the power-velocity curve (except when leg extension velocity was multiplied by the ground reaction force, see above). The peak power appeared to be included in the range of data available.

Figure 4.19. Knee power corresponding to the maximum knee angular velocity vs. normalized maximum knee angular velocity relationship (A) and knee power corresponding to the maximum leg extension velocity vs. normalized maximum leg extension velocity relationship (B).

Figure 4.20. Normalized power corresponding to maximum leg extension velocity vs. normalized external load (A) and normalized leg extension velocity (B) relationships.
Chapter 5  Discussion
There were several goals to be accomplished in this study. The first and most important goal was to determine the force-velocity and power-velocity relationships for a multi-joint movement, in this case a noncountermovement vertical jump. Secondly, a goal was to find the best parameter combinations and methods for determining the force- and power-velocity curves. The third goal was to investigate how favorably multi-joint force-velocity relationships compare to the classic Hill force-velocity relationship, i.e. are the curves ‘Hill-like?’ (the expression Hill-like will be used to describe negative sloped, upward concavity of the curves, with $0 < a/F_o < 1$). The final goals were to determine whether the experimental power-velocity curves corresponded with the ascending or descending portion of the theoretical power-velocity curve and at what load and center of mass velocity the maximum power occurred.

This discussion will be presented as follows: A general discussion of topics concerning this investigation will be presented initially. Secondly, the individual parameters involved in this study and how they vary with changes in load will be analyzed. Next, the various force-velocity parameter combinations and methods for data collection will be discussed and compared to previous research. After that, the discussion will center on the Hill-like properties of the force-velocity relationships in this study, followed by a theoretical mechanical analysis to attempt to explain why there may be differences from the classic curves. Finally, the discussion will move to the power-velocity relationship in multi-joint movements.

5.1 General Considerations

When studying concentric force-velocity relationships, it is ideal to have the widest possible range of experimental conditions. The experiment ideally has a condition where zero velocity is measured, another condition where zero force is measured, and as many conditions as possible between those extremes. This situation allows for the study of a ‘complete’ concentric force-velocity curve.

This experiment had several limitations due the experimental protocol. Because of safety concerns, it was not possible to load the subjects until they could produce no movement. Likewise, it was impossible to fully unload the subjects past their body
weight since they would have been suspended before the attempt and jumping would have been impossible. The range of force and velocity data in this study is larger than in any other published study, but the range is still only a part of the theoretical force-velocity curve.

The height of the jumps across the experimental conditions seemed to change, as might be expected. As the load was increased, the height of the jump decreased. However, some of the perceived changes in jump height may have been artifacts of the experimental protocol. The barbell in the loaded conditions was “caught” in mid flight to add to the comfort and safety of the subjects upon landing. Therefore the subjects became two separate masses during the flight phase and the subject landed with a different mass than at takeoff. Similarly, the subjects in the unloaded conditions were suspended from above with elastic tubing, which affected the natural flight of the subjects. Therefore, no attempt was made to calculate the height of the jump or make any evaluation of the flight time. Everything after the time of takeoff was ignored and was not included in the discussion.

The choice of the maximum center of mass velocity as the time when most of the variables were compared has some interesting ramifications. As mentioned briefly in the Methods, the instant of maximum center of mass velocity corresponds to the instant when the acceleration of the center of mass is zero. This means that the ground reaction force is equal to the body weight and the additional load (or subtracted load in the unloaded cases). This moment in time was chosen in part because many of the classical force-velocity studies controlled the motion such that the acceleration was zero, perhaps making the current study and the classical studies more comparable. However, the classical studies ignored the mass of the muscle itself, and the force generated by the muscle against the external load was the only force involved. In the current study, even the no-load, bodyweight conditions required the leg muscles to overcome resistance.

There is a key difference between the analyses involving load and those involving ground reaction force. Although they have the same units, the load is simply the weight of the barbell or the force produced by the elastic bands. The ground reaction force was measured at the instants of maximum velocity and is a measure of the force produced by
the subject in addition to the load. Since the acceleration of the center of mass is zero at
the instant of measurement in some cases, the load and the ground reaction force are
different only by the body weight. Therefore, force-velocity and load-velocity curves in
this study are not the same and will be treated separately.

5.2 Individual Variables and Load

The goal of this section is to show that by changing the external load under which
the subjects jumped, satisfactory changes occurred in the other relevant variables
involved in studying force-velocity relationships and to point out some interesting
phenomena in these changes. In this experiment, all of the variables were ‘parametric’
(Zatsiorsky, 1995). Parametric in this context means that one instant in time from each
trial is isolated, and then all of the conditions are combined to form a ‘parametric’
relationship in which the same condition from each trial is compared over the entire range
of trials (except when the discussion focuses on the changes of a particular variable
during specific trials). The following variables will be discussed: 1) time and distance
moved, 2) velocity of the center of mass, 3) ground reaction force, 4) angular velocity,
and 5) leg extension velocity, 6) knee moment, and 7) EMG.

5.2.1 Time and Distance Moved

As might be expected, as the load was increased, the total time of movement in an
individual trial increased (Figure 4.1 B). If the general force-velocity relationship holds,
then as the load increases, the resulting velocity of the center of mass should decrease at
all times during the movement. This is generally true from looking at Figure 4.1 (A).
The time from the instant of initial movement to the instant of maximum center of mass
velocity vs. load curve closely resembled the total time vs. load curve (Figure 4.1 C).
This is not surprising since the majority of the movement occurred before the
achievement of maximum center of mass velocity.

The time from the instant of maximum center of mass velocity to the instant of
takeoff increased a small amount with increased load (Figure 4.1 D). However, the
relative change for the highest loads was as much as two times that for a bodyweight
vertical jump. This indicated that the body position at maximum center of mass velocity
may have been approximately constant over the loading conditions since a decrease in velocity and a constant distance from the maximum velocity position to the takeoff position causes an increase in time. In fact, the distance the center of mass moved from the beginning of the movement to the instant of its maximum velocity was not well correlated as illustrated by Figure 5.1 (A). However, the distance from the hip to the ankle decreased as load increased over the same time period (Figure 5.1 B), indicating that the center of mass had reached its maximum velocity before the legs were fully extended in the high-load conditions. This result is similar to previous work (Tsarouchas and Klissouras (1981).

![Figure 5.1](image.png)

**Figure 5.1.** Distance moved from initial posture to instant when maximum center of mass velocity was reached vs. load relationship: center of mass (A) and leg distance (B).

### 5.2.2 Center of Mass Velocity

When the maximum velocity of the center of mass was plotted against load, a nearly linear decreasing relationship was found, with a slightly concave downward trend (Figure 4.2). This was different from previous work in which velocity decreased in a more curved, or concave upward, way as load increased (Tsarouchas and Klissouras, 1981; Viitasalo, 1985). These studies found this relationship to be more Hill-like without fitting the curve mathematically. The magnitudes of the velocities in this study compared favorably to other studies in the literature (Bosco, et al., 1995; Tihanyi, et al., 1987), but
Tsarouchas and Klissouras (1981) reported much higher velocities for both loaded and unloaded jumps.

5.2.3 Ground Reaction Force

The maximum ground reaction force increased with increasing external load (Figure 4.5) and the force-time curves were similar to that reported in the literature (Tsarouchas and Klissouras, 1981). As illustrated by Figure 4.3, these maximums always occurred late in the movement after an initial rise, followed by a leveling off, or decrease, then the final rise to the peak. The maximum and average rates of force production both decreased with increased load (Figure 4.4). Maximum rate of force production occurred at different points in the movement for different subjects and for different conditions, sometimes during the first force rise, sometimes during the second. This may explain why the data is noisier for the maximum rate of force production as compared to the average. Maximum rates of force production occurring at different times may be an indication of different styles of jumping between subjects, or it could show that some of the subjects could not consistently perform the jumping task over the range of loads. Since the time of the performance and the maximum ground reaction force both increased as load increased, and the time of maximum force was always late in the movement, it makes sense that the average rate of force production should decrease with load as well.

The dual peak in the ground reaction force-time curve is a typical feature in many vertical jump studies with much discussion as to what causes this phenomenon. One popular explanation is that the initial peak is due to the extension of the hips (i.e. raising of the trunk) and that the other peak is due to the extension of the legs (Miller and East, 1976). This study lends some support to this idea since the double peak seems to decrease and eventually almost disappear with more and more unloading for some subjects. In the loaded conditions, the weight was placed on the subject’s shoulders. If the initial peak is due to the raising of the trunk, the initial peak should be more pronounced since the mass of the trunk plus the load is increased. In the unloaded conditions, the elastic bands were attached to the subject at the upper part of the back. More upward band force would assist the trunk in moving upward and would decrease the amount of force needed to raise the trunk when compared to the loaded jumps. The
initial peak should then be less pronounced. The data in the current study certainly does not show these trends conclusively, but may lend support to a common theory. However, the time of maximum center of mass velocity occurs later in the movement than the maximum leg extension velocity implying that the legs extend first, then the trunk (see Section 5.7.1). This ‘conflict’ could be a potential avenue for future research.

The ground reaction force that corresponded with the instant of maximum velocity increased linearly with increased load (Figure 4.5). This is similar to data reported by Nelson and Martin (1985). This is not surprising since the two parameters are closely related and in fact only differ by the body weight of the particular subject as stated earlier. The ranges for ground reaction force in the loaded configuration are consistent with other studies in the literature (Tihanyi, et al., 1987; Bosco and Komi, 1979; Viitasalo, 1985). Very few researchers have examined unloaded jumping, but the ranges in this study are greater than Tsarouchas and Klissouras (1981) who unloaded to about 50% of bodyweight.

5.2.4 Knee Angular Velocity and Leg Extension Velocity

The maximum knee angular velocity-load relationship was very scattered, but indicated a generally decreasing, concave downward shape (Figure 4.6). This was different than previously reported data in vertical jumping, which was reported to be more Hill-like, with concave upward curves (Bosco and Komi, 1979; Viitasalo, 1985); although Bosco and Komi (1979) fit their curve by hand and their data were not clearly concave upward and could possibly be favorably compared to this study. Leg extension velocity had not been previously reported in the literature, but it appeared to be an easy and reliable way to report extension velocity of the knee joint in a vertical jump. Leg extension velocity decreased nearly linearly with increased load, following the trend of the center of mass velocity (Figure 4.7 A). Once again, the curve was not as Hill-like as expected.

Knee angular velocity was expected to closely follow other velocity patterns since the knee extensors are sometimes considered the main muscle movers in the vertical jump. Bosco and Komi (1979) assumed this to be true and simply plotted average ground reaction force (assumed to be closely related to the knee extensor force) and average knee
angular velocity measured with a goniometer for vertical jumps with various additional loads.

5.2.5 Knee Moment

Knee moment corresponding to the maximum knee angular velocity did not appear to change in any discernable pattern as load increased (Figure 4.7 B). This result is probably due to the fact that knee moment is affected a great deal by the posture and the coordination of force production in a multi-joint movement such as the vertical jump. The subjects were most likely in a slightly different position at the instant of maximum knee angular velocity in each condition. Even if the maximum angular velocity occurred at a predictable time in the movement (which it did), the knee moment could vary a great deal. This effect can be greatly exaggerated by additional weight on the shoulders of the subjects (Zatsiorsky, 1995). Possibly because of this effect, knee moment has not been reported in the vertical jump force-velocity literature, but it is very common in the single-joint literature (see Section 2.1.3.3). Please refer to Section 5.6.3 for a more detailed discussion on joint moments and position affects.

5.2.6 EMG

EMG was originally planned to be a large part of this experiment, but full analysis proved to be beyond the reasonable scope of the project. Therefore only two subjects were analyzed and the data for the most loaded, bodyweight, and most unloaded conditions were presented in Figure 4.8 and Appendix A. The loading did not appear to change the activation patterns to a great extent within a subject, but the gluteus maximus seemed to be activated earlier and more continuously in Subject B. The leg extensors (rectus femoris and vastus lateralis) appeared to be the main muscles in use during the movement for Subject A. They were involved for nearly the entire movement, except near the end where the gastrocnemius, gluteus maximus and the biceps femoris became more prevalent. This seems to corroborate the work of several researchers who studied the role of biarticular muscles in the vertical jump (Prilutsky and Zatsiorsky, 1994; Bobbert, et al., 1986; van Soest, et al., 1993; van Leeuwen and Spoor, 1992; Jacobs, et al.,1996; Gregoire, et al., 1984). They have shown that the force produced in the knee
extensors can be transferred to the hip and ankle extensors and vice versa through the biarticular knee and flexors and ankle extensors (hamstring and calf muscles).

A common theory is that the force in vertical jumping is transferred from the proximal joints to the distal ones, i.e. the hip extensors act first, then the knee extensors, then finally the ankle extensors (Bobbert and van Ingen Schenau, 1988). From a cursory look at a limited amount of EMG data it appears that the knee extensors act first and then the hip and ankle extensors are used primarily near the end of the movement for one subject, but the knee and hip extensors seemed to work more concurrently in the other subject. There appears to be more than one effective pattern of activation in the vertical jump. This could be a direction for future research.

It is important to remember that EMG signal strength varies depending on the depth of the muscle, the skin thickness, and the quality of the electrode application so relative strength of signal does not necessarily indicate level of muscle activation. This discussion was limited to activation timing and sequences. A more detailed study of these patterns is required before more definitive conclusions can be made.

This section discussed how and why the main variables in this study varied with changes in external load. The experimental protocol and procedures were shown to be adequate enough to produce the widest range of forces and velocities seen in the literature for vertical jumping. Nearly all of the variables chosen for discussion varied in a predictable way as load varied, however, several variables were shown to be poor choices for force-velocity study due to possible joint position affects.

5.3 Parameter Combinations for Studying Force-Velocity Relationships

This section discusses the results of combining the previously discussed variables in the most informative ways that illustrate the force-velocity characteristics of the vertical jump under varied loading conditions. As has been mentioned previously, the choices for the selection of variables in parametric, multi-joint, force-velocity studies are innumerable. This study is no exception. Although many parameter combinations were examined in this study, several of them were rejected when no discernable trends were seen, e.g. knee moment vs. knee angular velocity, or if the parameters did not seem to
add to the analysis, e.g. maximum force vs. maximum velocity (see Results for details). These results will not be included in the discussion.

The only truly independent variable in this experiment was external load. All other calculated variables are dependent on external load, so combinations involving these parameters indicate relationships only; cause and effect cannot be implied. The following relationships will be discussed: 1) ground reaction force corresponding to the instant of maximum center of mass velocity vs. maximum center of mass velocity, 2) ground reaction force corresponding to the instant of maximum knee angular velocity vs. maximum knee angular velocity, and 3) ground reaction force corresponding with the instant of maximum leg extension velocity vs. leg extension velocity.

5.3.1 Ground Reaction Force vs. Center of Mass Velocity

The corresponding ground reaction force when plotted with the maximum velocity of the center of mass had two patterns across the subjects (Figure 4.9 A and B). Most of the subjects produced concave downward trends, which shows up in the normalized, combined plot (Figure 4.9 C). A few of the subjects had a concave upward trend. The concave upward trend is consistent with the results previously reported in the literature (Tsarouchas and Klissouras, 1981; Tihanyi, et al., 1987) and with Hill-like curves (see below), but the concave downward trend differs from most of the literature. Bosco, et al. (1995) found linear average force-velocity relationships. Wit, et al. (1993) found that the isokinetic knee flexion torque and angular velocity relationship was slightly concave downward for one group of subjects. This was the only favorably comparing result found in the literature.

5.3.2 Ground Reaction Force vs. Knee Angular Velocity and Leg Extension Velocity

Corresponding ground reaction force when plotted with maximum knee angular velocity and leg extension velocity both indicated decreasing, slightly concave upward relationships (Figure 4.11). These showed the same trends as previously reported results in the literature (Bosco and Komi, 1979) and were more Hill-like than the force-velocity relationship reported above. The selection of knee angular velocity assumed that the leg extensors were the primary mover in the vertical jump and that the angular velocity of the
knee joint was related to the linear velocity of shortening of the leg extensors. Since the leg extensors were assumed to be the primary muscle group, the vertical ground reaction force was assumed to be related to the force production in the leg extensors (Bosco and Komi, 1979). This is corroborated on some level by the EMG results discussed above.

This section showed that the corresponding ground reaction force vs. center of mass velocity relationship did not compare favorably with the prevailing literature, but that ground reaction force vs. knee angular velocity and extension velocity relationships compared favorably to much of the force velocity literature for multi-joint movements.

5.4 Force Plate and Video Method Comparison

This part of the project came about as a result of the need to combine force plate and video data in order to calculate parameters such as knee moment and knee power. Theoretically, all of the parameters calculated with either method should be the same. However, in actual practice it is rare for digitized video data to accurately reproduce force plate and velocity data except in very simple, slow movements. Data acquired from video tends to be comparatively noisy and therefore requires significant filtering which can sometimes clip off real data during the filtering process. This study was dependent on maximum values for the instant of analysis in each load condition, so the compromise solution outlined in the Methods was used because clipping of the data was undesirable. The analysis process required the data to be differentiated twice in order to calculate linear and angular acceleration. This process magnified any noise artifacts, making the acceleration data, and therefore the calculated ground reaction force, more unreliable than direct measurement from a force plate.

Despite the aforementioned complications, when single data points from each loading condition were selected and combined to form the parametric load-velocity and force-velocity relationships for each subject, both methods produced very similar results (most of the intra-class correlation coefficients were 0.9 or above). The video data was consistently lower at any given instant than the force plate data for both maximum center of mass velocity and ground reaction force; but when the data was normalized, the two methods produced nearly identical results. Interestingly, the video data produced a
ground reaction force-center of mass velocity relationship that was more Hill-like than the plot produced with the ground reaction force data. The fact that the two methods tended to agree was important because the methods involving the inverse dynamics analysis of the knee joint used a combination of force plate data and differentiated video position data to calculate knee moment.

5.5 Comparisons to Hill’s Curve

As has been mentioned in the previous sections, the force-velocity and velocity-load curves from this study in many cases differed qualitatively from those published in the literature. All of the authors of vertical jump force-velocity studies indicated that parametric force-velocity relationships in vertical jumping under varying load were Hill-like except Bosco, et al. (1995), who showed a more linear result. As previously described, Hill-like curves were defined as a descending, concave upward curve with the \( a/F_o \) ratio between zero and one. This relationship is often fit hyperbolically with Hill’s curve to analytically describe the data. All of the force-velocity relationships in this study were descending curves, as expected. However, an initial fit with simple second-degree polynomials indicated that several of the curves were not Hill-like in that they tended to be concave downward, not concave upward.

In spite of their visual differences from the classical Hill curves, all of the relevant curves in this study were approximated with Hill’s force-velocity equation and the appropriate constants were calculated and reported in the Results. In the cases where the initial polynomials were concave upward (corresponding ground reaction force vs. maximum center of mass velocity (video), vs. maximum angular velocity, and vs. leg extension velocity), the Hill’s fit was nearly identical to initial fit and the correlation coefficients reflected this (Table 4.1). In the other cases, the Hill fit was nearly linear and did not fit the data quite as well as the polynomial did, as reflected by the correlation coefficients (Table 4.1). The values for the constant \( a/F_o \) in these curves did not compare favorably with the literature. Most authors who calculated \( a/F_o \) found their ratio to be between zero and one (Jaric, et al., 1986; Tihanyi, et al., 1987; all other single muscle/fiber studies) with a few exceptions. Ameredes, et al. (1992) found \( a/F_o \) ratios of
1.0 to 1.6 in the gastrocnemius of dogs, and Chow and Darling (1999) found $a/F_o$ values to typically be between zero and one, but reported values as high as 5.05 for the wrist flexors. Therefore, it must be concluded that for the variable combinations described above as concave downward and/or with the ratio $a/F_o$ greater than one, Hill’s force-velocity equation was not the best way to describe the data. As mentioned in the previous chapters, Hill’s curve was just another way to describe the force-velocity relationship and did not have any additional intrinsic meanings. Therefore, any other fit that described the data best was satisfactory. The next section will discuss in detail the possible theoretical reasons why multi-joint force-velocity curves may not follow the classic force-velocity curve as described by Hill.

5.6 Theoretical Mechanical and Physiological Explanations

The previous sections have shown that the results of this study, for many of the parameter combinations, were qualitatively different than the classical Hill’s curve. The purpose of this section is to shed light on the possible reasons why this may have happened from a theoretical, mechanical point of view. It was beyond the scope of this project to attempt to create a model that would combine all of these factors to predict the current results; however, it was constructive to analyze potential reasons for the differences. Each critical point in the analysis will be discussed individually; then all parts of the analysis will be combined, and the force and velocity will be traced from the muscles to the effective force acting at the center of mass and the center of mass velocity.

Wilkie (1950) stated that some very strict guidelines should be followed when studying parametric force-velocity relationships in human muscle:

1) Geometrically simple joint.

2) Movement should involve as few muscles as possible with a small distance from the joint center to the muscle insertions.

3) The movement should not disturb the rest of the body.

4) The movement should be reproducible (i.e. easy task).
None of these requirements are fulfilled in loaded and unloaded vertical jumps: 1) not only is there more than one joint involved, but the joints are complex and are extended at different rates at different times, 2) the movement involves many muscles, both agonist and antagonist, 3) the entire body moves, and 4) the results of the pilot studies indicated that maximum center of mass velocity was reproducible over several trials, but that does not imply that all of the parameters leading to the maximum velocity were exactly reproduced from trial to trial (as can be seen by the distance that the center of mass traveled during each of the trials (Figure 5.1 A), for example).

Intuitively, force should generally decrease as velocity increases as seen in everyday experiences with weight lifting and other loaded movements. The results of the current experiment confirmed this general relationship. However, it was not surprising that the exact shape of the force-velocity curve was not the same as in single muscle/fiber or single joint studies because all of the requirements listed by Wilkie could not be controlled in multi-joint movements. Perhaps it is more surprising that, with a few exceptions, most authors studying multi-joint force-velocity relationships have compared their data favorably with Hill’s curve (see Section 5.5), despite the multi-joint problems.

The following sections identify and discuss the main factors in force-velocity relationships and how they might contribute to differences between Hill-like curves in simpler models and non-Hill-like curves in the present multi-joint movements. The main factors to be discussed are 1) muscular factors, 2) factors involving individual and multiple joints, and 3) factors involving joint position.

5.6.1 Muscular Factors

This analysis starts with the assumption that every muscle involved has a typical Hill-like force-velocity curve if tested in a controlled laboratory situation, and that this relationship would be manifested during a complex movement. Obviously, a vertical jump is not a controlled laboratory situation for each muscle, so potential complications exist that can change how the intrinsic force-velocity relationships are seen externally.
5.6.1.1 Muscle Length

A well-known property of muscles is that the force producing capability of a particular muscle changes with changes in muscle length (Askew, et al., 1998; Chapman, 1974; Granzier, et al., 1989; Kaufman, et al., 1989; Kornecki and Siemienski, 1995). Some factors that contribute to the force-length characteristics are the number of active actin and myosin cross bridges that are overlapped at a given muscle length, and the structure of the muscle (i.e. whether the muscle has a few long fibers or many short fibers arranged in parallel with each other, pennation angle, etc.) (Baratta, et al., 1995; Kaufman, et al., 1989). Force-velocity investigators often attempt to control this issue by collecting data at specific muscle lengths or joint angles so that the muscle is in the same configuration over the various loading conditions. The effect of joint configuration on muscle length is discussed in more detail in Section 5.6.2.1.

Controlling the precise body position and, therefore, the exact muscle lengths, was extremely difficult if not impossible in multi-joint movements. This was shown in this study by Figure 5.1 (B), which indicated that the knee joint was in a different position at the instant of maximum center of mass velocity under different loading conditions. Since the body position at the instant of maximum velocity was not consistent across all of the loading conditions, it could contribute to a change in the appearance of the force-velocity curve since the muscles would not have been in the same configuration for each trial.

5.6.1.2 Muscle Prestretch

If the muscle is stretched while it is stimulated, the force output of the contractile components increases. In addition, sudden additions of force can cause the passive structures of the muscle (the series elastic component) to stretch and slightly delay the force production of the muscle. Prestretch often occurs during movements in which the direction of motion is reversed, often characterized as the stretch-shortening cycle, and has been a hot topic for investigation (e.g. Fukashiro, et al., 1983; Cavagna, et al., 1968; Cavagna, 1977; Assmussen and Bonde-Petersen, 1974; Anderson and Pandy, 1993; Alexander and Bennet-Clark, 1977). Investigators studying force-velocity characteristics
of muscle have attempted to control prestretch by trimming off all connective tissue possible in single muscle/fiber preparations, and/or by only allowing concentric muscle actions during the experiments, thus avoiding prestretch or force delays altogether.

The present study attempted to control the prestretch issues by using noncountermovement vertical jumps. However, the muscles were statically loaded and were activated as the subjects assumed the jumping position, especially in the high loading conditions and not as much in the very unloaded conditions. This disparity could contribute to changes in the force-velocity relationship across loading conditions. This will be discussed in more detail in the following section.

5.6.1.3 Quick Release vs. No Quick Release Methods

A quick release technique was used in many single fiber/muscle and single-joint force-velocity experiments (see ‘QR’ column in Tables 2.1-2.5). In this type of procedure, the muscle was statically preactivated to a predetermined level in the starting position, then the fiber/muscle/limb was suddenly released to a lower force level during the movement. This technique is used for several reasons. First, it allows the fiber/muscle(s) to be more fully activated during the movement and reduces the acceleration to maximum velocity period as the fiber/muscle/limb is released from the resting position. In low load situations, the fiber/muscle(s) may actually move through the entire contraction distance before the maximum force and/or velocity is reached. Second, because the fiber/muscle(s) are preactivated it reduces the time for the fiber/muscle(s) to produce force, take up whatever slack there may be in the system, and reach maximum activation levels. Third, it prestretches the series elastic component to reduce force production delay.

Quick release techniques were not used in the present experiment. This means that the muscles were preloaded at different levels and presumably activated at different levels before the movement began. The subjects were forced to accelerate from zero velocity to maximum velocity without the benefit of similar muscular preactivation across the external loading conditions. This could affect the force-velocity relationship if the maximum force or velocity theoretically possible was not achieved during the time of the movement, as stated in the previous section. For a given level of unloading, an
increase in the center of mass velocity could make the velocity-load curves, and similarly the force-velocity curves since force and load are closely related, more Hill-like. A quick release-type procedure may have addressed some of these concerns, but it would add some complications to the calculations. This may be an avenue for future investigation.

5.6.1.4 Instantaneous Muscle Force and Velocity

In classic studies of muscle force-velocity relationships, either the force that the muscle must produce or the velocity of the contraction is controlled and ideally kept constant during a trial. This can be accomplished very nicely in single muscle fiber preparations with sophisticated machinery. In studies involving humans, constant muscle force and/or velocity is not valid, even in single-joint studies that control the external force or the movement velocity. As the joint configurations change, the lengths of the muscles vary, changing the force production. Additionally, because of the changes in joint configuration and activation, the velocity of shortening of the muscle itself may not be constant, even if the limb velocity (or joint angular velocity) is held constant. Joint configuration will be discussed in more detail below.

In this vertical jump study, joint configuration was not controlled, nor was joint velocity. This means that the force produced by individual muscles was not constant throughout the movement despite the maximum effort of the subjects, and the individual muscle shortening velocity was not constant throughout the movement. The choice of maximum center of mass velocity did not insure that the maximum shortening velocity of any given muscle in the legs occurred at that instant. These individual muscle force and velocity factors may have contributed to the differences seen in the results compared to Hill’s curve.

5.6.2 Individual and Multiple Joint Factors

Each joint in the body contributes a different set of conditions to the whole body during each instant in the movement because of several factors: the geometry of the joint and joint configuration, and the number and arrangement of muscles acting at the joint.
5.6.2.1 Joint Geometry and Joint Configuration

Each joint in the body has a particular geometry or shape. Where and how the muscles are attached around the joint affect how a given level of muscular force at a given instant in time will contribute to the joint moment. A certain muscle force should theoretically produce a joint moment that is proportional to the effective moment arm of the muscle in relation to the joint center. As the joint rotates, the configuration and the geometry of the joint changes, altering the angles of muscle attachment and the effective moment arms for the muscles. Therefore, even if a muscle produced a constant force throughout the movement, the changing joint configuration would cause the moment produced at the joint to change. As mentioned above, changing the joint configuration can also change the length of the muscles, altering the amount of force the muscles are able to produce at a given moment (e.g. An, et al., 1984; Brand, et al., 1982; Spoor, et al., 1990).

These relationships can be written mathematically. In a simple case, the muscle moment arm \( d \) is related in some predictable way as a function of joint angle \( \theta \),

\[
d = f_1(\theta),
\]

the force \( F \) produced by the muscle is also predicted by the joint angle,

\[
F = f_2(\theta),
\]

and the moment \( M \) is related to the force by the size of the moment arm,

\[
M = F \times d.
\]

Combining,

\[
M = f_2(\theta) \times f_1(\theta).
\]

Similarly, the velocity produced at the muscle \( V \) can be expressed as some function of the joint angle,

\[
V = f_3(\theta),
\]

and the velocity produced at the muscle is related to the joint rotation velocity \( \omega \) by the size of the moment arm \( d \), or

\[
\omega = V/d.
\]

Combining,

\[
\omega = f_3(\theta)/f_1(\theta).
\]
Equations (4) and (7) show that the joint moments that produce rotation and the resulting angular velocities at the joint are very complicated even for the simple situation of a single joint with a single muscle.

When multiple joints are involved, the situation is further complicated by the fact that the joint configurations of more than one joint are changing at once. This is especially true when muscles span two joints. One joint could be attempting to shorten the muscle, another could be attempting to lengthen the muscle, the overall length of the muscle could be shortening, lengthening or remaining constant. The muscle would then be contributing to the net moment of both joints.

Measuring the force and velocity at a specific joint angle for all conditions can minimize the complications due to joint configuration in single joint studies since the moment arm and the muscle length should be approximately the same. As mentioned above, the choice of the instant of maximum center of mass velocity in this multi-joint study did not guarantee constant joint configurations, nor did choosing a particular joint angle to study since other joints may not have been at the same relative configuration from trial to trial. It is easy to see how joint configuration could be a contributing factor to the differences in the current study to classic Hill’s curves.

5.6.2.2 Number and Arrangement of Muscles

The net moment produced at a particular joint is complicated by the fact that there are many muscles that cross most joints. Each one of these muscles has its own force-velocity relationship. Each muscle is attached differently around the joint, therefore, the same joint rotation can produce different effects in each muscle as the individual muscle moment arms and lengths change. Each muscle then contributes to the net joint moment differently at each instant in the movement.

An assumption that is often made in human force-velocity studies is that all of the muscles involved in the motion are maximally activated during the entire movement. Even for a single joint, this assumption is not always valid. Certain muscles can be activated at different times during the movement. In multi-joint movement, simultaneous activation almost never occurs. Many authors (Pandy and Zajac, 1991; van Ingen Schenau, et al., 1992; Bobbert and van Ingen Schenau, 1988; Pandy, et al., 1990, van
Soest and Bobbert, 1993) have shown that the patterns of muscle activation in jumping are very complex. Non-simultaneous activation is clearly seen with a cursory look at the EMG results in this study (See Appendix A).

Net joint moment is also complicated by the action of antagonistic muscles. These muscles produce forces that contribute moments in the opposite direction of the movement, effectively reducing the net moment and confounding the interpretation of the moment produced at the joint. Therefore, the force-velocity relationship of the individual muscle could be hidden and would not show up in the movement of the entire body. In loaded and unloaded vertical jumping, it is almost certain that antagonists are active for balance and control. This is seen in the EMG results as well (see Appendix A). Multiple joints complicate the matter even further. Two-joint muscles that seem to be antagonists at one joint may actually contribute to the movement at another joint. If the muscle is sufficiently activated to keep the muscle at nearly a constant length, movement at one joint can actually be transferred to the next joint through the two-joint muscle (Prilutsky and Zatsiorsky, 1994; Bobbert, et al., 1986; van Soest, et al., 1993; van Leeuwen and Spoor, 1992; Jacobs, et al., 1996; Gregoire, et al., 1984).

5.6.3 Joint Position Factors

Joint position refers to the angle that two segments form at a joint. This differs from joint configuration in this paper in that joint configuration refers to the changes internal to the joint (i.e. muscle moment arms, angles of attachments, etc), whereas the joint position ignores all of the internal properties. The positions of the joints, and therefore the limb segments, themselves contribute to how force and velocity are manifested at the end point of the limb. In this case, the endpoint that we are interested in is the center of mass of the body. Of course, the center of mass is not an actual point on the body, it is an imaginable point that can be calculated and is positioned in the body in the trunk in the case of the vertical jump. The velocity of the center mass can be calculated, and the force on the center of mass can be measured at the ground with a force plate. It is assumed that the force measured at the force plate is equal to the force acting at the center of mass.
The model that will be used in this section is called a basic body model or a stick figure model. This model assumes that the body consists of rigid segments connected by hinge joints. Moments at each joint are produced by an imaginary ‘torque motor’ that replaces the muscles and the moment arms discussed above. The effects of all of the factors discussed in the previous sections are lumped into the joint moment at each joint. The lengths of all of the segments are assumed to remain constant. A three-link chain will be assumed: the foot, the shank, and the thigh. The center of mass will be assumed to be located at the hip at the endpoint of the chain, and the magnitude of the ground reaction force at any given instant will be assumed to act at the hip. The instant of maximum center of mass velocity is the instant of interest in this study for a given trial.

The endpoint force depends on the joint torques, and the joint configuration. This can be shown mathematically by the equation,

$$T = J^T F,$$  \hspace{1cm} (8)

where $T$ is the matrix of joint torques, $J^T$ is the transpose of the Jacobian matrix, and $F$ is the matrix of endpoint forces. If the joint torques are known, the equation,

$$F = (J^T)^{-1} T,$$  \hspace{1cm} (9)

is useful, where $(J^T)^{-1}$ is the inverse of the transverse Jacobian matrix. In certain instances, the inverse of the Jacobian cannot be solved, for example when the legs are fully extended, i.e. the leg segments are in a straight line. When the body approaches this singular position, the legs can produce and support more force than at any other joint position. The ground reaction forces in this study were maximal late in the movement when the legs were approaching full extension. Tihanyi, et al. (1987) found that isometric force could be produced at a level predicted by Hill’s force-velocity equation only at joint positions that corresponded to the instant of maximum force production late in the movement.

In a similar way, the angular velocities at the joints are related to the endpoint velocity by the Jacobian matrix,

$$V = J \omega.$$  \hspace{1cm} (10)

There is no singularity in the Jacobian matrix in this instance since the inverse of the matrix is not required, but in the case of leg extension, the ability of the subject to
transfer angular velocity to endpoint velocity is reduced the closer the legs get to full extension.

If the force produced by the muscle is constant, and the effective moment arm of the joint are assumed to constant (may not be a bad assumption if the muscle insertions are a small distance from the joint center as Wilkie (1950) suggested) then the torque at the joint is equal to the equation,

\[ T = F \times d, \]

where \( d \) is the effective moment arm in the joint. This means that for this analysis, the joint torque at each joint is constant. This enables the effects of the joint torques to be separated from the joint position. If all of the joint torques are combined into one vector \( \mathbf{T} \) of constant magnitudes, the magnitude of this vector is, \( (T_1^2 + T_2^2 + \ldots + T_n^2)^{1/2} \), where \( T_1^2, T_2^2, \ldots, T_n^2 \) are the magnitudes of the joint torque in each joint. A simple case is when this magnitude is equal to one. Squaring eliminates the square root, and since the square of the magnitude of a vector is the dot product of itself,

\[ \mathbf{T}^T \mathbf{T} = 1. \]

Substituting equation (8) into equation (12) yields,

\[ \mathbf{F}^T \mathbf{J} \mathbf{J}^T \mathbf{F} = 1. \]

This is the equation of an ellipse and represents an envelope of possible force vectors at the endpoint of the segment chain. Actions in the direction of the major axis of the ellipse are most effective, or ‘easiest’ to produce. Actions in the direction of the minor axis of the ellipse are the most ‘difficult.’

Similarly, if the muscle shortening velocity is assumed to be constant and the moment arm continues to be constant than the angular velocities at the joints will be constant. Following the same logic as above, the magnitude of the angular velocity vector is,

\[ \mathbf{\omega}^T \mathbf{\omega} = 1, \]

and substituting equation (10),

\[ \mathbf{V}^T (J^{-1})^T (J^{-1}) \mathbf{V} = 1. \]

This represents the velocity ellipse with the same interpretation as above for the force ellipse. Examples of possible force and velocity ellipses at different joint positions are
illustrated in Figure 5.2. This analysis shows that even if the joint torques and velocities were constant at all of the joints throughout the entire movement (which they were not), the joint position can greatly influence the force and the velocity that a subject may be able to produce at various positions. Power in this analysis is invariant (a scalar), therefore the product of the force and the velocity (vectors) combination must remain the same.

![Figure 5.2. Illustration of force and velocity ellipses for a simple stick figure model at initial position and near takeoff. Note that the force and velocity ellipses change in opposite directions during the movement.](image)

The multiple joints and muscles and the joint positions cause significant complications to the interpretations of force-velocity relationships in the entire body. The fact that the involved muscles and joints do not all move and act in the same way at the same time sharply deviates from the classical experiments on force-velocity relationships. If, in spite of all of the factors mentioned in this section, the force-velocity characteristics established on single muscles and joints appeared in multi-joint movements, the finding would be impressive. However, at least in this study, most of the force-velocity relationships did not conform to the classical form of Hill’s curve.

### 5.7 Power-Load and Power-Velocity

Power is an attractive variable to investigate because it appears to be a parameter that is closely related to athletic performance. In addition, the vertical jump is a tool that is often used to measure and predict athletic power. It was beyond the scope of this
investigation to debate the efficacy of the vertical jump and power measurements to predict athletic power. In a nutshell, the commonly held theory is that the more power an athlete can produce, the faster the athlete will tend to be and/or be able to deliver more force in a shorter period of time. Thus, in athletic events that require these characteristics, the better that athlete will be.

5.7.1 Shape of the Power Curve

Parametric power measurements in conjunction with force-velocity or velocity-load experiments follow a typical shape. In force-velocity experiments that cover the entire concentric range of forces and velocities, the power curve begins at zero power at zero velocity/load/force, increases parabolically to a maximum value, and then decreases to zero power at the maximum velocity/load/force. Since power can be calculated by multiplying force and velocity, it is easy to see why this typical power curve exists. At maximum load levels, the ability of the muscles to overcome the load is overcome, thus there is no movement, i.e. zero velocity and zero power. At the other extreme, with no load, velocity will be at a theoretical maximum, but will require no force to produce it, thus zero power. Between these extremes intermediate values produce the increasing-decreasing shape of the power curve.

As mentioned previously, it was impossible to cover the entire range of theoretical loads, forces, and velocities for safety and other practical reasons in this experiment. Therefore, it was interesting to identify where on the power curve this experiment lay, i.e. on the ascending or descending portion of the curve. When looking at the power-force and power-load curves, it appears that the power curve is in the ascending range (Figure 4.17). This indicates that this experimental protocol included much of the most unloaded portion of the theoretical power-load curve, but left out much of the more loaded conditions that are theoretically possible. Logically, this makes the power-velocity curve descending, which agrees with the results (Figure 4.18). These curves seem to make sense, as the experiment could be safely undertaken in the unloaded ranges until the subjects could not reach the ground, i.e. the elastic bands produced more force than the weight of the subjects and they were suspended, unable to jump. However, the experiment was stopped at the other extreme before the additional loads became
dangerous to the subjects even though they could have theoretically lifted more weight (although probably not jumped with more weight). It is believed that no multi-joint force-velocity and power study using the vertical jump as the model has included so much of the theoretical range of power, load, force, and velocity.

An exception to these results was seen in the power calculated using maximum leg extension velocity and the corresponding ground reaction force (Figure 4.20). In this case the power-velocity curve was ascending, exactly the opposite trend from the maximum velocity of the center of mass power-velocity curve mentioned above (Figure 4.18). The only difference between the two power curves was that the relative instant when the maximum velocity occurred was not the same within a given trial. The general tendency was for the maximum leg velocity to occur before the maximum center of mass velocity (except at the highest loading conditions in some subjects) and for the time difference to decrease as the load increased (Figure 5.3 A). The maximum center of mass velocity occurred after the maximum ground reaction force had occurred when the ground reaction force-time curve had begun its steep drop to zero at takeoff in the center of mass calculations, but leg extension velocity occurred nearer to the maximum ground reaction force at low loads Figure 5.4). In addition, the difference between the two velocities increased with increased load (Figure 5.3 B). As load decreased, a higher relative leg extension velocity was multiplied by a higher relative ground reaction force. This explains why leg extension power was in the ascending portion of the power-velocity curve.

Since the maximum leg extension velocity occurred before the maximum center of mass velocity, this may imply that the legs extended before the entire body and/or the trunk reached full extension. This is corroborated by the fact that the EMG results indicated that the leg extensors were more active early in the movement whereas the hip extensors were primarily active late in the movement. This goes against the coordination patterns proposed previously (Bobbert and van Ingen Schenau, 1988).

5.7.2 Point of Maximum Power

The power curves were either primarily ascending or descending as mentioned above, but the curves in most cases appeared to reach a maximum. This was an
important feature of these curves, because several other force-velocity investigations have focused on this maximum power point (Wilson, et al., 1993; Newton, et al., 1997; Kraemer, and Newton, 1994). The velocity or force that corresponds to the maximum power point is often called the ‘optimal’ force or velocity, because it is at this force and velocity combination where maximum power is produced. This maximum power point has received a fair amount of attention because of the connection between power and athletic performance. It seemed to be especially important in sports such as cycling where a relatively constant force or velocity level is maintained. One theory based on
optimal load promotes training at the loading level that produces maximum power; the
idea being that training at the point of maximum power will increase the maximum power
production of the athlete. This ‘optimum’ training level has been reported to be
approximately 33% of the maximum amount the athlete can move (Kraemer and Newton,
1994; Wilson, et al., 1993).

It was not possible to directly compare the maximum power points of this study
with the theoretical optimum proposed above because the exact shape of the power curve
in the force and velocity ranges not covered in this study are unknown. However, based
on the self-reported maximum squat lifts of the subjects, and the fact that maximum
power occurred at approximately 85% of the subjects’ body weight in additional loading,
maximum power occurred at approximately 37-61% of the maximum squat lifts of the
subjects. In addition, maximum power occurred at approximately 70% of the velocity of
a normal bodyweight jump. When the maximum velocity predicted by the power curve
calculated with the force plate data was used, maximum power corresponded to a velocity
of 56% of maximum achievable velocity. These measures appeared to be higher than the
theoretical ‘optimal’ load level reported in the literature, but were similar to the results
reported by Thomas, et al. (1995). Remember that the calculations for optimal load done
for the current study were simply rough estimates and based on self-reported maximum
squats. Some of the subjects had not done a maximum squat in some time before the
experiment and were estimating their current abilities.

5.8 Conclusions

Based on the results of this study and the preceding discussion, the following
conclusions corresponding to the original hypotheses for the experiments were as
follows:

1. Hill-like curves were defined as descending, concave upward curves with
   the ratio \( a/F_o \) between zero and one. All of the velocity measures
   (maximum center of mass using both video and force plate methods,
   maximum knee angular velocity; and leg extension velocity) vs. load
   produced velocity-load plots that had concave downward polynomial fits
which were different than Hill’s curve and $a/F_o$ was greater than one as well. Therefore, it was concluded that these curves were not Hill-like.

When the appropriate ground reaction force was plotted against the same velocity variables, the ground reaction force vs. the maximum center of mass velocity using the force plate and video methods failed to produce curves that were Hill-like in form. Once again, force plate curve was concave downward and $a/F_o$ was greater than one. The video method produced a curve that was concave upward, but $a/F_o$ was greater than one. It was concluded that these curves were not Hill-like. Ground reaction force vs. maximum knee angular velocity and maximum leg extension velocity were both Hill-like in appearance and had $a/F_o$ ratios between zero and one. It was concluded that these curves were indeed Hill-like.

None of the plots with the knee moment produced any relationships at all and were decidedly not Hill-like.

2. When comparing the curves to each other, the ‘best’ force-velocity curves in this study were ground reaction force vs. maximum center of mass velocity (video) fit with either the Hill equation or the polynomial, the maximum center of mass velocity vs. load (force plate and video) fit with the polynomial, and maximum center of mass velocity vs. load (video) fit with Hill’s force-velocity equation. All of these relationships had intra-class correlation coefficients over 0.9. The best fit of all was the corresponding ground reaction force vs. maximum center of mass velocity (video) with the Hill fit.

3. Power calculated with the knee moment produced no relationship and therefore did not conform to the theoretical power-velocity curve.

Power calculated by multiplying ground reaction force by the maximum center of mass velocity and by the maximum leg extension velocity produced power-velocity and power-load curves that were similar to the theoretical parabolic ascending-descending curves. They did not cover the entire range of theoretical data, but they did reach a peak. The
power vs. load and vs. ground reaction force both had good correlation coefficients over 0.9.

4. The power-velocity curve calculated with the maximum center of mass velocity was on the descending portion of the theoretical power-velocity curve as expected, and the power-load curves were on the ascending portion of the theoretical power-load curve, as expected. The power-velocity curve calculated with the leg extension velocity was exactly the opposite from what was expected. This power-velocity curve was on the ascending portion of the curve and the power-load was slightly descending. A provisional, hypothetical explanation of this controversy has been suggested.

5. The load corresponding to the maximum power was calculated to be approximately 37-61% of the reported maximum squat lifts of the subjects and approximately 56% of the maximum velocity of the center of mass of the subjects. These were higher than the theoretical maximum power load of 33% of maximum strength and 33% of maximum velocity that was expected.
Reference List


Appendix A. EMG Plots for Two Subjects
Figure A.1. EMG for Subject A, most loaded condition.
Figure A.2. EMG for Subject A, bodyweight condition.
Figure A.3. EMG for Subject A, most unloaded condition.
Figure A.4. EMG for Subject B, most loaded condition.
Figure A.5. EMG for Subject B, bodyweight condition.
Figure A.6. EMG for Subject B, most unloaded condition.
Appendix B. Results of Pilot Studies
This appendix explains the pilot studies that were undertaken to determine the best solution to unload the subjects to produce higher center of mass velocities. This was not a simple problem; the requirements were that the subject should be assisted in the movement by a vertical force that was as constant as possible to mirror the loaded conditions.

**Original Apparatus**

The first attempt to build an assisting apparatus utilized a block and tackle system, with a mechanical advantage of three, to produce a counterweight in the unloaded trials. A stack of weights was attached to one side of the pulley system, and the subject in the harness was attached to the opposite side. The idea was to put the mechanical advantage of the pulley system on the side of the subject. This meant that the counterweight, which would have the maximum potential to fall with the acceleration of gravity, would only have to accelerate at one third of the acceleration of the subject. Since the subjects could not accelerate at a rate greater than three times gravity, there would always be an assisting force pulling on the subject as they jumped.

In fact, the mechanism worked. There was no slack in the rope as the subjects jumped (as determined by video analysis), indicating that there was always a pulling force on the subject. However, an analysis of the force readings from the transducer connected to the harness the subjects were wearing indicated that there was a decrease in force that went nearly to zero. Because the force did not drop completely to zero meant that there was still tension in the rope, but the effectiveness of the counterweight was lost nearly completely. There was almost no change in the maximum velocity during the unloaded jumps using this method. The inertia of the counterweight and the friction of the pulley system were believed to be the cause of the decrease in force during the jumps. Regardless, the system did not meet the experimental requirements and was rejected.

**Current Apparatus**

The velocity vs. time profile for one subject attained with the current apparatus utilizing filled elastic surgical tubing for the unloading mechanism was nearly the same as Figure 4.1. When the data from the loaded, bodyweight, and unloaded trials were combined, a wide range of peak velocities was produced. Two subjects were tested using
a protocol similar to the one presented in the Methods. The results of both subjects indicated that a range of peak velocities both higher and lower than a normal jump was attainable using the current apparatus. The force record from the transducer connected to the harness indicated that the force decreased up to about 25% as the subject jumped due to the elastic properties of the bands. However, the force difference was apparently small enough to produce changes in peak velocity of the center of mass between the conditions.

Three trials at each condition were performed. The trial to trial differences in the same condition for each subject were small. Therefore, to reduce data analysis time, only the trial with the highest peak velocity was chosen for the main experiment. The ground reaction force vs. maximum center of mass and the power vs. maximum center of mass plots showed the general trends expected for the experiment. Therefore, it was concluded that the apparatus and protocol were satisfactory to investigate force-velocity relationships in the vertical jump.
Appendix C. Informed Consent Form
Informed Consent Form
The Pennsylvania State University

Title of Project: Power and Force Production in the Vertical Jump with Varying Load

Principal Investigator: Vladimir Zatsiorsky, Ph.D.

Other Investigators: Andrew Hardyk, M.S.

This is to certify that I, ______________________, have been given the following information with respect to my participation as a volunteer in a program of investigation supervised by Dr. Zatsiorsky.

1. Purpose of the Study:
The purpose of this study is to examine the muscular power and force production in humans at various jumping velocities in the vertical jump.

2. Procedures to be Followed:
You will be asked to perform several vertical jumps from a stationary position. Three types of loading conditions will take place: no load, unloaded, and loaded. In the no load condition, you will place your hands on the broomstick that is provided and jump straight up without a countermovement (i.e. you will start from a lowered position and jump upward without first moving downward). In the loaded condition, you will hold a barbell with weights on your shoulders (similar to the squat in weight lifting) and jump upward. A special apparatus will take the weight away while you are in the air to protect you when you land. In the unloaded condition, you will be attached to a harness which will make you lighter. Once again, you will place your hands on the broomstick and jump straight upward. All of these jumps should be maximal attempts for maximum height.

EMG electrodes will be attached to your skin on your leg muscles to record the activity of your muscles during the jumps. You will also be videotaped to determine the positions of your body during the movements.

3. Discomforts and Risks
In the loaded condition, there is the possibility that you will perceive discomfort because of the weight on your shoulders. The bar has been padded to alleviate this discomfort. You will also be instructed in the proper technique to use, and you will be required to use a weight belt to protect your lower back from any potential injury. There is also a possibility that the barbell could come in contact with your body after it is taken from you in midflight. Once again, the padding on the barbell will prevent any serious problems. Upon removal, the EMG electrodes may
cause some slight skin irritation. Appropriate clean up materials will be provided to you which should alleviate any discomfort.

4. a. Benefits to You:
You will receive no direct benefits such as financial compensation or class credit for your participation in this study.

b. Potential Benefits to Society:
Society could potentially benefit from a greater understanding of power production in human movements. This could manifest itself in the area of sports performance or in everyday life in such movements as rising from a chair.

5. Time Duration:
This study will require approximately two hours of your time.

6. Statement of Confidentiality:
Your participation in this research is confidential. Only the investigators and his assistants will have access to your identity and to information that can be associated with your identity. In the event of publication of this research, no personally identifying information will be disclosed.

7. Right to Ask Questions:
You have the right to ask questions at any time before, during, or after the study. If you have questions in the future, please contact either:

Andrew Hardyk  Vladimir Zatsiorsky
200 Biomechanics Lab  200 Biomechanics Lab
University Park, PA 16802  University Park, PA 16802
(814) 865-3445   (814) 865-3445

I have been given an opportunity to ask any questions I may have, and all such questions have been answered to my satisfaction.

8. Compensation
I understand that in the event of injury resulting from research, neither financial compensation nor free medical treatment is provided for such injury by the University. Questions regarding this statement or your rights as a subject of this research should be directed to the Office for Regulatory Compliance in 212 Kern Building, University Park, PA (814-865-1775).

9. Voluntary Participation:
I understand that my participation in this study is voluntary, and that I may withdraw from this study at any time by notifying the investigator. My
withdrawal from this study or my refusal to participate will in no way reflect my care or access to medical services.

This is to certify that I consent to and give permission for my participation as a volunteer in this program of investigation. I understand that I will receive a signed copy of this consent form. I have read this form and understand the content of this consent form.

Volunteer Date

I, the undersigned, have defined and explained the studies involved to the above volunteer.

Investigator Date
Vita

Andrew Timothy Todd Hardyk

Andrew Hardyk is wrapping up his career as a professional student and plans to spend more time on the job that actually pays the bills. He is currently an Assistant Track and Field Coach at Penn State University and hopes to incorporate his research skills in developing better coaching techniques to develop the next generation of track and field superstars. He sees a huge divide between scientists and coaches and sincerely hopes that he can help to bridge that gap by working from both sides.

Hardyk graduated from the University of Cincinnati in 1992 with a Bachelor of Science degree in Aerospace Engineering and a Master of Science in Engineering Mechanics one year later, specializing in orthopedic biomechanics. At Cincinnati, he was a team co-captain for the track and field team and received the ‘Jimmy Nippert Award’ for the most outstanding all-around senior male athlete. Post-collegiately, he competed in the United States Track and Field Championships four times, twice in the long jump and twice in the 100m and was selected to represent the North team at the U.S. Olympic festival in 1995. In five years as a coach at Penn State, he has produced 10 All-Americans and has had numerous NCAA qualifiers.

Hardyk currently has limited publications to his name, but hopes that this dissertation will be a source for several in the near future. He gave a presentation in 1994 at the American Society of Biomechanics Annual Meeting entitled, “Strategies Used By Elite Male Gymnasts to Generate High Forward Angular Momentum During Takeoff in Vaulting: A Cluster Analysis,” and was a co-author of a chapter on sprint training entitled, “Sport Speed,” that was published earlier this year.

Hardyk is very happily married to the former Angela Showalter, who is an Obstetrics and Gynecology resident at the Penn State/Hershey Medical Center.