NEUTRINO FLAVOR OSCILLATIONS IN THE ICECUBE DEEPCORE ARRAY

A Dissertation in
Physics
by
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Abstract

A large number of experiments of different types have provided strong evidence for neutrino oscillations and thus for physics beyond the Standard Model of Particle Physics. New experiments are being built and designed to further investigate neutrino oscillations, to obtain precision measurements of dominant oscillation parameters and discover sub-dominant effects. On the other hand, neutrino telescopes, like IceCube and the IceCube DeepCore Array, are using neutrinos as a means of learning about astrophysical sources, discovering dark matter and other high energy phenomena. Atmospheric neutrinos constitute a background for these searches. This dissertation shows that the large number of atmospheric neutrinos that the IceCube DeepCore detector will accumulate can be used in order to extract useful information about neutrino oscillations.

Atmospheric neutrino interactions within the IceCube DeepCore array are examined in the context of neutrino flavor oscillations. The detection of an appearance of a tau-flavored neutrino flux, not present in the original atmospheric neutrino flux is calculated and quantified using the statistical interpretation of the neutrino-induced electromagnetic and hadronic cascades within the detector. The “track signal” of Cherenkov light created by muons created from either charged current interactions of muon-type neutrinos with detector volume nuclei or the decay of tau leptons within the detector’s instrumented volume are also examined. Using a chi-squared analysis, new precision bounds are determined for the main atmospheric neutrino oscillation parameters and shown to drastically reduce the current parameter space. The implications of these new precision measurements are discussed in the context of currently open questions in the area of neutrino flavor oscillation studies. Future work and the impact of future measurements on this analysis are also discussed. Full three-flavor neutrino oscillation transition probabilities with matter effects are calculated and employed in the study. A discussion of the calculation of the full differential cross sections for neutrino-nucleon
deep inelastic scattering interactions in both the limit of a negligible mass final state lepton and in the situation of a non-negligible mass final state lepton are discussed and employed with the latter being necessary for the calculation of the tau-type neutrino charged current interaction induced cascades. Physics motivated assumptions about the systematic error present in the IceCube DeepCore detector and the effective volume of the detector are made for the analysis.

In summary, it is shown that cascade measurements in the IceCube DeepCore Array can provide evidence for tau neutrino appearance in atmospheric neutrino oscillations. A statistically significant (3σ) $\nu_\tau$ appearance signal could be obtained in only a few months of observation and is important for future $\nu_\tau$ interaction studies. Furthermore, the very high statistics atmospheric muon neutrino data can be used to obtain precise measurements of the main oscillation parameters, significantly improving present results and aiding in the solution to open questions in the neutrino sector.
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Dedication

To my love, Diana.
Chapter 1

Introduction

1.1 History of Neutrinos

An interesting problem had arisen in nuclear physics in the early 20th century [1]. A study of nuclear beta decay had revealed that the beta particle itself was an electron, but the energy of that electron was exhibiting mysterious properties. In its most basic form, the beta decay of 1930 involved a neutron (in the nucleus of an atom, e.g., Nitrogen or Lithium) decaying into a proton and an electron which was ejected from the nucleus:

\[ n^0 \rightarrow p^+ + e^- \]  \hspace{1cm} (1.1)

For a basic two-body decay such as the one in Eq. (1.1), the energy of the electron, \( E_e \), assuming the neutron to be at rest, should be

\[ E_e = \left( \frac{m_n^2 - m_p^2 + m_e^2}{2m_n} \right) c^2. \]  \hspace{1cm} (1.2)

Thus, the electron’s energy should be a fixed constant based solely on the masses of the particles involved. Instead, the measurements in the early 20th century revealed that this was actually the maximum of an energy curve for the beta particle.

After considering the possibility that the nucleus may not conserve energy, Pauli suggested that the two body decay may actually be a three body decay, and
introduced a new particle that would become the neutrino (actually, the antineutrino):

\[ n^0 \rightarrow p^+ + e^- + \bar{\nu}. \]  

(1.3)

Using the experimental results and Eqs. (1.1) and (1.2), two conclusions about the mysterious neutrino particle can be drawn immediately. First, Eq. (1.1) demands that this new particle is uncharged. Second, because Eq. (1.2) does represent the maximum energy that the emitted electrons possess, the extra particle should have a very small mass, maybe even zero. Now in equation Eq. (1.3), the new particle involved in the beta decay of a neutron is actually an antineutrino. At first, it was unclear that this was the case. After all, the neutrino was uncharged and distinguishing between a particle and an antiparticle is somewhat subtle for a neutral particle. Photons and neutral pions, for example, are their own antiparticles; what about the neutrino? A series of discoveries would demonstrate the need for an antineutrino. Before that could happen though, the neutrino needed to be shown to be more than a bookkeeping device.

In the 1950’s, Cowan and Reines set out to verify the neutrino’s existence by searching for inverse beta decay [2]:

\[ \bar{\nu} + p^+ \rightarrow n^0 + e^+. \]  

(1.4)

Setting up large tanks of water mixed with CdCL₂ in between scintillator layers near a nuclear reactor (an excellent source of \( \bar{\nu} \)'s), they looked for, and found, the double coincidence of the gamma produced from positron annihilation and the gamma produced in neutron capture by Cd confirming the above reaction. Reines was awarded the Nobel Prize in 1995 for his work on finding the neutrino.

About the same time, Konopinski and Mahmoud had proposed the law of conservation of lepton number [3]. The basic rule states that each electron, muon, tau and neutrino has a lepton number of +1 and their antiparticles have a lepton number of -1. The only physical processes allowed are ones that conserve lepton number. This introduced a way of demonstrating the difference between a neutrino and an antineutrino. A similar process to Eq. (1.4) (in fact, called the crossed
reaction) had been demonstrated to take place:

\[ n^0 + \nu \rightarrow p^+ + e^- \]  \hspace{1cm} (1.5)

Davis and Harmer proposed that if indeed lepton number is conserved and antineutrinos are different from neutrinos, then the process

\[ n^0 + \bar{\nu} \rightarrow p^+ + e^- \]  \hspace{1cm} (1.6)

ought to be forbidden and it is [4]. Therefore, the easiest way to differentiate between a neutrino and an antineutrino is by observing their lepton number, which experimentally amounts to observing the charged lepton that is sometimes produced after a neutrino’s weak interaction.

The story of lepton number has another interesting bit. It seemed that not only is lepton number conserved to high accuracy, but the individual types of leptons also conserved their relative flavors. For example, the following reaction conserves both charge and lepton number but is still forbidden:

\[ \mu^- \rightarrow e^- + \gamma \]  \hspace{1cm} (1.7)

It was thus proposed that there are three separate lepton numbers, one each for the electron, muon and tau, positive for particles and negative for antiparticles, and these individual lepton numbers should be conserved as well. Neutrinos, being leptons, needed to obey this law, and so, it was proposed (and confirmed experimentally) that there must be three separate types, or flavors, of neutrino (and antineutrino), one each for electrons, muons, and taus.

Pontecorvo had already suggested in 1959 that the neutrino involved in \( \beta^- \) decay may be different from the one emitted during pion decay [5], but it was not until 1962 that Lederman, Schwartz and Steinberger produced the first conclusive evidence for the different neutrino flavors of \( \nu_e \) and \( \nu_\mu \) for which they were awarded the 1988 Nobel Prize [6]. After the discovery of the tau lepton at SLAC in 1975, a situation of missing energy in the tau decay much like the missing energy in \( \beta^- \)-decay suggested that a tau neutrino also existed. It was found by the DONUT experiment in 2000 [7].
For this dissertation, perhaps the most interesting twist in the history of the neutrino came in 1958 when Goldhaber et al. [8] set out to measure the helicity of the neutrino via electron capture of $^{152}\text{Eu}$:

$$^{152}\text{Eu} + e^- \rightarrow \nu_e + ^{152}\text{Sm}^* \rightarrow ^{152}\text{Sm} + \gamma.$$ (1.8)

It was shown in Ref. [8] that since $J(^{152}\text{Eu}) = 0$, measuring the helicity of the resulting photon is equivalent to measuring the helicity of the neutrino. After several measurements, it was determined that the helicity of the neutrino was always -1. Even though neutrinos are spin-$\frac{1}{2}$ particles, it is experimentally confirmed that only left-handed, helicity = -1, neutrinos, and right-handed, helicity = +1, antineutrinos, are observed. Since no boosted frame of reference has been observed where a left-handed neutrino could be rendered right-handed, it was assumed that neutrinos must be traveling at the speed of light and therefore must be massless. Although the evidence of left-handed-only neutrinos is immense, new experimental results reveal that neutrinos possess an interesting property that suggests they are massive, although that mass is expected to be extremely small.

### 1.2 Recent Developments

Since neutrinos seem to be present in nuclear reactions, it became interesting to use them for observational astronomy. A great many of the objects that occupy astronomer’s interests, e.g., stars, produce neutrinos in great abundance. Being that the neutrino is a neutral, possibly massless, lepton means that it only communicates with other particles through weak interactions. This means that neutrinos will be free from the burdens felt by other astronomical probes such as charged particles and photons and can be used, among other things, to “see” far away objects and even the interior of stars.

Stars, like the Sun, are modeled as enormous fusion reactors, producing incredible amounts of energy and copious numbers of electron neutrinos in the nuclear reactions of their cores. By observing these neutrinos, astronomers are actually seeing the interior of the star. The Sun, for example, produces enough electron neutrinos that there is a flux at Earth of $7 \times 10^{10}$ neutrinos per cm$^2$ per second.
As will be discussed in greater detail in Chapter 4, the number of observed electron neutrinos from the Sun is about 1/3 of what is expected. The observations were at first disturbing because they suggested that the energy production method theorized for stars was incorrect. An interesting solution to the so called “Solar Neutrino Problem” would have to wait until another unexpected disappearance of neutrinos (this time of atmospheric origin) was discovered and analyzed with the Super-Kamiokande detector, in Japan, 1998.

Built 15 times bigger than the previous Kamiokande particle detector, Super-Kamiokande (SK) was constructed primarily to look for proton decay. Atmospheric neutrinos from cosmic ray interactions were a large background to the extremely small signal expected from proton decay. As will be explained in greater detail in Chapter 4, by sorting the types of these neutrinos detected in the 50,000 ton experiment, SK provided the first evidence that atmospheric neutrinos may change their flavors during propagation from source to detection [9]. Using a similar idea as presented by SK but applied now to solar neutrinos rather than atmospheric neutrinos, measurements of solar neutrinos were made, and presented in 2001, by the Sudbury Neutrino Observatory (SNO) that included all flavors of neutrinos [10]. These measurements determined that the total flux of all neutrino flavors matched the expected flux of solar electron neutrinos. The interpretation of this result, in analogy to SK, is that the solar electron neutrinos had changed flavor to include the muon and tau flavor neutrinos.

The proposed mechanism of these neutrino flavor oscillations follows a similar path to the mixing of quarks by assuming that the flavor states of the neutrino particle are made of linear superpositions of massive neutrino states. This will be discussed in detail in the next chapter, but there are two very important implications: first, that the massless neutrinos actually do have mass in this theory, and second, that the conservation of individual lepton flavor numbers that was used to prohibit certain interactions and gave rise to the different neutrino flavors is violated.
1.3 Summary of Dissertation

I will begin in Chapter 2 with the plane wave derivation of neutrino flavor oscillations assuming that neutrino flavor eigenstates can be written as a linear combination of neutrino mass eigenstates. Chapter 3 continues a theoretical discussion of detection methods of the elusive neutrino using weak force interactions. The experimental evidence of neutrino flavor oscillations will be discussed in Chapter 4, followed by a detailed discussion of the IceCube Neutrino Observatory and its low energy extension DeepCore in Chapter 5. Chapters 6 and 7 conclude by examining atmospheric neutrinos in IceCube and DeepCore. This particular flux of neutrinos is considered a background to astronomical neutrino searches, IceCube’s primary objective, but as this dissertation demonstrates, there is much “signal” to be found in this “noise.”
Neutrino Oscillations: Theory

The plane wave derivation of the standard theory of neutrino oscillations is well suited for this study. I will begin outlining the framework for this derivation as discussed by Kim and Pevsner [11]. Just like quark flavor states are a superposition of mass eigenstates, we will consider that a neutrino with a flavor of $\alpha$, $\nu_{\alpha}$, can be described as a superposition of neutrino mass eigenstates $|\nu_i\rangle$ and vice versa:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$  \hspace{1cm} (2.1)

and

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i}^* |\nu_\alpha\rangle,$$  \hspace{1cm} (2.2)

where the unitary matrix, $U$, is the lepton sector analog of the CKM matrix in the quark sector and is called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix with a useful parametrization (to be discussed later) given by the Particle Data Group [12]. While in certain situations it may be more useful to derive a description of neutrino flavor states from quantum field theoretical considerations, in the limit that the production and detection processes are not sensitive to the different masses of the neutrino mass eigenstates, the quantum field theory description and the quantum mechanical description are the same. This is the case for all neutrino oscillation experiments. Since we know that neutrinos are extremely light, we can use the ultrarelativistic limit such that the dispersion relation for the energy eigenvalues, $E_i = \sqrt{\vec{p}^2 + m_i^2}$, for the massive neutrino states $|\nu_i\rangle$ with momentum, $\vec{p}$,
becomes $E_i \simeq E + \frac{m_i^2}{2E}$. The lowest energy probed by neutrino oscillation experiments is on the order of 100s of keV while the upper bound on the mass of the heaviest neutrino mass eigenstate is 1 eV, i.e., five orders of magnitude below the neutrino’s energy, so this is a safe assumption. In this situation, the propagation time, $t$, and the propagation length, $L$, are equal, i.e., $t = L$. Lastly, it will make the math a bit cleaner to assume that each of the massive neutrino eigenstates has an equal momentum and so the flavor state has a definite momentum; it turns out to be irrelevant for the neutrino oscillation probability.

### 2.1 Neutrino Oscillations in Vacuum

The driving question in neutrino flavor oscillation physics is if I somehow produce a neutrino of flavor, $\alpha$, what is the probability that I detect a neutrino of flavor, $\beta$, after the neutrino propagates for a distance, $L$ or time, $t$. To answer this question, we begin with massive neutrino states, which are eigenstates of the Hamiltonian and evolve as plane waves. Using the Schrödinger equation,

$$i\frac{d}{dt} |\nu_i(x,t)\rangle = H_{\text{vacuum}} |\nu_i(x,t)\rangle,$$

and

$$H_{\text{vacuum}} |\nu_i\rangle = E_i |\nu_i\rangle,$$

we get the evolution of the massive neutrino states as

$$|\nu_i(x,t)\rangle = e^{-iE_it} |\nu_i(x,0)\rangle$$

and assuming that a neutrino with momentum $p$ was produced at $x=t=0$,

$$|\nu_i(x,0)\rangle = e^{ipx} |\nu_i\rangle.$$

This allows us to revisit Eq. (2.1) above and say that

$$|\nu_\alpha(x,t)\rangle = \sum_i U_{\alpha i} |\nu_i(x,t)\rangle = \sum_i U_{\alpha i} e^{-iE_it} |\nu_i(x,0)\rangle = \sum_i U_{\alpha i} e^{-iE_it} e^{ipx} |\nu_i\rangle.$$
Next, substituting Eq. (2.2) into the last step gives

$$| \nu_\alpha(x,t) \rangle = \sum_{i,\beta} U_{\alpha i} U_{\beta i}^* e^{-iE_i t} e^{ipx} | \nu_\beta \rangle.$$  \hfill (2.8)

In answering the driving question, examining a flavor transition from $| \nu_\alpha \rangle \rightarrow | \nu_\beta \rangle$ gives a time-dependent transition amplitude of

$$A(\alpha \rightarrow \beta)(t) = \langle \nu_\beta | \nu_\alpha(x,t) \rangle = \sum_i U_{\alpha i} U_{\beta i}^* e^{-iE_i t} e^{ipx}.$$  \hfill (2.9)

Or assuming the ultrarelativistic limit for the energy eigenvalues and $t = L$,

$$A(\alpha \rightarrow \beta)(L) = \sum_i U_{\alpha i} U_{\beta i}^* e^{-i \frac{m_i^2 L}{2 E}}.$$  \hfill (2.10)

Finally, by squaring this transition amplitude and substituting $m_i^2 - m_j^2 = \Delta m_{ij}^2$ we get the probability that a neutrino flavor transitions from flavor $\alpha \rightarrow \beta$

$$P(\alpha \rightarrow \beta)(L, E) = \sum_{i,j} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} e^{-i \frac{\Delta m_{ij}^2 L}{2 E}}.$$  \hfill (2.11)

Or again using the unitarity of the mixing matrix, $U$, we can separate the real and imaginary parts of $U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}$ and write $P(\alpha \rightarrow \beta)(L, E)$ as

$$P(\alpha \rightarrow \beta)(L, E) = \delta_{\alpha \beta} - 4 \sum_{i>j} Re[U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}] \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4 E} \right)$$

$$+ 2 \sum_{i>j} Im[U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}] \sin^2 \left( \frac{\Delta m_{ij}^2 L}{2 E} \right).$$  \hfill (2.12)

These general formulae for the oscillation probabilities become quite complex when the elements of the mixing matrix, $U$, are inserted in terms of mixing angles. For the purpose of the analysis that follows later in this dissertation’s discussion, these probabilities (along with the addition of the matter effects) were used. However, in many experimental situations which will be discussed in the next chapter, simplified versions of these equation can be used to great precision. These will be
discussed in the subsequent sections.

2.1.1 Three-Flavor Neutrino Oscillations in Vacuum

Although several experimental results point to only three active neutrino flavors, electron, muon and tau, the only such restriction on the number of massive states is that there must be at least three distinct states, but one may have a mass of zero, i.e., there must be two mass differences. If there are more than three, then the new corresponding flavor states must not participate in the weak interaction and are labeled as sterile. Unless otherwise noted, I will assume three massive neutrino states represented in Eq. (2.1) as \( |\nu_i\rangle \) where \( i \) takes on the values 1, 2, 3 and three flavor neutrino states represented in Eq. (2.1) as \( |\nu_\alpha\rangle \) where \( \alpha \) takes on the values e, \( \mu \), \( \tau \) representing the electron, muon and tau flavors, respectively. I will also assume the massive states are orthonormal and, by the unitarity of \( U \), the flavor states are also orthonormal. In the case of 3 neutrino mixing, the Particle Data Group’s parametrization of \( U \) is given by [12]:

\[
U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}\cos\delta_{13} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{13}\sin\delta_{13} \\
 s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13}
\end{pmatrix} ,
\]

(2.13)

with \( c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}, \theta_{ij} = [0, \frac{\pi}{2}] \), and the Dirac CP-Violating-Phase, \( \delta = [0, 2\pi] \).

2.1.1.1 Parameters Used for Three Flavor Mixing

Neutrino data from solar, atmospheric, reactor and accelerator experiments is well understood in terms of three-flavor neutrino oscillations. Two \( \Delta m^2 \) values and two (large) mixing angles are well determined, while the third mixing angle is limited to be very small. The CP-violating phase (\( \delta \)) is completely unconstrained. In addition, the sign of \( \Delta m^2_{31} \) is also unknown. The best fit oscillation parameter values obtained from present (at the time of the analysis in Chapters 6 and 7) data are [13]:

\[
|\Delta m^2_{31}| = 2.56 \times 10^{-3} \text{ eV}^2
\]
\[ \Delta m_{21}^2 = 7.6 \times 10^{-5} \text{ eV}^2 \]
\[ \sin^2 2\theta_{23} = 0.99 \]
\[ \tan^2 \theta_{12} = 0.47 \]

(2.14)

and \( \sin^2 2\theta_{13} \leq 0.15 \) for \( \Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2 \). At the three sigma level \( |\Delta m_{31}^2| \) can vary between \( 2.0 - 2.83 \times 10^{-3} \text{ eV}^2 \), while \( \theta_{23} \) varies between \( 35.5^\circ - 53.5^\circ \). In the near future, long baseline experiments like MINOS [14] and T2K [15] will improve the current precision on \( \Delta m_{31}^2 \) and possibly discover a non-zero value of \( \theta_{13} \), if this is close to the present upper limit. In a few years, reactor experiments like DoubleChooz [16], RENO [17] and Daya Bay [18] will provide improved sensitivity to \( \theta_{13} \).

### 2.1.2 Special Cases of Neutrino Flavor Oscillations

Many of the experiments discussed in Chapter 4 are not sensitive to the influence of three neutrino mixing and can therefore be studied using a two flavor model theory with much simplified oscillation formulae. The 2 neutrino flavors \( \nu_\alpha, \nu_\beta \) are considered as linear superpositions of 2 massive neutrino eigenstates \( \nu_1, \nu_2 \). In this case there is only one \( \Delta m^2 \) and one mixing angle. The mixing matrix takes on the form

\[
U = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}.
\]

(2.15)

The probabilities given in Eq. (2.12) become

\[
P_{\nu_\alpha \rightarrow \nu_\beta}(L, E) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) (\alpha \neq \beta)
\]

(2.16)

and

\[
P_{\nu_\alpha \rightarrow \nu_\alpha}(L, E) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) (\alpha = \beta).
\]

(2.17)

The \( \Delta m^2 \) for atmospheric oscillations is much larger than its solar counterpart, i.e., \( \Delta m^2_{\text{atm}} = \Delta m^2_{31} \approx \Delta m^2_{32} \gg \Delta m^2_{21} = \Delta m^2_{\text{sol}} \). In this case, the probabilities
also greatly simplify to the following:

\[ P(\nu_\mu \rightarrow \nu_\tau) = \sin^2(2\theta_{23}) \cos^2(\theta_{13}) \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right), \quad (2.18) \]

\[ P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right), \quad (2.19) \]

\[ P(\nu_e \rightarrow \nu_\tau) = \sin^2(2\theta_{13}) \cos^2(\theta_{23}) \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right). \quad (2.20) \]

### 2.2 Neutrino Oscillations in Matter

Neutrinos propagating in matter are subject to a potential due to coherent forward elastic scattering with the nucleons and electrons that are present in the matter. The Feynman diagrams (See Figs. 2.1 & 2.2) for these processes lead to a charged current (CC) potential of \( V_{CC} = \sqrt{2} G_F N_e \) through W exchange and a neutral current (NC) potential of \( V_{NC} = -\frac{1}{2} \sqrt{2} G_F N_n \) through Z boson exchange.

![Feynman diagram of CC interaction leading to \( V_{CC} \) through W boson exchange.](image)

> Figure 2.1. Feynman diagram of CC interaction leading to \( V_{CC} \) through W boson exchange.

In the CC potential, \( N_e \) represents the density of electrons in the medium through which the neutrino is propagating. For coherent forward elastic scattering, only electron neutrinos will participate in these CC interactions. To participate in this type of interaction, a charged lepton of the corresponding flavor needs to be present and since the only charged lepton present in “normal” matter is the
electron, only electron neutrinos will be allowed to interact this way. For the NC potential, \( N_n \) represents the density of neutrons in the medium. In coherent forward elastic scattering, all neutrinos participate, but electrical neutrality where the number of protons and electrons are equal implies that the NC potentials of these charged particles will cancel each other and only neutrons will participate. Furthermore, when examining the evolution of the neutrino flavors, the NC potential amounts to a phase common to all flavors and so is irrelevant for the calculations of flavor transitions. All together, the potential can be represented by

\[
V_{\beta} = V_{CC}\delta_{\alpha\epsilon} + V_{NC}. 
\]  

(2.21)

The transition probability can be most easily modified by examining the evolution of flavor states in the Schrödinger picture. In that case, we can think of the flavor transition amplitudes as

\[
\langle \nu_{\beta} | \nu_{\alpha}(x, t) \rangle = \psi_{\alpha\beta}(t). 
\]  

(2.22)

By modifying the vacuum Hamiltonian, \( H_{\text{vacuum}} \) from Eq. (2.4) with an additional part \( H_{\text{matter}} \) that obeys

\[
H_{\text{matter}} | \nu_{\alpha} \rangle = V_{\alpha} | \nu_{\alpha} \rangle, 
\]  

(2.23)

the time evolution of the flavor transition amplitude will become

\[
i \frac{d}{dt} \psi_{\alpha\beta}(t) = \sum_\eta \left( \sum_{i} iU_{\beta i}E_i U_{\eta i}^* + \delta_{\beta\eta} V_{\beta} \right) \psi_{\alpha\eta}(t). 
\]  

(2.24)
Again assuming ultrarelativistic neutrinos and eliminating the NC potential since it generates a phase common to all flavors and is irrelevant to flavor transitions, we obtain

\[ i \frac{d}{dx} \psi_{\alpha \beta}(x) = \sum_{\eta} \left( \sum_{i} U_{\beta i} \frac{\Delta m^2_{12}}{2E} U_{\eta i}^* + \delta_{\beta \eta} V_{CC} \right) \psi_{\alpha \eta}(x). \]  

(2.25)

It is the solution to this differential equation that is squared to produce the transition probabilities used later in Chapters 6 and 7. The graphs in Figs. 2.3 - 2.6 show the oscillation probabilities with matter effects for the energy range up to 30 GeV for straight upward going neutrinos (zenith angle = 180°) traveling through the Earth. The PREM model was used for the Earth’s density profile during the calculations. It should be noted that above 10 GeV, the electron neutrino oscillations constitute a tiny effect, dropping to nearly 0 after 30 GeV, while the \( \nu_\mu \rightarrow \nu_\tau \) transition probability is quite large. This is very important for the IceCube Deep-Core experiment since this is the energy range where it is highly sensitive. For higher energies see Fig. 7.1 in Chapter 6.

\[ P^{-1.3}_{\nu_e \rightarrow \nu_\mu} \text{ with matter effects for a zenith angle of 180° and } \sin^2 2\theta_{13} = 0.1. \]
Figure 2.4. $P_{\nu_e \rightarrow \nu_e}$ with matter effects for a zenith angle of $180^\circ$ and $\sin^2 2\theta_{13} = 0.1$.

Figure 2.5. $P_{\nu_\mu \rightarrow \nu_\tau}$ with matter effects for a zenith angle of $180^\circ$ and $\sin^2 2\theta_{13} = 0.1$. 
Figure 2.6. $P_{\nu_e \rightarrow \nu_\mu}$ with matter effects for a zenith angle of $180^\circ$ and $\sin^2 2\theta_{13} = 0.1$. 
Principles of Neutrino Detection

3.1 Neutrino Weak Interactions

Neutrinos are only subject to weak interactions (apart from gravitational interactions). For neutrinos in the energy range of 10 GeV to 100 GeV, only deep inelastic scattering (DIS) processes are important for neutrino interactions with nuclei. For these types of interactions, the total cross section for neutrino-nucleon interactions can be approximated by Ref. [20]

\[ \sigma_N \approx 0.677 \times 10^{-38} E_\nu \text{[GeV]} \text{cm}^2 \]  \hspace{1cm} (3.1)

and

\[ \sigma_{\bar{\nu}N} \approx 0.334 \times 10^{-38} E_\nu \text{[GeV]} \text{cm}^2. \]  \hspace{1cm} (3.2)

The mean free path, or interaction length, of neutrinos, \( \lambda_{\text{free}} \) is therefore energy dependent and can be calculated as

\[ \lambda_{\text{free}} = \frac{1}{N_A \sigma_{\text{CC}} \rho} \]  \hspace{1cm} (3.3)

where \( N_A \) is Avogadro’s Number (\( N_A = 6.022 \times 10^{23} \)) and \( \rho \) is the density of the medium through which the neutrino is traversing. Ice, for example, has \( \rho_{\text{ice}} = 0.9 \text{ g cm}^{-3} \). This small cross section means that neutrinos of 100 GeV possess an extremely large interaction length in water/ice, measured in centimeters of water equivalent (cm.w.e), \( \lambda_{\text{free}} \approx 10^{12} \text{cm.w.e} \). This is very useful for neutrino telescopes,
since this means that a large overburden can be used to filter out unwanted charged particles and leave the neutrino flux intact.

A detailed discussion of the calculation of the differential cross sections can be found in Chapters 6 and 7.

### 3.2 Energy Losses of Charged Particles

For particles with very high energies, the total energy loss is dominated by the Bremsstrahlung process and the other processes that depend linearly on the particles energies, e.g., electron-positron pair production and photo-nuclear effects [23]. At the energies explored in this dissertation, this remains true for electrons, but ionization and excitation are the dominant energy loss mechanism for muons. Tau leptons are an interesting case, because, in general, at these energies, they decay before energy loss becomes an issue.

#### 3.2.1 Ionization and Excitation

The mechanism that dominates charged-particle interactions (at least until TeV energies for muons) is the energy loss by ionization and excitation, which is basically elastic scattering off of atomic electrons. This energy-loss process is described by the Bethe-Bloch formula [24]:

\[
- \frac{dE}{dx_{\text{ion}}} = K z^2 Z \frac{1}{\bar{A}^2 \beta^2} \left[ 1 - \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I^2} - \beta^2 - \frac{\delta(\beta \gamma)}{2} \right],
\]

with

- \( K \approx 0.307 \text{ MeVg}^{-1} \text{cm}^2 \),
- \( N_A \) — Avogadro’s Number,
- \( r_e \) — Classical Electron Radius \( \approx 2.82 \text{ fm} \),
- \( m_e c^2 \) — Electron Rest Mass \( \approx 0.511 \text{ MeV} \),
- \( z \) — Charge number of the incident particle,
- \( Z, A \) — Target charge number and target mass number,
- \( \beta \) — \( \frac{v}{c} \), with \( v \) the velocity of the incident particle,
- \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \).
\[ T_{\text{max}} = \text{Maximum energy transferred to an electron,} \]
\[ = \frac{2m_e p^2}{m_0^2 + m_e^2 + 2m_e E/c^2}, \]
\[ m_0, p, E \text{ – mass, momentum and energy of the incident particle,} \]
\[ I = \text{Average ionization energy of the target,} \]
\[ I \approx (10 \text{ GeV}) \cdot Z, \]
\[ \delta = \text{Takes into account polarization effects which reduce the energy losses.} \]

The energy loss of charged particles, according to the Bethe-Bloch relation, exhibits a \(1/\beta^2\) increase at low energies starting at \(\beta\gamma \approx 3.5\) where a minimum in the ionization rate occurs. After this minimum, the rate increases slightly at higher energies and is fairly constant in the energy range of interest, 10 GeV to 100 GeV. The minimum energy loss (occurring near 1 GeV lepton energy) of singly charged particles, like the electron, muon and tau particles of interest to this discussion is about 2.0 MeV/(g/cm\(^2\)) in ice and rises to about 2.5 MeV/(g/cm\(^2\)) at the high end of our energy range (100 GeV).

### 3.2.2 Bremsstrahlung

For high energies, the Bremsstrahlung process becomes significant. At the energies explored in this dissertation, the energy loss of electrons due to the Bremsstrahlung process is most significant. The energy loss for electrons from the Bremsstrahlung process can be described by [23]

\[ - \frac{dE}{dx_{\text{Brems}}} = 4\alpha N_A \frac{Z^2}{A} \nu_e^2 E \ln \frac{183}{Z^2} \]

where \(\alpha\) is the fine-structure constant \((\alpha \approx 1/137)\). The other quantities in Eq. (3.6) have the same meanings as in Bethe-Bloch. As mentioned earlier, energy loss due to Bremsstrahlung is of particular importance for electrons. For heavy particles, the Bremsstrahlung energy loss is suppressed by the factor \(1/m^2\). This also means that electrons will be greatly influenced by Bremsstrahlung radiative losses at much lower energies than, for example, a muon. The energy loss, however,
increases linearly with energy, and is therefore important for all particles at high energies.

### 3.2.3 Electron-Positron Pair Production and Nuclear Interactions

In addition to Bremsstrahlung, charged particles can also lose some of their energy by direct electron-positron pair production, or by photo-nuclear interactions where a photon is exchanged with the nucleus of an atom. The energy loss due to these two interaction processes also varies linearly with energy and so it is important at high energies where it is comparable to Bremsstrahlung losses.

### 3.2.4 Cherenkov Radiation

For the results discussed in this dissertation, perhaps the most interesting light emission mechanism for charged particles involves Cherenkov radiation. When a charged particle moves in a dielectric medium with refractive index $n$, and has a velocity $v$ that exceeds the phase velocity of light in that medium, $c_n = c/n$, it will emit electromagnetic radiation known as Cherenkov radiation. The threshold for this effect depends on the particle’s speed. Cherenkov radiation only occurs if $v \geq c/n$ or, $\beta = v/c \geq 1/n$. It is important to note, that the index of refraction, $n$, is really a function of wavelength behaving like

$$n = 1.55749 - 1.57988\lambda + 3.99993\lambda^2 - 4.68271\lambda^3 + 2.09354\lambda^4 \quad (3.7)$$

in ice. For muons in ice and considering photons with a wavelength of 400nm, $n$ is $\sim 1.32$ and the energy threshold for the Cherenkov effect will be

$$E_C = \frac{m_{\mu}}{\sqrt{1 - \frac{1}{n^2}}} \approx 161 \text{ MeV}. \quad (3.8)$$

This means that we can safely assume the leptons will exhibit this effect since our neutrino energies are always greater than 1 GeV. To better visualize this light emission, consider the Cherenkov radiation of a particle of charge number $z$ in the experiment that will be examined further in this dissertation, the IceCube
DeepCore (ICDC) array. The particle creates a certain number of photons in the range where the photo-multiplier tubes of the experiment are most sensitive ($\lambda = 300$ nm up to $\lambda = 500$ nm). The number of photons can be calculated from the Frank-Tamm-formula [26]:

$$\frac{d^2 N}{dx d\lambda} = \frac{2\pi \alpha}{\lambda^2} \left( 1 - \frac{1}{\beta n} \right).$$

(3.9)

Cherenkov radiation is emitted at an angle of $\theta_C = \arccos 1/n\beta$ relative to the direction of the particle velocity. For relativistic particles ($\beta \approx 1$), the Cherenkov angle is $\sim 42^\circ$ in water, and $1.4^\circ$ in air. In water, around $2.6 \times 10^4$ photons per meter are produced between 300nm and 500nm by a singly charged relativistic particle. The corresponding number in air is 30 photons per meter. The Cherenkov effect is utilized in large water/ice Cherenkov detectors for neutrino astronomy. For charged particles in the energy range considered in this dissertation, 10 GeV to 100 GeV, the energy loss due to Cherenkov radiation is negligible compared to the ionization and Bremsstrahlung processes.

### 3.3 Propagation of Light

Besides providing an excellent material for both constructing a $\sim$ km$^3$ detector and serving as the source of nuclei for neutrino weak interactions, the deep glacial ice found at the South Pole is one of the most transparent materials on the planet. Due to the pressure of the accumulating overburden of ice, the ice below $\sim 1500$m is almost devoid of light scattering air-bubbles and is thus ideal for detection of Cherenkov radiation from charged particles. Most of the light attenuation in the ice is therefore caused by tiny micron sized pieces of dust. The scatterings and absorptions in the Antarctic ice have been very well measured, (e.g., by AMANDA) and are summarized in the graphs below.

The effective scattering length, $\lambda_e$ is described by

$$\lambda_e = \frac{\lambda_s}{1 - <\cos \theta>},$$

(3.10)

where $\lambda_s$ is the average distance between scatters and $<\cos \theta>$ is the average
scattering angle. For Antarctic ice, the accepted value of the average scattering angle is $\sim 0.94$, which indicates that most scatters are in the forward direction increasing the effective scattering length. The effective scattering coefficient, $b_e = \lambda_e^{-1}$ is dependent on the light’s wavelength, $b_e \sim \lambda^{-0.9}$ [28]. The fraction of light that is absorbed per unit length is the absorptivity, or absorption coefficient. The reciprocal value, $a^{-1} = \lambda_a$ is the absorption length and can be thought of as the distance at which the survival probability of a photon drops by a factor of $e^{-1}$. For Antarctic ice, at the depth explored by this dissertation’s experimental focus, IceCube DeepCore, the scattering length is approximately 40m and the absorption length is approximately 135m [28].
Neutrino Oscillations: Experiment

There are two different classes of neutrino oscillation experiments. One type is an appearance experiment that begins with a flux of one flavor of neutrino at the source and measures the flux of a different flavor neutrino at a detector a distance, \( L \), away. The other type of experiment is a disappearance experiment where the same flux of neutrino that is produced at the source is measured at the detector. In general, besides the specifics of the experimental setup, the two most important aspects of these experiments are an accurate knowledge of the initial flux and the effect that neutrino oscillation mechanism will have on that initial flux. As discussed in Chapter 2, the oscillation probabilities are set by the phase of the sine in Eq. (2.12). This phase depends on three factors that set up the different types on neutrino flavor oscillation experiments:

- \( L \): The distance between the source of neutrinos and the detector of neutrinos sometimes called the baseline.
- \( E \): The energy of the neutrinos produced at the neutrino source.
- \( \Delta m^2 \): The mass squared difference (or splitting) between the relevant mass eigenstates.

Examining the relationship between these parameters in the phase of the sine in the oscillation probabilities presents three distinct areas to be considered:

- \( \frac{L}{E} \ll \frac{4}{\Delta m^2} \): In this case, either \( L \) is too small or \( E \) is too large. In most
cases, the detector is too close to the source of neutrinos. The oscillations
don’t have enough time to actually manifest themselves.

- $\frac{L}{E} \gg \frac{4}{\Delta m^2}$: In this case, either $L$ is too large or $E$ is too small. In most
cases, the detector is too far away from the source of neutrinos, such as in
astrophysics. Many oscillations have happened before the neutrino is de-
tected in the experiment. Although $L/E$ cannot be measured accurately, the
oscillations can still manifest themselves as an average transition probability.

- $\frac{L}{E} \approx \frac{4}{\Delta m^2}$: This is the sweet spot for measuring neutrino oscillations and
fitting parameters. Reactor, accelerator and atmospheric neutrino oscillation
experiments all fit into this category.

The last bit of Eq. (2.12) that is important for this discussion is actually more
readily seen in Eq. (2.16) and that is the $\sin^2 2\theta$ which sets the amplitude of the
oscillation.

Eq. (2.12) and experimental results have led to an understanding that neutrino
oscillations are primarily controlled by two independent mass splittings and three
mixing angles. Currently, these considerations have led to two areas of experi-
mental study. The first is a low frequency oscillation driven by a small mass splitting
that is primarily explored by solar neutrino experiments. The other is a higher
frequency oscillation driven by a much larger mass splitting and is studied pri-
marily by atmospheric neutrino experiments. In principle, reactor and accelerator
experiments can measure parameters related to either domain since the distance,
$L$, from the source to detector, can be adjusted. However, as will be discussed
further below, it is still difficult to probe the small mass splitting associated with
solar neutrino oscillations.

4.1 Solar Neutrino Experiments

Bahcall et al. go into great detail in Ref. [29] describing the Standard Solar Model
(SSM) of thermonuclear energy production and the resulting pure flux of electron
neutrinos to be expected on Earth. A summary of the eight primary neutrino
fluxes coming from the Sun is presented in Fig 4.1. Two of the most important

Fluxes for neutrino oscillation experiments are the neutrinos that come directly from the fusion of protons into helium (the p-p and pep neutrinos) and the $^8B$ neutrinos that come from the fusion of helium. The electron neutrinos that come directly from the fusion of protons are by far and away the greatest source of solar neutrinos on Earth, encompassing about 98% of all solar neutrinos. The trouble with these neutrinos is that they are low energy, with a peak in energy of only about 1.4 MeV for pep neutrinos and 0.42 MeV for the p-p neutrinos. This low energy means that the Cherenkov light measuring techniques explained in the previous chapter are quite difficult to implement. To study these low energy neutrinos, use of another type of experiment called a radiochemical experiment in which neutrinos interact with nuclei to create chemically separable species is often used. The other important flux of neutrinos, the $^8B$ neutrinos, are more energetic, with a peak energy of about 15 MeV, which opens them up to Cherenkov experiments that can provide added information not present in radiochemical experiments, such as neutrino direction and energy spectra.

Following a roughly chronological time-line, the Homestake experiment first began taking data in the late 60s and ran for almost 30 years [31]. It implemented the radiochemical method to measure the total flux of electron neutrinos coming
from the Sun using the reaction

\[ \nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^- . \quad (4.1) \]

This reaction has a threshold of 0.814 MeV and so the greatest flux of electron neutrinos from the p-p reaction is not included in the measurement. The rate that Homestake reported, in units of \(10^{-36}\) events per atom per second called a Solar Neutrino Unit (SNU) was \([31]\) 2.56 ± 0.16 (stat.) ± 0.16 (sys.) SNU. This should be compared with an expected value of 7.6 SNU \([29]\) predicted by the SSM. This deficit of electron neutrinos became known as the Solar Neutrino Problem (SNP). Similar radiochemical experiments in the 90s and early 2000s would replace the chlorine in the above reaction with gallium.

\[ \nu_e + ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} + e^- . \quad (4.2) \]

With a threshold of only 0.233 MeV, these experiments, the GALLEX and GNO experiments in Italy and the SAGE experiment in Russia, would now include the p-p neutrinos and act to confirm Homestake’s observed electron neutrino deficit \([32,33,34]\). GALLEX’s results were 77.5 ± 6.2 (stat.) \(\pm 1.3\) (sys.) SNU, GNO’s results were 69.3 ± 4.1 (stat.) ± 3.6 (sys.) SNU and SAGE’s results were 69.1 \(\pm 1.3\) SNU all in good agreement and all to be compared with an SSM prediction of 128 \(\pm 7\) SNU. By measuring the neutrinos from the p-p reactions, not only did these experiments confirm the SNP, they confirmed that hydrogen fusion is the Sun’s energy source.

A real breakthrough to the SNP came in the late 90s and on to today with water Cherenkov detectors providing the best proof of the neutrino flavor oscillation solution to the SNP. As mentioned in the introduction, the two big players in the solution to the SNP are the SK detector in Japan and the SNO detector in Canada, please see Figs. 4.2 and 4.3. Both detectors are Cherenkov detectors and make use of a number of interactions to measure neutrino fluxes. SK is an enormous 50kton water Cherenkov detector with a fiducial volume of 22kton for solar neutrino searches. Its principle detection method is elastic scattering (ES) of neutrinos with electrons,

\[ \nu_e + e^- \rightarrow \nu_e + e^- . \quad (4.3) \]
SK observed recoiling electrons with energies greater than $\sim 4.5$ MeV which correspond to neutrinos with energies greater than $\sim 4.7$ MeV and so only the $^8B$ solar neutrinos were included. The observed $^8B$ solar neutrino flux is $2.35 \pm 0.02\text{(stat.)} \pm 0.08\text{(sys.)} \times 10^6\text{cm}^{-2}\text{s}^{-1}$ \cite{37}. It is worth noting that the smaller precursor to SK, called simply Kamiokande, measured a similar flux of $^8B$ solar neutrinos $2.80 \pm 0.19\text{(stat.)} \pm 0.33\text{(sys.)} \times 10^6\text{cm}^{-2}\text{s}^{-1}$ \cite{38}. Comparing these to the SSM prediction of 5.6 reveals about a 50% discrepancy. The ES is primarily produced by $\nu_e$, but can also occur for $\nu_{\mu,\tau}$ so the numbers reported above include both. The count is dominated by $\nu_e$ since the cross section for $\nu_{\mu,\tau}$ ES is much smaller than for $\nu_e$'s.

$$\Phi_{ES} = \Phi_{\nu_e} + 0.1553\Phi_{\nu_{\mu,\nu}} \quad (4.4)$$

As stated earlier, one of the advantages of SK is that it can measure the $^8B$ solar neutrino spectra and directional information and therefore annual, night and day effects, which are very useful for neutrino flavor oscillation examinations. SNO took SK's capabilities further, by not only measuring ES between neutrinos and
electrons, but also charged current (CC) and neutral current (NC) interactions of neutrinos with deuterium nuclei,

\[ CC : \nu_e + d \rightarrow e^- + p + p, \]  

\[ NC : \nu_{\text{any}} + d \rightarrow \nu_{\text{any}} + p + n. \]  

Note in the above equations, that only the electron neutrinos participate in the CC interactions while all flavors participate in the NC interactions. This means that SNO can measure separately the electron neutrino flux and the total neutrino flux. Using 1kton of \( D_2O \) (later doped with salt) and almost 10,000 Photo Multiplier Tubes (PMTs), SNO measured the following fluxes [39]:

\[ \Phi_{CC} = \Phi_{\nu_e} = 1.68 \pm 0.06(\text{stat.})^{+0.08}_{-0.09}(\text{sys.}) \times 10^6 \text{cm}^{-2}\text{s}^{-1}, \]
\[ \Phi_{NC} = \Phi_{\nu_{\text{total}}} = 4.94 \pm 0.21(\text{stat.})^{+0.38}_{-0.34}(\text{sys.}) \times 10^6 \text{cm}^{-2}\text{s}^{-1}, \]
\[ \Phi_{ES} = 2.35 \pm 0.22(\text{stat.})^{+0.15}_{-0.15}(\text{sys.}) \times 10^6 \text{cm}^{-2}\text{s}^{-1}. \]  

The last of these is in good agreement with SK. The first two fluxes can be used to show that the electron neutrinos make up roughly one-third of the total flux, in agreement with neutrino oscillation expectations. The electron neutrino flux
can be subtracted from each of the ES (see Eq. (4.4)) and NC fluxes to reveal the flux of tau and mu neutrinos, which are in good agreement with each other and with the neutrino flavor oscillation theoretical predictions. Fig. 4.4 shows the fluxes of the neutrinos with band widths at the 1-σ confidence level. The intersection of the bands serves as proof that the solar neutrino flux is not composed of only electron flavor neutrinos when detected on Earth. It should be noted that

**Figure 4.4.** Graph of $\nu_{\mu,\tau}$ fluxes vs. $\nu_e$ as measure by SNO. Source: Q. R. Ahmad et al., Phys. Rev. Lett. 89, 011301, (2002).

**Figure 4.5.** Graph of allowed regions obtained from global analysis of solar neutrino data. Source: B. Aharmim et al., Phys. Rev. C72, 055502, (2005).
SNO and SK used their capabilities of measuring night, day and annual seasonal fluxes separately and found no real discrepancy. This suggests that the neutrino oscillation effect that we observe is based more on oscillations that occur within the Sun and less on oscillations that occur on the propagation from the Sun to Earth.

It can be seen from a quick calculation that with an average Earth-Sun distance of $L = 1.496 \times 10^8$ km and neutrinos of energy $\sim 10$ MeV, the square mass splitting should be about $\approx 10^{-10}$ eV$^2$ for observable effects, but this is in contradiction to the global fit of all the mentioned solar oscillation parameter data which is $\Delta m^2 = 6.5^{+1.4}_{-2.3} \times 10^{-5}$ eV$^2$ and $\tan^2 \theta = 0.45^{+0.09}_{-0.08}$ with 1-\(\sigma\) uncertainties [39], see Fig. 4.5. Although it is beyond the scope of our discussion, current consensus is that matter effects, similar to those discussed in Chapter 2, are important for these oscillations and need to be considered in addition to the vacuum effects.
4.2 Atmospheric Experiments

Historically, atmospheric neutrinos provided the first clear evidence of neutrino oscillations with measurements by the Kamiokande experiment in Japan and the IMB experiment in Ohio demonstrating an “atmospheric neutrino problem” as early as the 1980s. Unlike the reactor and accelerator experiments that have also diligently verified atmospheric neutrino oscillation parameters, both the energy of the neutrinos, \( E \), and the source-detector distance, \( L \) are largely out of experimentalist’s control. Furthermore, the flavor of the neutrinos is a mixture of \( \nu_e \)'s, \( \bar{\nu}_e \)'s, \( \nu_\mu \)'s and \( \bar{\nu}_\mu \)'s instead of a pure flavor.

4.2.1 Origins of Atmospheric Neutrinos

Atmospheric neutrinos originate in the upper atmosphere when cosmic rays interact with the atmosphere and produce pions, muons and kaons that later decay creating electron and muon neutrinos, see Fig. 4.6. These cosmic rays have been studied extensively using ground arrays, satellite and balloon experiments and a composition of roughly \( \sim 85\% \) proton, \( \sim 11\% \) \( \alpha \)-particle, \( \sim 2\% \) electron and \( \sim 2\% \) heavier nuclei has been observed, although the composition is energy dependent [42]. The energy spectrum of these cosmic rays from 200 MeV to \( 10^{20} \) eV follows a series of power law forms [43]:

- For \( 10^8 \text{ eV} < E < 10^{15.5} \text{ eV} \), the spectrum is proportional to \( E^{-2.7} \).
- For \( 10^{15.5} \text{ eV} < E < 10^{17.7} \text{ eV} \), the spectrum is proportional to \( E^{-3.0} \).
- For \( 10^{17.7} \text{ eV} < E < 10^{18.5} \text{ eV} \), the spectrum is proportional to \( E^{-3.3} \).
- For \( 10^{18.5} \text{ eV} < E < 10^{20} \text{ eV} \), the spectrum is proportional to \( E^{-2.7} \).
- Above \( 10^{20} \text{ eV} \), the cosmic ray flux is believed to be suppressed by the GZK cutoff [44].

Please see Fig. 4.7. For atmospheric neutrino oscillation experiments, the most important neutrino creating cosmic rays are those between \( 10^9 \text{ eV} \) and \( 10^{13} \text{ eV} \), where the intensity of nucleons is approximately \( 1.8E^{-2.7} \) nucleons \( \text{cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ GeV}^{-1} \). The dominant decay chains that lead to the flux of observed atmospheric neutrinos
come from the decay of secondaries produced from the interactions of the cosmic rays with the atmosphere. Those decays include:

\[
\begin{align*}
\pi^+ & \rightarrow \mu^+ + \nu_\mu \\
\pi^- & \rightarrow \mu^- + \bar{\nu}_\mu \\
\mu^+ & \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \\
\mu^- & \rightarrow e^- + \bar{\nu}_e + \nu_\mu \\
K^+ & \rightarrow \mu^+ + \nu_\mu \\
K^- & \rightarrow \mu^- + \bar{\nu}_\mu \\
K_L & \rightarrow \pi^\pm + e^\pm + \nu_e (\bar{\nu}_e)
\end{align*}
\]

It is important to note that the flux of neutrinos is ultimately due to the number of nucleons not nuclei meaning that heavier nuclei although fewer in number do contribute significantly. Currently, the absolute flux of neutrinos still carries with
it a 20-30% error, but the ratio $R = \frac{\nu_e + \bar{\nu}_e}{\nu_\mu + \bar{\nu}_\mu}$ is claimed to be known to better than 5% since the ratio is based on a very good understanding of pion/muon decay and eliminates overall flux normalizations. Fig. 4.8 shows the flux as calculated by Agrawal, Gaisser et al. [45], which is used later in the calculations of Chapters 6 and 7.

The last bit of information that is critical for atmospheric neutrino oscillation experiments is the distance $L$, from the production of neutrinos to the detector. Models presented in Ref. [45] find that the most probable height, $h$, of production for $\nu_\mu$'s coming from pion decays is approximately 15km and for $\nu_\mu$'s and $\nu_e$'s from muon decay is a bit lower at 13km (due to the muon’s longer lifetime). The distance $L$ can then be found as

$$L = \sqrt{(R_\oplus + h)^2 - (R_\oplus - d)^2 \sin^2 \theta_z + (R_\oplus - d) \cos \theta_z}, \quad (4.9)$$
4.2.2 Atmospheric Neutrino Oscillations

The origin of the atmospheric neutrino problem began with the Kamiokande experiment in Japan. The detector itself was a 3000 ton (1500 ton fiducial) water Cherenkov detector that had 1000 PMTs and ran in the late 80s. Kamiokande measured an interesting discrepancy between the observed atmospheric neutrino events and the expected number. For $\mu$-like events, there were approximately 40% fewer events than expected, yet for the electron-like events, the number of observed matched the number expected. By measuring the ratio $R$ (described in the previous section) for observed events and $R$ for expected events and then finding the double ratio (or ratio of ratios) of those numbers, Kamiokande laid the groundwork for the atmospheric neutrino problem. In the sub-GeV range, the ratio of the ratios was $0.60^{+0.07}_{-0.06}$ (stat.) $\pm 0.05$ (sys.) and in the multi-GeV range, the ratio of the ratios was $0.57^{+0.08}_{-0.07}$ (stat.) $\pm 0.07$ (sys.) [46]. The results were later confirmed by the IMB water Cherenkov detector in Ohio and the iron tracking-calorimeter experiment Soudan-2 in Minnesota [47, 48].

Figure 4.8. Atmospheric Neutrino Fluxes using data from Refs. [45].
In 1998, SK (previously described, but operating with a 22.5 kton fiducial volume for atmospheric neutrinos) reported a startling discovery. By using the directional information available in SK’s measurements, the neutrino events can be separated by zenith angle. This means neutrinos traveling upward, through the entire earth, can be differentiated from neutrinos traveling downward. SK reported the asymmetry between up and down events (in the multi-GeV range where directional information is best) as $A_{\mu} = -0.296 \pm 0.048({\text{stat.}}) \pm 0.01({\text{sys.}})$ for muons and $A_e = -0.036 \pm 0.067({\text{stat.}}) \pm 0.012({\text{sys.}})$ for electrons. The asymmetry is in good agreement with $\nu_{\mu} \leftrightarrow \nu_{\tau}$ transitions. One of the most exciting contributions from SK came in the form of a full zenith angle distribution. By comparing the measured zenith angle dependent deficit with no oscillations expectations, the collaboration demonstrated that the data was consistent with neutrino oscillations with $\sin^2 2\theta > 0.82$ and $5 \times 10^{-4} \text{eV}^2 < \Delta m^2 < 6 \times 10^{-3} \text{eV}^2$ at the 90% CL [49]. More recently, by combining analysis to include fully-contained, partially-contained (described in the next chapter in more detail) and upward going muon atmospheric neutrinos, in 2010 SK improved the limits on the measurements of the atmospheric oscillation parameters to $0.583 \geq \sin^2 \theta \geq 0.407$ and $1.9 \times 10^{-3} \text{eV}^2 < \Delta m^2 < 2.6 \times 10^{-3} \text{eV}^2$ at the 90% CL [50].

### 4.3 Reactor and Accelerator Experiments

Reactor and accelerator neutrino oscillation experiments are free from the predetermined baselines, $L$, of solar and atmospheric neutrino oscillation experiments. With the freedom to choose, $L$, these terrestrial neutrino oscillation experiments can be designed to examine specific parameter spaces and so are most commonly used to examine either the solar parameters or the other set of oscillation parameters called atmospheric oscillation parameters. The experiments are often divided into groups depending on the length of their baseline, with short baseline (SBL) experiments usually having a baseline, $L$, of $\sim$10-100m and long baseline (LBL) experiments usually with an $L$ of more than $\sim$1km.
4.3.1 Reactor Experiments

Reactor experiments make use of the pure flux of $\bar{\nu}_e$ provided by nuclear reactors and by choosing $L$ carefully, look exclusively for disappearances of these fluxes. In terms of neutrino oscillations, they look for oscillations of $\bar{\nu}_e$ to $\bar{\nu}_{\mu,\tau}$. As discussed in Chapter 2, looking for neutrino flavors amounts to looking for the charged leptons left behind after a charged current interaction of a neutrino. However, in order to produce the corresponding lepton, the original neutrino needs to have sufficient energy for the lepton’s creation, and the flux of $\bar{\nu}_e$ from nuclear reactors provides $\bar{\nu}_e$’s with a peak energy of only about 3.6 MeV (and extends up to $\sim$8 MeV) far below the threshold for $\mu$ or $\tau$ lepton production. Therefore, any $\bar{\nu}_e$ that oscillates into a $\bar{\nu}_\mu$ or $\bar{\nu}_\tau$ will be unable to produce a charged lepton and suppress the observed event rate.

For disappearance oscillation experiments an accurate knowledge of the original flux is imperative. The flux density of $\bar{\nu}_e$ coming from a nuclear reactor is related to $P$, the thermal power of the nuclear reactor in MW, usually monitored to better than 1% and $L$, in m, the baseline distance from reactor to detector as

$$\Phi_\nu = 1.5 \times 10^{12} \frac{P}{\text{MW}} \frac{\text{cm}^{-2}\text{s}^{-1}}{L^2/\text{m}^2}. \quad (4.10)$$

It is worth noting that recently, Mueller et al. [51] have noted that approximately a $+3\%$ shift in the normalization of this flux needs to be made.

Many reactor experiments make use of the inverse beta decay previously used by Cowan and Reines,

$$\bar{\nu} + p^+ \rightarrow n^0 + e^+. \quad (4.11)$$

The reaction has a threshold of 1.804 MeV meaning that only $\sim$25% of the $\bar{\nu}_e$ reaching the detector are accessible. It is therefore important to reduce the background as much as possible. The most important shielding is against the hadronic and muonic components of cosmic ray interactions. This is fortunate because it requires only a minimum overburden, something that is often difficult to find near nuclear reactors.

KamLAND is one of the most important reactor oscillation experiments because its setup allows verification and further constraint on the solar parameters.
Built ~200km away from 85 local reactors in the mine previously occupied by the Kamiokande experiment, KamLAND uses 3000 tons of liquid scintillator and almost 2000 PMTs to look for the double coincidence event that is characteristic of the inverse beta decay reaction. After 764 ton-years of exposure, 258 $\bar{\nu}_e$'s were found versus expectations for the non-oscillated flux of about $365.2 \pm 23.7$ [52]. The observed deviations from the expected spectrum suggest neutrino oscillations with $\Delta m^2 = 7.9^{+0.6}_{-0.5} \times 10^{-5}$ eV$^2$ as seen in Fig. 4.9. Although the mixing angle is less constrained by KamLAND, a combination of this data with previous solar data gives Fig. 4.9 and a best fit angle of $\tan^2 \theta = 0.40^{+0.10}_{-0.07}$. The combination of KamLAND’s data with SNO and other solar data provide very compelling evidence that solar $\nu_e$’s oscillate to $\nu_\mu$’s and $\nu_\tau$’s as the solution to the SNP.

Whereas KamLAND’s design allowed it to explore the solar parameters and therefore the SNP, other reactor experiments have the ability to examine the analogous atmospheric neutrino problem. As discussed in the previous section, a deficit of $\nu_\mu$ neutrinos is observed in the flux of neutrinos coming from the cosmic ray interactions in the atmosphere. The question then becomes, do the $\nu_\mu$ become $\nu_e$, $\nu_\tau$ or $\nu_S$. Two LBL reactor experiments, CHOOZ in France and Palo Verde in Arizona, were constructed to measure $\bar{\nu}_e$ disappearance and explored the $\nu_e \leftrightarrow \nu_\mu$ oscillation channel explanation of atmospheric oscillations.

![Figure 4.9. Graph of allowed regions obtained from global analysis of solar neutrino data including KamLAND Reactor experiment. Source: T. Araki et al., Phys. Rev. Lett. 94, 081801 (2005).]
Both CHOOZ and Palo Verde used several tons of liquid scintillator, approximately 1km away from their respective nuclear power plants. Both experiments made observations that are consistent with no $\bar{\nu}_e$ oscillations as summarized in Fig. 4.10. In fact, CHOOZ has placed the tightest limit on any mixing angle involving electron type neutrinos such as the third mixing angle, called $\theta_{13}$ with $\sin^2 \theta_{13} < 0.12$ (90% CL) at $\Delta m^2 \approx 3 \times 10^{-3}$ eV$^2$ [53, 54]. These measurements combined with SK’s measurements provide favorable evidence that the missing $\nu_\mu$’s have oscillated to $\nu_\tau$’s rather than $\nu_e$’s.

Accurately determining $\theta_{13}$ is the goal of the next generation of reactor neutrino oscillation experiments. Three experiments mentioned earlier in Chapter 2, Double CHOOZ (in France), Daya Bay (in China) and RENO (in Korea), are under construction, with Double CHOOZ expecting data from its far detector in the summer of 2011. All of these experiments make use of near and far detectors to measure the neutrino flux at separate locations and drive down the systematics that have made measuring $\theta_{13}$ so difficult. When completed, expected sensitivities to $\sin^2 2\theta_{13}$ at 90% CL are better than 0.02 for Double CHOOZ (after 5 years), better than 0.01 for Daya Bay (after 3 years), and better than 0.02 for RENO (after 3 years) [16, 17, 18].
Figure 4.10. 90% CL exclusion plots for the $\nu_e \leftrightarrow \nu_\mu$ channel for the atmospheric oscillation parameters for the two reactor experiments CHOOZ on the top and Palo Verde on the bottom with Kamiokande allowed region results and other experimental limits included for comparison. In the Palo Verde plot, “swap” and “reactor power” refer to different methods for handling the analysis explained in [54]. Source for top: M. Apollonio et al., Phys. Lett. B 420, 397, (1998). Source for bottom: F. Boehm et al., Phys. Rev. D 64, 112001, (2001).
4.3.2 Accelerator Experiments

Similar to reactor oscillation experiments, accelerator experiments have the ability to set the baseline, $L$, to a length appropriate for the oscillation parameter space they are exploring. Additionally, by using neutrinos from accelerators, they can also set the energy and flavor of the neutrinos they wish to examine. One such experiment was the K2K experiment in Japan. K2K used SK as the detector, and an accelerator at the KEK laboratory as a neutrino source. Close to the neutrino source was another neutrino detector, as well, which aided in measuring the initial flux before the neutrinos traveled the 250km to SK. Built to check for atmospheric neutrino oscillations, the neutrinos delivered by the accelerator are an almost entirely pure beam of $\sim 1.5$ GeV $\nu_{\mu}$'s. They were created using the decays of positive pions from proton collisions with an aluminum target. K2K observed 112 $\mu$-like events compared with the expected $158.1^{+9.2}_{-8.6}$ events [55]. The energy distribution of the events are well described with $\nu_{\mu} \leftrightarrow \nu_e$ oscillations and $\Delta m^2 \approx 2.8 \times 10^{-3}$ eV$^2$.

A more recent LBL accelerator experiment is the MINOS detector using the accelerator at Fermilab as a neutrino source. It also utilizes a nearly pure beam of $\nu_{\mu}$'s this time at an energy of 3 GeV created from hadronic decays. Again utilizing a near detector to measure the original flux and a far detector (735km away) to check for atmospheric oscillation parameters, it observed 215 events compared to an expected $336 \pm 14$ events [56]. MINOS favors maximal mixing at $\Delta m^2_{atm} = 2.74 \times 10^{-3}$ eV$^2$. Please see Fig. 4.11. With twice the data in 2008, MINOS released new results favoring oscillations with $\Delta m^2 = 2.43 \pm 0.13 \times 10^{-3}$ eV$^2$ at the 68% CL [57] and again in 2010 with $\Delta m^2_{atm} = 2.35^{+0.11}_{-0.08} \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_{32} > 0.91$ [58]. Lastly, and perhaps most interestingly, by reversing the polarity of the detector magnets and inverting the current at the accelerator source, MINOS has been able to examine antineutrinos as well. Best fit parameters in that case were $\Delta m^2_{32} = 3.36^{+0.45}_{-0.40} \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_{32} = 0.86 \pm 0.11$ which constitutes a $2.3\sigma$ difference between neutrinos and antineutrinos [58]. Future accelerator experiments are also focusing on measuring the last mixing parameter, $\theta_{13}$. The T2K experiment in Japan looks for $\nu_e$ appearance in $\nu_{\mu}$ beams using SK, but this time with a new beam source at the JPARC accelerator facility in Tokai. The T2K experiment has a 295km baseline and has already begun taking data.
Figure 4.11. $\nu_\mu \leftrightarrow \nu_e$ parameter space allowed regions for the atmospheric neutrino oscillation parameters for the accelerator experiments K2K on the top, and MINOS on the bottom with Super Kamiokande allowed region results included for comparison. Source for top: M. H. Ahn et al., Phys. Rev. D74, 072003 (2006). Source for bottom: M. Kordosky et al., Phys. Rev. D 77, 072002, (2008), [arXiv:hep-ex/0711.0769].

After 5 years of data, T2K should be able to measure $\theta_{13}$ to an accuracy of an order of magnitude better than CHOOZ’s limit [62]. A very similar experiment called the NuMI Off-axis $\nu_e$ Appearance (NO$\nu$A) experiment at Fermilab will look for $\nu_\mu \rightarrow \nu_e$ but will also examine $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$. With a baseline of 810km, and beam energies peaked at 2 GeV, 6 years of data will allow NO$\nu$A to not only measure $\theta_{13}$, but may also allow it to resolve the neutrino mass hierarchy [63]. NO$\nu$A will
begin taking data in 2013.

4.3.3 Latest Global Fits to Oscillation Data

Combining the results of MINOS, SK, KamLAND and the corrections to the reactor flux spoke about previously, as well as the newest results from T2K (measuring $\theta_{13}$), leads to the global 3 flavor analysis best fit neutrino oscillation parameters (except for $\theta_{13}$ which is only based on T2K) with $\pm 2\sigma$ and for the normal hierarchy [64, 65]:

$$
\begin{align*}
|\Delta m^2_{31}| &= 2.28 - 2.64 \times 10^{-3} \text{ eV}^2 \\
\Delta m^2_{21} &= 7.24 - 7.99 \times 10^{-5} \text{ eV}^2 \\
\sin^2 \theta_{23} &= 0.41 - 0.61 \\
\sin^2 \theta_{12} &= 0.28 - 0.35 \\
\sin^2 \theta_{13} &= 0.03 - 0.28.
\end{align*}
$$

(4.12)
Chapter 5

The IceCube Neutrino Observatory and the DeepCore Array

5.1 Detector Overview

As discussed in Chapter 4, cosmic ray interactions in the atmosphere give rise to a natural beam of neutrinos. These atmospheric neutrinos in the GeV range have been used by the Super-Kamiokande (SK) detector to provide evidence for neutrino oscillations [66]. The large size of neutrino telescopes such as AMANDA, IceCube and KM3NeT make possible the detection of a large number of atmospheric neutrino events with a higher energy threshold, $\sim 100$ GeV, even though the neutrino flux decreases rapidly with energy ($\sim E^{-3}_\nu$). Built to detect neutrinos from astrophysical sources, or from the decay of Weakly Interacting Massive Particles (WIMPs) annihilations [67], at the high detection threshold energies of these ice/water Cherenkov detectors, neutrino oscillation effects would be small.

IceCube (IC) has been optimized for the detection of neutrinos at energies of a few TeV and above. Following from that, IC’s energy detection threshold is at $\approx 100$ GeV. In the energy range of a few hundreds of GeV the number of hit optical modules in IC is typically small and the capabilities to reconstruct such events are limited. On the other hand, there are various very interesting physics cases at these lower energies. These cover both astro-physical and particle-physics topics.

Recently, a low energy extension of the IC detector, the IceCube DeepCore
array (ICDC) has been proposed and deployed [68]. It consists of 8 densely instrumented strings (7 m spacing among optical modules) located in the deep center region of the IC detector plus the seven nearest standard IC strings. Originally consisting of 6 new strings, 2 additional strings added to ICDC. Its goal is to significantly improve the atmospheric muon rejection and to extend the IC neutrino detection capabilities in the low energy domain. The instrumented mass is 15 Mton. Such a low threshold array buried deep inside IC will open up a new energy window on the universe. It will search for neutrinos from sources in the Southern hemisphere, in particular, from the galactic center region, as well as for neutrinos from WIMP annihilation, as originally motivated. In [70] it was proposed that neutrino oscillation physics is a further motivation for building such an array. In particular, [70] analyzed the sensitivity of ICDC to the neutrino mass hierarchy.

ICDC can detect up to 100,000 atmospheric neutrino events per year, orders of magnitude beyond the present data sample, providing rich opportunities for detailed oscillation studies. In the same spirit of Ref. [70], I will concentrate here on the neutrino oscillation analysis in ICDC, focusing first on the track signal and then on the cascade signal. By exploiting the cascade channel, ICDC could provide strong evidence for tau neutrino appearance from oscillations of atmospheric neutrinos, greatly improving previous SK results on tau neutrino appearance evidence [71]. By exploiting the track signal, tighter constraints on neutrino oscillation parameters can be obtained.

The detection energy threshold of a neutrino telescope depends mainly on [72] the optical properties of the detection medium as discussed in Chapter 3, the geometry of the Cherenkov light detecting modules in the instrumented volume, with interactions at lower energies requiring a denser instrumentation, and the detection efficiency of the individual modules. ICDC takes each of these into account with closer module and string spacing, more efficient modules and instrumentation in the clearest ice.
5.2 Design and Geometry

The Cherenkov light detection modules on each of IC’s strings house a 25 cm diameter Hamamatsu PMT in which a pulse-digitizer mainboard is also placed earning it the name Digital Optical Module (DOM) because of the in-ice digitization. The DOMs of ICDC are much improved over IC’s DOMs. Each contains a 25 cm high quantum efficiency (HQE) Hamamatsu R7081MOD PMT that gives a 40% increased optical efficiency\[72\]. Simulations show that the effect these improved PMTs have on ICDC is enhanced signal efficiency at all energies especially at low energy, $\sim 10$ GeV, where the gain is orders of magnitude.

Figure 5.1. Diagram of ICDC’s layout with other local experiments. Source: T. DeYounge, 2011, “Neutrino telescopes: Catching Images of Ghost Particles,” Photonics Imaging Workshop, Boston, MA, MIT Lincoln Laboratory.
The new HQE PMTs mean that better signal resolution at lower energies is possible with a minimum number of new strings. ICDC adds 8 strings to the central region of IC in the clearest ice between 1450m and 2450m with 60 DOMs per string as shown in Fig. 5.2. Six of the strings use the new HQE PMTs while the two strings taken from IC use the original DOMs. 10 of the DOMs on each string are placed at depths between 1750m and 1850m, above a major dust-layer and the remaining 50 are in the clear ice at depths between 2100m and 2450m [68].
As shown in Fig. 5.2, another interesting design of ICDC is that IC’s 125 meter interstring spacing has been changed to less than 72 meters in ICDC. Furthermore, the DOMs themselves are only 7 meters apart in ICDC compared to 17 meters for IC. In total, ICDC is 6 times more photosensitive than IC. Lastly, while IC requires 8 DOMs to register an event, ICDC can distinguish an event with only 3 lowering the energy threshold even further [68].
5.3 Distinguishing Neutrino Flavors

Chapter 3 went into some detail about the light given off by charged particles as they move through ice. ICDC can use this light to distinguish the different neutrinos by looking for the signature light event that each lepton produces within the detector.

For NC interactions, the exchange of a Z boson between the original neutrino and a nucleon creates a hadronic shower. More details on hadronic showers can be found in Chapter 7. These NC induced hadronic showers are basically spherical and only about 5m in diameter so that for a detector like IC with DOM spacing of 17m or even ICDC with DOM spacing of 7m, the NC interaction needs to occur within the detector volume for an accurate energy reading. Although it has been reported that at high enough energy, the light emission will be stronger in the direction of the original neutrino’s movement, most of the directional information will be lost. It should also be noted that since NC interactions produce no charged lepton, hadronic showers of each neutrino flavor are indistinguishable.

Charged current (CC) interactions produce more interesting results for IC and ICDC. Since a CC interaction between a neutrino and a nucleon results in the creation of a charged lepton, it is possible to differentiate between lepton flavors and so accurately describe the original neutrino flavor. In the CC interactions of a $\nu_e$, a hadronic shower at the point of interaction, called the vertex point, is followed by the creation of an electron. As Chapter 3 points out however, the electron quickly loses energy and interacts electromagnetically producing an electromagnetic shower of particles. More details on electromagnetic showers can be found in Chapter 7. These two showers happen almost on top of each other and are indistinguishable. Furthermore, the overlapped hadronic and electromagnetic showers created by the CC interactions of the $\nu_e$ and the resulting electron are indistinguishable from the hadronic showers created by NC interactions.

Perhaps the most interesting CC interaction for IC and ICDC is that of the $\nu_\mu$. The $\mu$ lepton created after one of these interactions can travel quite far (see Chapter 6 for details), 100s of meters at 100 GeV. The Cherenkov radiation given by the decelerating muon leaves a track-like signature in the detector and so this type of event earns the name track. Good energy and directional information can
be gathered from these types of events especially at high energy.

![Figure 5.3. Types of μ tracks. From left to right, fully contained, partially contained, stopping, through going.](image)

Several categories exist for these track events depending on where the initial interaction takes place and where the track finally ends. If the interaction vertex is inside the detector, the initial hadronic shower will be recorded as well as the resulting track of Cherenkov light given off by the muon. If the entire event (initial interaction and track) occurs inside the detector it is referred to as a fully contained event and gives the best energy and directional reconstruction. If any part of either the original interaction vertex or the track occurs outside the detector volume then the event is only partially contained. In this case, energy information is lost in the initial hadronic shower and the unobserved track. If the track passes completely through the detector (possible at high energy) it is called through going and if it stops within the detector it is called a stopping event. ICDC is maximized to observe the tracks left by muons.

The last lepton, $\tau$, resulting from a CC interaction of a $\nu_\tau$ is interesting because it can exhibit qualities similar to both electrons and muons. At the energies relevant to this discussion, the short lifetime of the $\tau$ lepton means that the original hadronic interaction and the tau’s decay result in an overlap of the two events just as in the $\nu_e$ case. In $\sim 17\%$ of cases, the tau decay will result in a muon that can
then leave a track. If the tau has sufficiently high energy, however, a track will be left by the tau lepton after the initial hadronic shower. The tau will then decay and if the initial interaction, track and final decay shower all occur within the detector, a so called double bang event will be recorded resulting in clear identification of a $\nu_\tau$. 
Chapter 6

Atmospheric Neutrinos in IceCube DeepCore: Analysis of the Track Signal

In the past, atmospheric neutrinos in the Super-Kamiokande detector have shown evidence for neutrino oscillations and the first measurements of the parameters $|\Delta m^2_{31}|$ and $\sin^2 2\theta_{23}$ [66], providing compelling evidence for neutrino oscillations versus more exotic phenomena [75]. While facing more systematics than accelerator/reactor experiments due to the uncertainties in the (natural source) neutrino fluxes, atmospheric neutrinos provide great opportunities for exploring oscillation physics due to the large range of energies and path lengths that they span. The ICDC detector will collect a data sample which is a few orders of magnitude larger than that of the Super-Kamiokande (SK) experiment and can also measure energy and directional information, like SK, such that many of the systematic errors associated with unknown normalizations of fluxes, cross sections, etc. can be much better understood and reduced by using the data itself. The MINERVA [76] experiment will also provide important information about cross sections in the relevant energy range. The downward going neutrinos are largely unaffected by oscillations, so they can be used for determining the atmospheric neutrino flux.
6.1 The Importance of Precision Measurements of the Neutrino Oscillation Parameters

For all the problems that neutrino oscillation theory addresses, it raises at least an equal number of interesting questions. First, and probably foremost, is addressing the questions of how massless standard model neutrinos acquire the mass necessary for neutrino oscillations, why that mass is so fleetingly small and why neutrino mixing angles are so large in contrast to quark mixings which are very small. A number of theories have been presented from minimal extensions to the standard model that allow for Dirac neutrinos generating mass via the Higgs mechanism, to granting right-handed neutrinos the ability to move in extra dimensions as a way of explaining their extremely weak interactions with other standard model particles, to giving up the distinction between matter and antimatter for neutrinos and declaring the neutrino a Majorana particle (See Ref [77] and the references therein). Invariably, these theories are all at the very least extensions to the standard model of particle physics and owe their introduction into the scientific community’s consciousness to the discovery of neutrino oscillations. The best understanding of the correct models and the best information for future models therefore depends to a great extent on the best understanding of neutrino oscillations and this begins with precision measurements of the oscillation parameters that govern them.

Tied up in these models is also a hint of a solution to the reason why our universe is dominated by matter instead of having equal amounts of matter and antimatter. It was proposed in 1967 by Andrei Sakharov that to account for the extra baryons that make up the universe, there are three necessary conditions: (1) Conservation of baryon number must be violated, (2) The rate of pair annihilation must be reduced by demanding that the reactions violating the conservation of baryon number occur slower than the expansion rate of the universe and (3) CP-symmetry is violated allowing some matter producing reactions to occur more readily than the antimatter counterparts. It is with this last condition that neutrino oscillations offer a chance for experimental investigation by offering a solution to the baryon asymmetry in the universe through leptogenesis. For example, the same mechanism that may allow for the smallness of the neutrino’s mass may also allow for an asymmetry in leptons by demanding right-handed neutrinos and allowing for
neutrino decays generating more leptons which through non-perturbative effects enable the observed asymmetry in baryons. A better understanding of CP violation in weak interactions will greatly aid in the determination of the mechanism that leads to a matter dominated universe. Since no experimental evidence exists for CP violation in the string sector, observation depends upon weak interactions which are exemplified by the neutral neutrino. Neutrinos offer a unique chance to examine the hypothesis linking baryon asymmetry and leptogenesis by providing a means to create leptons without violating charge conservation. Here again, neutrino oscillations have been recognized as a powerful experimentally current avenue of CP-violation examination by studying the differences in the behaviors of neutrinos and antineutrinos. Determining the CP-violating phase present in the neutrino mixing matrix depends on sorting out genuine CP-violations and wrongly assessed CP-violations due to matter effects. It is here, too, that a precise knowledge of the oscillation parameters becomes imperative.

To sort through these various ideas and aid in model building, the most precise parameters available are indispensable. At the forefront of the examination of physics beyond the standard model is the theoretical and experimental study of neutrino oscillations. An enormous effort is being dedicated to the next generation of neutrino oscillation experiments, all in the pursuit of the best possible oscillation parameters. However, construction of these experiments and processing of the data takes time. There are three LBL accelerator experiments that are running and completely functional, MINOS, T2K and OPERA, one SBL experiment, MiniBOONE, one reactor experiment, KamLAND, and one atmospheric detector, SK. NOνA, Daya Bay, LBNE, Double CHOOZ and RENO will not begin taking for years. There are two take away points here: 1) The only detectors that currently have the ability to more precisely define the atmospheric oscillation parameters in neutrino oscillation theory are MINOS and SK, both of which dominate the global fits of these parameters and, 2) The ICDC experiment has the capability to see 100,000 events per year, is completed and running, and examines a wide range range of oscillation lengths and energies, some of which no other experiment looks at (and does so with neutrinos that were at first considered a background, just like Homestake and just like SK.)

The analysis that follows indicates (1) that the number of events available for
examination of oscillation effects is staggering, (2) that even within some very conservative assumptions, the parameter space can be extensively shrunk, especially for $\Delta m^2_{\text{atm}}$ oscillation parameter, aiding in neutrino hierarchy studies, (3) that asymmetries in that parameter space can be used to examine the octant ambiguity for $\theta_{23}$ and determine whether it is larger or smaller than $\pi/4$, (4) that the range of examinable $L/E$’s available to ICDC can offer complementarity to the fixed baseline experiments that measure only one $L/E$, and (5) that the incredibly large volume and statistics of ICDC can aid in studying CP-violations leading to our matter dominated universe by refining our knowledge of areas least effected by them, all with an experiment that is built, currently taking data and outpacing experiments that will not examine similar areas for at least 10 years.

The concentration of this analysis will, first, be on the very high statistics track signal expected in ICDC. This is dominated by muon events originating from the charged current (CC) interactions of muon neutrinos. Additional small contributions come from $\nu_e \rightarrow \nu_\mu$ oscillations for non-zero values of $\theta_{13}$, as well as $\nu_\mu \rightarrow \nu_\tau \rightarrow \tau \rightarrow \mu$ events. At low energies cascade events from various interactions of all neutrino flavors might not be distinguished from tracks, so they also contribute to the same event rate.

An outline of the steps followed in the analysis is as follows:

1. The calculation of the differential cross section for neutrino-nucleon deep inelastic scattering.

2. The energy dependent volume of ICDC.

3. The three-flavor oscillation probabilities with matter effects for all channels, discussed in Chapter 2.

4. The calculations of the event rates and relative contributions to the overall rate.

5. A discussion of the systematic uncertainties and the ways they may be improved by future measurements.

6. A discussion of the chi-squared analysis and the inclusion of the uncertainties.

7. The results of the analysis and application to the aforementioned motivation.
6.2 Deep Inelastic Scattering Interactions

In the energy range of interest, 10 GeV to 100 GeV, neutrino weak interactions with atomic nuclei are dominated by deep inelastic scattering where the nucleon is broken apart since \( E_\nu \gg m_N \), where \( m_N(N = p, n) \) is the mass of the struck nucleon. Neutrino weak interactions can be split into two types depending on which weak gauge boson \( W^\pm \), or \( Z^0 \) is exchanged during the interaction. For the purposes of calculating the correct observable spectra, it was necessary to use the differential cross sections instead of the total cross sections available in the literature.

6.2.1 The Muon and Electron Neutrino Deep Inelastic Scattering Charged Current Interactions

The interactions of importance for the CC interactions at the energies discussed here have the form

\[
\nu_l + N \to l^- + X \tag{6.1}
\]

and

\[
\bar{\nu}_l + N \to l^+ + X. \tag{6.2}
\]

At lowest order, these processes can be represented by the Feynman diagram in Fig. 6.1.

Neglecting the mass of the final (and initial) lepton in each of these interactions, I shall denote the four-momentum of the incoming neutrino as \( p_\nu \), the four-momentum of the outgoing charged lepton as \( p_l \), the four-momentum of the nucleon as \( p_N \) and the four-momentum of the final state hadrons as \( p_X \). The four-momentum transfer (final lepton momentum minus initial lepton momentum) carried by the \( W \) is \( q \), given by

\[
q = p_\nu - p_l = p_X - p_N. \tag{6.3}
\]

Using these definitions, we can write down five important kinematic variables:

\[
s = (p_\nu + p_N)^2 = m_N^2 + 2p_\nu \cdot p_N,
\]

\[
Q^2 = -q^2 = 2p_\nu \cdot p_l \geq 0,
\]
Figure 6.1. Feynman diagram of $\nu_l + N \rightarrow l^- + X$ and $\bar{\nu}_l + N \rightarrow l^+ + X$ CC interaction through W boson exchange.

\[ x = \frac{Q^2}{2p_N \cdot q}, \]
\[ y = \frac{p_{N \cdot q}}{p_{N \cdot p_{\nu}}}, \]
\[ W^2 = (q^2 + p_N)^2, \]  

(6.4)

where $s$ is the Lorentz-invariant squared center-of-mass energy of the lepton-nucleon system, $x$ is the fraction of the nucleon’s momentum carried by the struck quark, also called the Bjorken scaling variable, $y$ is the fraction of the lepton’s energy lost and $W$ is the mass of the recoiling system, $X$. The deep inelastic scattering gets its name when one takes $Q^2 \gg m_N^2$ (deep) and $W^2 \gg m_N^2$ (inelastic). Since $x$ and $y$ range between 0 and 1, in the massless lepton limit, $s \gg m_N^2$ and the kinematic relation $xy = Q^2/(s - m_N^2)$ becomes more simply $xy = Q^2/s$.

In the laboratory frame, where the nucleon is assumed at rest, the kinematic variables can be rewritten in a slightly more practical form as

\[ Q^2 = 4E_{\nu}E_l \sin^2 \frac{\theta}{2}, \]
\[ x = \frac{Q^2}{2m_N(E_\nu - E_l)}, \]
\[ y = 1 - \frac{E_l}{E_\nu}. \]  

(6.5)

where \( \theta \) is the lepton scattering angle measured with respect to the direction of the incoming neutrino. Using this notation, the charged current DIS neutrino and antineutrino cross sections are given by

\[
\frac{d^2\sigma^{\nu (\bar{\nu})}}{dx \ dy} = \frac{G_F^2 M_N E_\nu}{\pi(1 + Q^2/M_W^2)^2} \left[ (y^2 x) F_{1W^\pm}^1(x, y) + \left[ (1 - y) \right] F_{2W^\pm}^2(x, y) \right. \\
\left. + \left[ xy(1 - \frac{y}{2}) \right] F_{3W^\pm}^3(x, y) \right],
\]

(6.6)

where \( F_{1W^\pm} \) are the structure functions, generally taken to be functions of both \( q \) and \( x \) although when using Bjorken scaling, \( x \) is the dominant kinematic quantity at tree-level. The structure functions are sums of the probability densities called a parton distribution functions that make up the nucleon. Together, these form a description of the structure of the nucleus. The parton distribution functions in the DIS scheme tabulated by the CTEQ6 collaboration were used to calculate the CC cross section and are incorporated into the structure functions as outlined in Ref. [22].

### 6.2.2 The Muon and Electron Neutrino Deep Inelastic Scattering Neutral Current Interactions

For the NC interactions, however, a charged lepton will not be produced and the interactions of importance instead have the form

\[ \nu_l + N \rightarrow \nu_l + X \]  

(6.7)

and

\[ \bar{\nu}_l + N \rightarrow \bar{\nu}_l + X. \]  

(6.8)
At lowest order, these processes can be represented by the Feynman diagram in Fig. 6.2.

Figure 6.2. Feynman diagram of $\nu_l + N \rightarrow \nu_l + X$ and $\bar{\nu}_l + N \rightarrow \bar{\nu}_l + X$ NC interaction through Z boson exchange.

The kinematics and assumptions from the CC case still apply with the CC’s final lepton’s four-momentum being replaced by the final neutrino’s four-momentum. The NC is mediated by a Z boson, so in the above expression for the charged current DIS neutrino and antineutrino cross sections, the $M_W$ is replaced by the $M_Z$. Additionally, the structure functions will pick up factors of the neutral-current vector and axial couplings as outlined in [11].

For illustrative purposes, the total cross sections were also calculated and are presented in graphical form in Figs. 6.3 and 6.4. Note that the antineutrino CC/NC cross sections are about half of the value for neutrinos in the energy range explored. The neutral current (NC) neutrino-nuclei interaction (associated with $Z^0$) cross section is somewhat less than the charged current (CC) (associated with $W^\pm$) cross section. It should also be noted that neutrino interactions with electrons are also possible, but, in general, possess a much smaller interaction cross section and can safely be neglected at these energies.
Figure 6.3. Energy Dependent Graph of Neutrino (and Antineutrino) Nucleon CC interaction Cross Section.
Figure 6.4. Energy Dependent Graph of Neutrino (and Antineutrino) Nucleon NC Interaction Cross Section.
6.3 Description of the Track Analysis

The investigation focuses on the neutrino energy range between 10 GeV and 100 GeV. We present results in terms of observable muon energy. Note that the energy of secondary muons from CC interaction in the 10 − 100 GeV neutrino energy range of interest here is \( \langle E_\mu \rangle = 0.52E_\nu \) and \( \langle E_\bar{\mu} \rangle = 0.66E_\bar{\nu} \), respectively for neutrinos and antineutrinos [22]. For our numerical calculations, the differential neutrino interaction cross-sections as given by [22] with CTEQ6 parton distribution functions are used.

As previously mentioned, upward-going neutrinos in the analysis for determining the neutrino oscillation parameters are included. The down-going neutrino events are not sensitive to oscillations, but could improve uncertainties in the normalization of the atmospheric neutrino flux.

6.4 Muon Neutrino Charged Current Interaction Events

The number of muon events from muon neutrino interactions as a function of observable energy and zenith angle is evaluated as:

\[
N_{i,j;\mu}(E_\mu, c_\nu) = 2\pi N_A \rho t \int_{E_i}^{E_i+\Delta_i} dE_\mu \int_{c_\nu,j}^{c_\nu,j+\Delta_j} dc_\nu \int_{E_\nu}^{\infty} dE_\nu V_\mu \frac{d\sigma^{CC}}{dE_\nu}(E_\nu, E_\mu) \left( \frac{d\phi_{\nu\mu}(\theta, E_\nu)}{dE_\nu d\Omega} P_{\nu\mu \rightarrow \nu\mu}(E_\nu, \theta) + \frac{d\phi_{\nu\bar{\mu}}(\theta, E_\nu)}{dE_\nu d\Omega} P_{\nu\bar{\nu} \rightarrow \nu\bar{\nu}}(E_\nu, \theta) \right) + \nu \rightarrow \bar{\nu} ,
\]

(6.9)

where \( N_A \) is Avagadro’s number, \( t \) is the time for which the detector runs, \( \rho \) is the density of the material, which is 0.9 g/cm\(^3\). In this equation, \( \Delta_i \) and \( \Delta_j \) are, respectively, the bin widths of the \( i \) energy bin and \( j \) zenith angular bin (defined by \( c_\nu = \cos \theta_{\text{zenith}} \)), \( t \) is the exposure time, \( d\phi_{\nu\nu} \)’s are the atmospheric (anti)neutrino differential spectra and \( d\sigma^{CC}/dE \) are differential CC (anti)neutrino cross sections. The energy bin widths are taken to be 5 GeV observable energy and the angular bin
widths considered, when such angular information is considered available, are four different angular bins: \( c_\nu \in (-1, -0.9), (-0.9, -0.8), (-0.8, -0.7) \) and \((-0.7, 0)\). In the analysis, we examined other energy and angular bin widths as explained below.

The oscillation probabilities \( P_{\nu_\alpha \rightarrow \nu_\beta} \) have been obtained in a full three flavor scenario, including matter effects. While the signal is dominated by muon disappearance, given the very high statistics, sub-dominant effects become observable if \( \theta_{13} \) is within reach of the near future reactor and accelerator experiments.

The vast majority of the track events come from the surviving \( \nu_\mu \) flux in the above calculation, however to improve sensitivity to oscillation parameters, additional sources of tracks were also evaluated, including \( \nu_\tau \rightarrow \tau \rightarrow \mu \) (to be discussed in the next section) and \( \nu_e \rightarrow \nu_\mu \rightarrow \mu \) (which is largely controlled by the size of smallest mixing angle, \( \theta_{13} \)). The resulting muon tracks constitute a small contribution to the overall flux, since the atmospheric flux is already quite low for \( \nu_e \) when compared to the \( \nu_\mu \) flux and the branching ratio for tau decays to muons is only 17\% (see Chapter 7 for a detailed discussion of tau decays in our analysis). These events amount to less than 5\% of the total number of tracks, but nevertheless were included in the analysis. As T2K and MINOS have begun to show, the size of \( \theta_{13} \) may actually be closer to the larger values that we examined in the analysis that follows which means these events will become more important and numerous.
6.5 Tau Neutrino Charged Current Interaction Events: Tau to Muon Decay Channel

The rate for tau charged current events that result in a decay to a muon is given by:

\[
\frac{dN^\mu}{dE_{\text{obs}}} = 2\pi n_T \int d\cos \theta \int_{E_{\text{obs}}}^{\infty} dE_\nu \int_{E_{\text{obs}}}^{E_\nu} dE_\tau \left( \frac{d\phi_{\mu\nu}(\theta, E_\nu)}{dE_\nu d\Omega} P_{\nu_\mu \rightarrow \nu_\tau}(E_\nu, \theta) + \frac{d\phi_{\mu\nu}(\theta, E_\nu)}{dE_\nu d\Omega} P_{\nu_\tau \rightarrow \nu_\nu}(E_\nu, \theta) \right) \frac{d\sigma^{\text{CC}}}{dE_\tau}(E_\tau, E_\nu) \frac{dn^\mu}{dE_\tau}(E_\tau) + (\nu \rightarrow \bar{\nu}). \tag{6.10}
\]

In terms of dimensionless variables typically used for these cross-sections the number of events is evaluated as: \(d\sigma^{\text{CC}}/dE_\nu \sim 1/E_\nu d\sigma^{\text{CC}}/dy\) with \(y = 1 - E_\tau/E_\nu\) and \(dn^{\text{had}}/dE_\tau \sim E_{\text{obs}}/E_\tau^2 dn^{\text{had}}/dz\) with \(z = 1 - E_{\text{obs}}/E_\tau\). These events also result in a muon and are included in the event rate if the energy of the resulting muon is high enough to be included, 5 GeV. One final note is that the charged current DIS cross section used for the tau neutrino needs tau lepton production effects to be incorporated. The details of how this was done can be found in the next chapter.

Because of the kinematics and decay relationships, to produce, via tau decay, a muon of the same energy, the energy of a \(\nu_\tau\) needs to be about 2.5 times higher than a \(\nu_\mu\) (on average). This addition seems like it should be small due to the rapidly decreasing flux spectrum of the \(\nu_\mu\)'s, but it turns out to be non-negligible (\(\sim 10\%\)) because, as seen in Fig. 2.5, the first maximum in the \(\nu_\mu \rightarrow \nu_\tau\) oscillation probability (minimum in the \(\nu_\mu \rightarrow \nu_\mu\) survival probability as seen in Fig. 2.6) is in the energy range of interest and for a large range the \(\nu_\tau\) flux can be significantly larger than the \(\nu_\mu\) flux. These events change the energy spectrum of the measured muon-like events. Furthermore, the added events contain information about the main oscillation parameters, \(\Delta_{31}\) and \(\theta_{23}\) which is the goal of this study [70].
6.6 The Atmospheric Flux and Normalizations

For the atmospheric (anti)neutrino fluxes, $d\phi_{\nu_e}/dE_d\Omega$, results from Ref. [45] are used. The atmospheric neutrino fluxes from Refs. [81] give similar results. The absolute electron and muon atmospheric (anti)neutrino fluxes have errors of $10\% - 15\%$ in the energy region of interest here [82]. Those errors are mostly induced by our incomplete understanding of hadron production, although the situation is expected to improve with HARP and MIPP data. In addition, a calibration of this overall systematic uncertainty can be performed by looking at different angular apertures/energy ranges where the oscillatory signal is not present. For the neutrino-antineutrino flavor ratios $\phi_{\nu_e}/\phi_{\bar{\nu}_e}$ and $\phi_{\nu_\mu}/\phi_{\bar{\nu}_\mu}$, the uncertainty is reduced to $\sim 7\%$ in the energy range explored in the current study \(^1\). Even smaller uncertainties are expected when the muon-to-electron flavor ratio $(\phi_{\nu_\mu} + \phi_{\bar{\nu}_\mu})/(\phi_{\nu_e} + \phi_{\bar{\nu}_e})$ is considered. Comments on the impact of the atmospheric neutrino flux uncertainties on the results below will be discussed, when the discussion on systematic uncertainties is made in the numerical analysis.

6.7 The Energy Dependent Volume of IceCube

DeepCore

The factor $V_\nu$ in Eq. (6.9) accounts for the effective detector volume. Unlike in many other detectors that use fixed geometric cuts and sharp detection thresholds, neutrino telescopes employ an energy-dependent effective volume, which depends on the type of observable considered, in order to maximize the number of events in their data analysis. Preliminary ICDC estimates [68] show that this effective volume is of the order of 4 Mtons for 10 GeV neutrinos, increasing with energy. Here results will be given for an effective volume estimate based on several different physics-motivated assumptions regarding the detector. An optimistic configuration consists in using the full $\sim 15$ Mtons detector mass, corresponding to the 250 m diameter, 350 m height detector.

For a fixed detector configuration, two different scenarios to define the effective

\(^1\)In general, the different available computations of the atmospheric neutrino fluxes [45, 81] predict almost the same neutrino-antineutrino ratios.
volume are considered: (1) allow only fully contained events, and (2) allow some of the muons to exit the detector. In both cases, it is required that the initial neutrino interaction occurs inside the detector volume. Note that ICDC uses the large IceCube detector as veto, in order to exclude events that originate outside the ICDC region. For the muon tracks, it is especially important to take into account the energy-dependent muon range in ice. For the fully contained events, the volume is given in Eq. (6.11). For the fully contained events, for a detector with cylindrical shape of radius $r$ and height $h$, $V_\mu$ is given by [73]

$$V_\mu(E_\mu, \theta) = 2hr^2 \arcsin \left( \sqrt{1 - \frac{R_\mu^2(E_\mu)}{4r^2} \sin^2 \theta} \right) \left( 1 - \frac{R_\mu(E_\mu)}{h} |\cos \theta| \right), \quad (6.11)$$

where $R_\mu(E_\mu)$ is the energy-dependent muon range in ice described in more detail below. The fully contained events would yield a better energy measurement. On the other hand, the atmospheric neutrino flux decreases very fast with energy and retaining muons that exit the detector helps improve the statistics at higher energies. These two possibilities turn out to give equivalent results in the final analysis, as most of the information about oscillation parameters is contained in the low energy data, where all events are fully contained without imposing further constraints. ICDC aims to detect the cascade from the initial neutrino interaction in addition to the muon track, which would provide a better measurement of the total initial neutrino energy. Availability of such information would improve our analysis, which only relies on the measurement of the muon energy. Requiring the neutrino interact inside the detector volume allows for the improved analysis if the separate identification of the cascade and track becomes possible.

### 6.7.1 Energy Loss of Muons

Muons, like other leptons, are not subject to strong interactions and they can consequently travel relatively large distances. This makes them important for particle detection in astroparticle physics. The total energy loss of muons can be described by:

$$-\frac{dE}{dx} = a + bE$$

(6.12)
where \( a \) describes the ionization energy loss and is approximately 0.26 GeV/m.w.e. (m.w.e. is meters water equivalent), and \( bE \) summarizes the processes of muon Bremsstrahlung losses, electron-positron production, and photonuclear interactions, with \( b \) approximately equal to \( 0.36 \times 10^{-3} \)/m.w.e. [27].

This can be used to find an energy dependent range of a muon,

\[
R_\mu = \frac{1}{b} \ln \left( 1 + \frac{b}{a} E \right),
\]  

(6.13)

with \( E \) the original energy of the muon. This range, in ice, is about 25m at 5 GeV and 400m at 100 GeV.

**Figure 6.5.** Muon Range in Ice using Eq. (6.13).

### 6.8 Track events in IceCube DeepCore

Before describing the statistical analysis that examines the use of the high number of events in the precision measurements, I would like to point out a computation
of the expected numbers of events as a function of observable muon energy and neutrino direction. The Figs. 6.6 and 6.7 show the computed events *per year* for the first angular bin, with \( \cos \theta = (-1.0, -0.9) \), and the first energy bin, with \( E_\mu = (5 \text{ GeV}, 10 \text{ GeV}) \), respectively. The high number of events is very promising for a statistical analysis with the integrated number of events computed to be on the order of 100,000 per year. Additionally, I’d like to point out that this analysis makes use of the first 6 strings that make up ICDC. The additional 2 strings that were added will improve energy threshold and directional information and aid in the analysis that follows below.

### 6.9 The Statistical Analysis and Systematic Uncertainties

Since no detailed ICDC simulations of the expected systematic uncertainties are presently available, a discussion of results for different assumptions regarding these
uncertainties is required. Systematics that affect the detector observables include angular reconstruction, muon track length reconstruction (directly related to the energy measurement), uncertainties introduced by the modeling of light propagation and light detection efficiency of the optical modules. The uncertainty on the energy and angular measurements is related to the estimation of the effective area/volume, which is a function of both the muon energy and the muon direction. As previously mentioned, ICDC does not have a sharp energy threshold and the ICDC analysis shows that the detector can trigger muons with energies as low as 1 GeV and reach an effective volume of 4 Mtons for 10 GeV neutrinos [68]. In our analysis we used different assumptions about possible energy thresholds. 5 GeV muon energy is the lowest energy threshold we considered. Given the detector properties, an estimate of the energy threshold to be given by a 25m track length uncertainty is used, which translates into about 5 GeV muon energy.

The atmospheric neutrino data is explored in terms of sensitivity to the main oscillation parameters, $\Delta m^2_{31} = \Delta m^2_{\text{atm}}$ and $\theta_{23}$ using a $\chi^2$ statistical analysis in the $(\sin \theta_{23}; \Delta m^2_{\text{atm}})$ parameter space, which contains both statistical and systematic uncertainties.
uncertainties. For a particular handling of inclusion of the $\theta_{13}$ effects (represented by the subscript “t”), and the $(\sin \theta_{23}$ and $\Delta m^2_{atm}$) parameters chosen by nature (indicated by the red dot in Figs. 6.8 - 6.11 and the discussion of $\theta_{13}$ that follows), we consider the number of $\mu$-like events $N_{ij}^{ex}(\sin \theta_{23}; \Delta m^2_{atm})$ measured by an experiment in the $i$- and $j$-th muon energy and $c_{\nu}$ bins (see Eq. (6.9)). These events include the $\nu_\mu$ and $\bar{\nu}_\mu$ signal, as well as the background secondary muons from $\nu_\tau$ and $\bar{\nu}_\tau$’s and the small contribution from $\nu_e(\bar{\nu}_e) \rightarrow \nu_\mu(\bar{\nu}_\mu)$ which is directly related to the included $\theta_{13}$.

The $\chi^2$ statistics, for a “theoretical” model including $\theta_{13}$ effects and the parameters $(\sin \theta_{23}; \Delta m^2_{atm})$ is defined as

$$\chi^2(\sin \theta_{23}'; \Delta m^2_{atm}) = \sum_{i=1,9} \sum_{j=1,4} \left[ \frac{N_{ij}^{ex}(\sin \theta_{23}'; \Delta m^2_{atm}) - N_{ij}^{th}(\sin \theta_{23}; \Delta m^2_{atm})}{\sigma_{ij}^{ex}(\sin \theta_{23}'; \Delta m^2_{atm})} \right]^2. \quad (6.14)$$

Here $N_{ij}^{th}(\sin \theta_{23}; \Delta m^2_{atm})$ is the expected event number from both signal $\nu_\mu$’s and background $\nu_\tau$’s and $\nu_e$’s given by a “theoretical” model using the best fit parameters and $N_{ij}^{ex}(\sin \theta_{23}'; \Delta m^2_{atm})$ represents the experimental value of the number of events given by the choice of parameters currently being examined. The variance $\sigma_{ij}^{ex}$ is calculated from experimental events with systematic uncertainties (as discussed below). We minimize $\chi^2$ in Eq. (6.14) for $(\sin \theta_{23}; \Delta m^2_{atm})$ parameters (i.e., 2 d.o.f). Displayed in Figs. 6.8 - 6.11 are the 90% confidence levels (CL) defined for $\chi^2 > 4.61$ for 2 d.o.f statistics.

To account for systematic uncertainties, an inclusion of an overall error computed as a fraction $f_{sys}$ of the number of events in each bin was made: $f_{sys} \cdot N_{i,j}(E_\mu, c_\nu)$, added in quadrature with the statistical error. For the analysis, $f_{sys}$ values of 5% and 10% are considered. This systematic uncertainty can account for the issues discussed above, including uncertainties in fluxes, cross-sections and total detector volume. Given that many of the uncertainties discussed can be reduced once data becomes available and the angular and energy regions not sensitive to the oscillation signal are used, a 10% overall systematic uncertainty is likely an overestimate of what can be achieved. Other types of uncertainties, concerning angular resolution, energy resolution and energy thresholds are explored in more
detail below, as different scenarios in the analysis are considered, since no detailed ICDC simulations of the expected systematic uncertainties are presently available.
6.10 Results

Figure 6.8 shows the 90% CL contours for the allowed parameter values for two degrees of freedom, with the atmospheric neutrino data collected in ICDC after a 10 years exposure. Note that our choice of 2 dof statistics is rather conservative as explained in the Appendix of Ref. [85]. The three plots correspond to three different reference values of $\theta_{23}$: $45^\circ$, $43^\circ$ and $41^\circ$, all within the 2 sigma presently allowed region. The very large statistics, of up to 100,000 events per year, provide very good sensitivity to oscillation physics and it can be seen that the precision will be significantly improved compared with the present one. Maximal mixing could be potentially excluded for values of $\theta_{23}$ below $42^\circ$.

The final sensitivity to the parameters of interest is however strongly influenced by various types of systematic uncertainties and, to a smaller extent, our knowledge of $\theta_{13}$. These effects are explored in some detail and show that under reasonable assumptions oscillation parameters can be measured with much higher precision than present data allows.

For Fig. 6.8 a reference value of $\sin^2 2\theta_{13} = 0.01$ is assumed. In order to understand the influence of this angle on the final sensitivity, results for three different assumptions, represented by different types of lines are shown. With our present knowledge of $\theta_{13}$ the results would be those represented by solid lines, which correspond to $\theta_{13}$ varying freely in its presently allowed range. The dashed lines show the results for the unrealistic case of a fixed $\theta_{13}$ at the assumed true value. The dotted lines corresponds to free $\theta_{13}$, but with a gaussian prior centered around the true value and with a 1 sigma uncertainty of 0.02. This is a very realistic situation, which assumes having an independent measurement of $\theta_{13}$ from a different experiment, which is expected on the relevant time scale. DoubleChooz should reach an uncertainty around 0.02 and Daya Bay around 0.01 or even smaller, which would improve this analysis even further. It can be seen, however, that even with our conservative assumption, the sensitivity is very close to the ideal situation of fixed $\theta_{13}$.

The different colors on the same plot depict the effects of the energy threshold and angular information. The blue lines assume 5 GeV muon energy bins and four

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Footnote 2: At the time of the calculations, T2K and MINOS had not yet released their June, 2011 results for $\theta_{13}$. 
Figure 6.8. The top left, top right and bottom panels show the 90% CL contours for 2 dof, for atmospheric mixing angles $\theta_{23} = 45^\circ, 43^\circ$ and $41^\circ$, respectively. The different colors account for different energy/angular thresholds (please see the text for details). The solid lines allow $\theta_{13}$ varying freely in its presently allowed range. The dashed lines show the results for a fixed $\theta_{13}$ at the assumed true value $\sin^2 2\theta_{13} = 0.01$. The dotted lines corresponds to free $\theta_{13}$, but with a gaussian prior centered around the true value $\sin^2 2\theta_{13} = 0.01$, with a 1 sigma uncertainty of 0.02. A systematic uncertainty of 5% has been included.

different angular bins: $c_{\nu} \in (-1, -0.9), (-0.9, -0.8), (-0.8, -0.7)$ and $(-0.7, 0)$. The green lines assume that in the lowest energy ((5 GeV, 10 GeV) observed energy) bin, directional reconstruction is very hard and a single angular bin of $c_{\nu} \in (-1, 0)$ is considered. The red lines correspond to eliminating the two lowest energy bins (below 15 GeV observed energy or 30 GeV incident neutrino energy) from the data,
thus setting the neutrino energy threshold around 30 GeV.

It can be seen that poor angular resolution will deteriorate the results somewhat, but the energy threshold is crucial for a good measurement of \( \theta_{23} \). For astrophysical source searches, a 30 GeV energy threshold would be sufficient and potentially even desired, in order to reduce the atmospheric neutrino background. The detector setup allows however for a much lower energy threshold, with an estimated effective volume of the order of 4 Mton at neutrino energies of 10 GeV. The oscillation effects become very small above 40 GeV so it is important to analyze the data at the lowest energies available in order to obtain the maximum amount of information about neutrino oscillation parameters. It is also very important for the measurement of atmospheric neutrino parameters to have events with energies both above and below the oscillation peak and have information about the shape of the energy spectrum, which is most sensitive to the oscillation parameters. For bins with energy above the peak, the allowed regions in the \((\sin \theta_{23}, \Delta m^2_{31})\) plane curve upwards, while for energies smaller than the peak they curve downwards. The combination of the two regions has a small intersection and provides a very good measurement. For higher energy threshold, for most path lengths, only the events above the peak would be observed and the sensitivity to \( \theta_{23} \) is greatly degraded.

In addition to the effects of \( \theta_{13} \), energy threshold and angular information effects, a 5% systematic error is added in the above results to account for flux/cross-section and other uncertainties, as previously discussed. This uncertainty can be greatly reduced by using the data itself to fit for a flux parameter and by including down-going and higher energy events.

Fig. 6.9 shows the equivalent to Fig. 6.8 but with a reference value of \( \sin^2 2\theta_{13} = 0.08 \). For the larger value of \( \theta_{13} \) strongly suggested by MINOS' and T2K's latest results, illustrated in Fig. 6.9, the contours become slightly asymmetric, as expected. This asymmetry in \( \theta_{23} \) could in principle be used to get sensitivity to the octant, i.e., determine whether \( \theta_{23} \) is larger or smaller than \( \pi/4 \). Our analysis shows that it may be possible to determine the octant for small \( \theta_{23} \), large \( \theta_{13} \) and favorable assumptions about energy threshold and angular information and systematics errors. For \( \sin^2 2\theta_{13} = 0.08 \) there is sensitivity to the octant at the 90% CL if \( \theta_{23} \) is smaller than 41°, while for \( \sin^2 2\theta_{13} = 0.1 \) it might be possible to have octant sensitivity for \( \theta_{23} \) smaller than 43°. These values are still in the 1 sigma
Figure 6.9. Same as Fig. 6.8 but assuming $\theta_{13}$ to be centered at $\sin^2 2\theta_{13} = 0.08$.

presently allowed regions for the atmospheric mixing angle $\theta_{23}$. The octant degeneracy \cite{86} is very hard to resolve and future neutrino facilities such as neutrino factories and/or superbeams \cite{87, 88, 89, 90, 91}, the combination of superbeams with atmospheric neutrino data \cite{92, 93}, or future iron calorimeter atmospheric neutrino detectors \cite{94} have been shown to provide powerful setups to resolve it. High statistics atmospheric neutrino data in ICDC could be a first step in addressing this degeneracy, if $\theta_{13}$ turns out to be large, which it seems to be and $\theta_{23}$ non-maximal. The detailed quantitative analysis of the octant sensitivity can be further refined. We have not yet done this, as the strong indication of a large $\theta_{13}$ were not present at the time of our original analysis, but it constitutes a future
area of interest to be investigated.

It is important to note that a three flavor analysis considering the physical quantity $\sin \theta_{23}$ can yield much better sensitivity than a two flavor one in terms of $\sin^2 2\theta_{23}$. In Ref. [95] it has been shown that going from $\sin^2 2\theta_{23}$ to $\sin \theta_{23}$ one can introduce errors of $10 - 20\%$ for $\theta_{23}$ values close to $\pi/4$.

All the above plots assume a reference value of $\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV$^2$. Changing the reference value of $\Delta m^2_{31}$ essentially moves the allowed regions up and down, without a significant change in the size and shape of the contours. However, it should be noted that regardless of the reference value the parameter space of $\Delta m^2_{31}$ is reduced dramatically.

Since the track signal is dominated by the muon neutrino disappearance channel, a non-zero CP-violating phase has a very small effect on the parameter sensitivity. For the largest values of $\theta_{13}$, the effect of the phase would be to slightly enlarge the $\Delta m^2_{31}$ allowed range towards lower values.

Considering smaller values for $\theta_{23}$ within the 1, 2 or 3 sigma presently allowed regions makes a better measurement even easier, allowing for both the exclusion of maximal mixing and, for relatively large values of $\theta_{13}$, for octant discrimination. Values of $\theta_{23}$ larger than 45° are not shown, as the results for $\theta_{23} = \pi/4 + \epsilon$ look like “mirror images” of those for $\theta_{23} = \pi/4 - \epsilon$. 
Figure 6.10. The top left, top right and bottom panels show the 90% CL contours for 2 dof assuming $\theta_{13} = 0.01$ and $\theta_{23} = 45^\circ$, 43° and 41°, respectively. The different colors account for different values of the systematic uncertainties, see the text for details. We have assumed free $\theta_{13}$ with a gaussian prior centered around the true value $\sin^2 2\theta_{13} = 0.01$, with a 1 sigma uncertainty of 0.02.

Figs. 6.10 and 6.11 depict the effects of systematic uncertainties, which are often a limiting factor in disappearance experiments. The blue contours assume no systematic error, green correspond to 5% and red to 10% systematic errors, for nine energy bins and four angular bins. Additionally, $\theta_{13}$ is assumed free with a gaussian prior centered around the true value $\sin^2 2\theta_{13} = 0.01$ and $\sin^2 2\theta_{13} = 0.08$ respectively, with a 1 sigma uncertainty of 0.02.

Notice that, while systematic errors from flux/cross-section uncertainties, etc. affect the parameter measurement at some level, they still allow for a fairly precise
Figure 6.11. Same as Fig. 6.10 but for $\theta_{13}$ centered around $\sin^2 2\theta_{13} = 0.08$.

determination of $\Delta m^2_{31}$ and $\theta_{23}$. The parameter space for $\Delta m^2_{31}$, for example, has shrunk by nearly half, even for the worst case scenario considered. For the cases assuming better energy threshold and angular information, the reduced parameter space is truly remarkable. As previously discussed, many of the systematics will improve with data, since normalization factors can be included as free parameters in the fit and determined with good precision, especially using the additional energy and angular bins where oscillations are not important. Information from near future experiments can also be used to reduce some of the uncertainties.

Good muon track reconstruction would be extremely useful, but relatively precise measurements can be achieved even with reduced angular resolution. The most important factor in the determination of the main oscillation parameters is
the energy threshold, as most of the sensitivity comes from neutrino energies be-
 tween 10 and 40 GeV. With the 10 GeV neutrino energy threshold expected to be
 reached with the 6 string detector considered here and with realistic assumptions
 about other parameters and uncertainties, the parameter determination will be
 very good. Two additional strings were deployed this year in the center of the
detector. These will likely improve the quality of the directional reconstruction
and the energy threshold and resolution and will thus lead to even better results.
Chapter 7

Atmospheric Neutrinos in IceCube DeepCore: Analysis of the Cascade Signal

Figure 7.1. $\nu_\mu$ survival probability and $\nu_\mu \rightarrow \nu_\tau$ oscillation probability as a function of the neutrino energy (in GeV), assuming upward going neutrinos and $\sin^2 2\theta_{13} = 0.1$.

The IceCube detector and its DeepCore extension are optimized for detecting
muon tracks from the charged current interactions of $\nu_\mu$. It is, however, possible to also detect cascades [78]. While these type of events no longer provide directional information, their energy can be measured quite precisely. Here the cascade events in the ICDC array are analyzed. There are several contributions to the cascade signal: charged current interactions of $\nu_e$, neutral current interactions of all neutrino flavors and electromagnetic and hadronic decays of tau leptons produced in charged current interactions of $\nu_\tau$.

An outline of the steps followed in the analysis is as follows:

1. A brief discussion of the importance of appearance of a flux of $\nu_\tau$ in the atmospheric flux.

2. The three-flavor oscillation probabilities with matter effects for all channels, discussed in Chapter 2.

3. An examination of the different types of cascade events that will be examined: electromagnetic and hadronic.

4. A detailed discussion of the calculation of the differential cross section for charged current neutrino-nucleon deep inelastic scattering for the tau neutrino which needs to include tau lepton production effects.

5. The effective volume chosen for the cascade events and justification of such.

6. A detailed discussion of the calculation of events in the separate cascade channels including the various decay modes of the tau lepton.

### 7.1 The Importance of a $\nu_\tau$ Flux Appearance in the Atmospheric Neutrino Flux

Nearly all neutrino oscillation experiments make use of a disappearing neutrino flux, e.g., SK looks at $\nu_\mu$ disappearance, reactor experiments look at $\bar{\nu}_e$ disappearance, etc. By comparison, relatively few experiments look for an appearance of flux, e.g., T2K looks at $\nu_e$ appearance in a $\nu_\mu$ beam and MiniBOONE does the same including anti-neutrinos. The observations of $\nu_\tau$ fare even worse. Indirectly,
SK was able to determine that the disappearing flux of $\nu_\mu$ they observe favors oscillations of $\nu_\mu \rightarrow \nu_\tau$. This was later, also indirectly, confirmed by the CHOOZ and Palo Verde experiments. The OPERA experiment is the first experiment constructed to look for $\nu_\mu \rightarrow \nu_\tau$ appearance and in June of 2010 reported their first candidate event, although it remains unconfirmed. The direct observation of a flux appearance of $\nu_\tau$ in the atmospheric neutrino flux has yet to be observed to unambiguously prove that $\nu_\mu \rightarrow \nu_\tau$ oscillation is the dominant transition channel at the atmospheric scale.

The results of this investigation show that ICDC will be able to statistically demonstrate a $\nu_\tau$ flux appearance by examining the atmospheric flux and effectively separating the types of observed events into either tracks or cascades. It has been pointed out that the number of tau neutrinos present in the non-oscillated atmospheric flux is orders of magnitude below the electron and muon type neutrino fluxes [96].

There are several important observations which suggest that the $\nu_\tau$ signal can be significant and can provide evidence for $\nu_\tau$ appearance from oscillations of $\nu_\mu$:

- First, the atmospheric electron neutrino and electron antineutrino fluxes at the relevant energies are significantly lower than the muon neutrino flux, such that the $\nu_e$ charged current interactions do not completely overwhelm the event rate.

- In addition, the energy range covered by ICDC corresponds to a maximum of $\nu_\mu \rightarrow \nu_\tau$ oscillations (minimum of $\nu_\mu$ survival), as can be noticed from Fig. 7.1.

- Therefore, the large flux of muon-type neutrinos ensures that a large flux of $\nu_\tau$ should be present.

These appearing $\nu_\tau$'s therefore erode the number of muon tracks recorded and increase the number of cascades detected. It is therefore possible to determine the additional number of cascades that will be generated by these $\nu_\tau$ and compare them to the irreducible background of cascades present from other neutrino-nucleon interactions and thus demonstrate the appearance of a $\nu_\tau$ flux that would be absent save for neutrino oscillations. The opportunity to study such a large number of
\(\nu_e\)'s is important, for example, in \(\nu_e\) interaction cross section studies and other aspects of the physics beyond the standard model leading to neutrino mass such as non-standard neutrino interactions.

## 7.2 Description of the Cascade Analysis

The neutrino energy range between 10 GeV and 100 GeV is investigated, assuming bins of 5 GeV width in the shower energy. Cascades have very little directional information, especially at these low energies, so the results were integrated over all upward going directions\(^1\). The downward going neutrinos are largely unaffected by oscillations, so they can be used for determining the atmospheric neutrino flux and thus the contribution of the \(\nu_e\) charged current interactions to the overall cascade rate. In the numerical calculations, the full three flavor oscillations were taken into account, as discussed in Chapter 2. It is however straightforward to see that solar parameters do not play an important role in the analysis due to the rather high energy threshold of ICDC. Also, \(\theta_{13}\) effects, while in principle observable for values of \(\theta_{13}\) close to the present bound, do not affect any conclusions regarding \(\nu_e\) rates, which are determined by the (maximal) atmospheric mixing angle \(\theta_{23}\).

## 7.3 Distinguishing Electromagnetic Cascades from Hadronic Cascades

There are two different types of cascades that are present in the analysis of the cascade channel in ICDC, electromagnetic and hadronic, each appearing in different situations by different mechanisms and with different light yields relative to the total energy of the shower. However, at the energies that we are discussing, the ICDC experiment will see each as a point source of light since the inter-DOM spacing is larger than the visible shower. Nevertheless, the light yield of each is slightly different as discussed here [97].

\(^1\)The integral was over the zenith angle direction \(\theta\) of the incident neutrinos, from \(c_\nu = -1\) (vertically upward going) up to \(c_\nu = 0\) (horizontally incident), \(c_\nu\) being the cosine of \(\theta\).
7.3.1 Electromagnetic Cascades

For electrons, Bremsstrahlung radiation is most important since the electron is so light. At the energies in question, an electron produced in a charged current interaction of an electron neutrino with a nucleon or an electron produced in a tau lepton decay will begin emitting high energy photons which will then materialize through pair production and create more electrons and positrons. These secondary particles will go on to do the same creating more particles each sharing the initial energy and thus individually possessing less energy. The resulting collection of produced particles and photons is referred to as an electromagnetic (EM) cascade, or shower. All of the incident electron neutrino energy is assumed to go into the cascade for the electron neutrino CC interactions. For the tau decays, the amount of energy in the cascade can be written as $E_\nu (1 - y) z'$, where $E_\nu$ is the energy of the incident neutrino, $(1 - y)$ is the fraction of this energy that is given to the tau lepton and $z'$ is the fraction of remaining energy given to the electron (which is approximately $(1 - z)$ where $z$ is the fraction of the tau lepton’s energy retained by the tau neutrino). The observable energy in the form of photons is approximately $0.96 \times 10^5$ Cherenkov photons per GeV for an EM cascade [97].

7.3.2 Hadronic Cascades

Strong and weak interactions are involved in the hadronic cascade process which results from the initial NC and CC interaction vertices of the neutrino and some of the decay channels (discussed in more detail below) of the tau leptons. On average, half of the incident neutrino energy goes into the initial hadronic cascade at the interaction vertex. Hadronic showers typically have a larger lateral spread than EM showers, although it should be noted that EM showers can be components of hadronic cascades. Energy resolution is slightly worse than for EM showers; for example, basically invisible neutrinos can leak out carrying energy as can other neutral particles. Muons may be created during tau decay and these are discussed below. Some secondary muons may also be produced after the hadronic decay during the shower evolution, but typically at much lower energies. At these energies, charged particle multiplicities are greater than 10 particles, meaning individual muon energies from charged pion and kaon decays will be less than 5%
of incident neutrino energy and may be below the threshold for Cherenkov light production. The observable energy in the form of photons is approximately $0.77 \times 10^5$ Cherenkov photons per GeV for a hadronic cascade meaning that for the same amount of energy a hadronic cascade will produce about 20% less light than an EM cascade. For this reason, the event rates for both the background and signal cascade events presented in the analysis below were kept separate since they are plotted vs cascade energy [97].

7.4 The Tau Neutrino Deep Inelastic Scattering Cross Section

In the previous chapter, the DIS differential cross sections for muon and electron neutrino-nucleon interactions neglected the mass of the initial neutrino and final charged lepton due to the fact that they are much smaller than the energy. It should be noted, that at the lowest energies, tau lepton production effects become important for the CC interactions, since the tau lepton is so massive. The NC interactions are unaffected by this problem and follow from the previous argument unchanged. An excellent discussion of the DIS tau neutrino nucleon cross section can be found in Ref. [21]. This was, of course, incorporated in the analysis of the cascade signal where production of tau leptons is possible. The differential cross sections were again calculated this time adding in the terms that contain either the mass of the tau lepton or the mass of the nucleon resulting in

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx \ dy} = \frac{G_F^2 M_N E_\nu}{\pi (1 + Q^2/M_W^2)^2} \left[ y^2 x + \frac{m_\nu^2 y}{2 E_\nu M_N} \right] F_1^{W\pm}(x, y) + \left[ (1 - \frac{m_\nu^2}{4 E_\nu^2}) (1 + \frac{M_N x}{2 E_\nu}) y \right] F_2^{W\pm}(x, y)$$

$$\pm \left[ xy (1 - \frac{y}{2}) - \frac{m_\nu^2 y}{4 E_\nu M_N} \right] F_3^{W\pm}(x, y) + \frac{m_\nu^2 (m_\nu^2 + Q^2)}{4 E_\nu^2 M_N x} F_4^{W\pm}(x, y) - \frac{m_\nu^2}{E_\nu M_N} F_5^{W\pm}(x, y) \right]. \quad (7.1)$$

The kinematic variables remain similarly defined as they were in the previous discussion, but the range for both $x$, the Bjorken scaling variable and $y$, the fractional
energy loss of the incoming neutrino need to be adjusted. The integration over x needs to be adjusted from its normal limits of 0 and 1 to

\[
\frac{m_r^2}{2M_N(E_\nu - m_r)} \leq x \leq 1
\]  

(7.2)

and \( y \) is now limited to run between \([21]\),

\[
a - b \leq y \leq a + b
\]  

(7.3)

where

\[
a = \frac{1 - m_r^2 \left( \frac{1}{2M_N E_\nu x} + \frac{1}{2E_\nu^2} \right)}{2 \left( 1 + \frac{M_N x}{2E_\nu} \right)}
\]

\[
b = \frac{\sqrt{\left( 1 - \frac{m_r^2}{2M_N E_\nu x} \right)^2 - \frac{m_r^2}{E_\nu^2}}}{2 \left( 1 + \frac{M_N x}{2E_\nu} \right)}.
\]  

(7.4)

A comparison of the \( \nu_\tau \) CC cross section to the \( \nu_\mu \) CC cross section, in the relevant 10 GeV - 100 GeV energy range is most easily seen by comparing the total cross sections. The ratio of these two total cross sections is provided in Fig. 7.2. Note that at the lowest energy, the \( \nu_\tau \) CC cross section is only 20% of the \( \nu_\mu \) CC cross section, and even at 100 GeV, the \( \nu_\tau \) CC cross section is still only \( \sim 76\% \). It should further be noted that even at 1000 GeV, the \( \nu_\tau \) CC cross section remains less than the \( \nu_\mu \) CC cross section. Once again, the parton distribution functions in the DIS scheme tabulated by the CTEQ6 collaboration were used to calculate the CC cross section and are incorporated into the structure functions as outlined in Ref. [22].
Figure 7.2. Ratio of the $\nu_\tau$ nucleon CC total cross section to the $\nu_\mu$ nucleon CC total cross section.
7.5 The Effective Volume for Low Energy Cascades

The instrumented volume of ICDC is basically a cylinder of 250 m diameter and 350 m height, i.e., the total physical mass is around 15 Mton. When analyzing cascades in ICDC, the low energy events however are mostly single-string events and the conservative option is to consider only a 40 m radius around each of the 6 densely instrumented strings (at the ICDC depth, the light attenuation length is 40-45m). This reduces the total mass to \(\sim 7\) Mtons, but avoids overestimating the observable low energy events.

7.6 Calculation of Different Types of Cascade Contributions

7.6.1 Electron Neutrino Charged Current Interaction Events

The first contribution to the irreducible background of cascade events are the cascades that result from \(\nu_e\)-nucleon CC interaction events. The initial interaction results in the production of a hadronic cascade and production of an electron which at these energies almost immediately produces an overlapping electromagnetic cascade. As a function of the kinematic variable \(y\) expressed earlier in the DIS differential cross section discussion, a fraction \(y\) of the incident electron neutrino’s energy goes into the initial hadronic decay and \((1-y)\) goes into the electron and resulting electromagnetic shower. These events constitute the majority of the background cascades, about 75%. The spectrum of \(\nu_e\) induced CC events as a function of shower energy is given by:

\[
\frac{dN_e}{dE_{\text{s}h\text{w}r}} = 2\pi n_T \int d\cos \theta \, V \, \sigma_{CC}^{(\nu_e)}(E_\nu) \left( \frac{d\phi_{\nu_e}(\theta, E_\nu)}{dE_\nu \, d\Omega} \right) P_{\nu_e \rightarrow \nu_e}(E_\nu, \theta) + \frac{d\phi_{\nu_\mu}(\theta, E_\nu)}{dE_\nu \, d\Omega} \, P_{\nu_\mu \rightarrow \nu_e}(E_\nu, \theta) \right) + (\nu \rightarrow \bar{\nu}) \, ,
\]  

(7.5)
where \( n_T \) is the number density of targets, \( V \) is the volume of the detector, \( \theta \) is the zenith angle direction of the neutrino and \( t \) is the observation time.

The second term of Eq. (7.5), which contains the contribution from oscillations of \( \nu_\mu \rightarrow \nu_e \), is negligible in practice. This contribution is very small even for the maximum currently allowed value of \( \sin^2 2\theta_{13} \) and after considering enhancements in the oscillation probabilities due to matter effects inside the Earth. It could, however, become relevant in the presence of non-standard neutrino interactions. For charged current electron neutrino interactions, the entire energy of the incident neutrinos is transferred to the aforementioned overlapping cascades and thus measured.

### 7.6.2 Electron Neutrino Neutral Current Interaction Events

For neutral current interactions, only a fraction \( y \) of the initial neutrino interaction is transferred to the cascade. Due to the steep energy dependence of the atmospheric neutrino fluxes, their contribution is thus expected to be smaller. The NC DIS cross sections discussed in Chapter 6 were used to ensure the proper shower energy spectrum. The spectrum for neutral current (NC) interactions of the electron neutrino flux as a function of the hadronic cascade energy is given by:

\[
\frac{dN_{NC,\nu_e}}{dE_{shwr}} = 2\pi n_T t \int d\cos \theta \int_{E_{shwr}}^{\infty} dE_{\nu} \frac{d\phi_{\nu}}{dE_{\nu}d\Omega} \frac{1}{E_{\nu}} \left( \frac{d\phi_{\nu}(\theta, E_{\nu})}{dE_{\nu}d\Omega} P_{\nu_e \rightarrow \mu_e}(E_{\nu}, \theta) + \frac{d\phi_{\nu}(\theta, E_{\nu})}{dE_{\nu}d\Omega} P_{\nu_e \rightarrow \mu_e}(E_{\nu}, \theta) \right)
\]

\[
\frac{d\sigma^NC}{dy}(y, E_{\nu}) \bigg|_{y = E_{shwr}/E_{\nu}} + (\nu \rightarrow \bar{\nu}) \tag{7.6}
\]

These NC induced hadronic cascades constitute about 10% of the total number of background cascades.
7.6.3 Muon Neutrino Neutral Current Interaction Events

The charged current events produced by muon-type neutrinos will result in a hadronic cascade, but will also produce a muon lepton which, in principle, will leave a track of Cherenkov radiation as discussed previously. It is assumed that if this track is long enough, i.e., the muon has enough energy, the muon will be identified and the event will be distinguishable from cascade type events resulting from the other discussed interactions. The NC interactions however will result in a hadronic cascade similar to the electron neutrino cascades examined previously.

For neutral current interactions, again, only a fraction \( y \) of the initial neutrino interaction is transferred to the cascade. Due to the steep energy dependence of the atmospheric neutrino fluxes, their contribution is thus expected to be smaller. The spectrum for NC interactions of the muon neutrinos as a function of the shower energy is given by:

\[
\frac{dN_{NC,\nu}}{dE_{\text{shower}}} = 2\pi n_T t \int d\cos \theta \ V \int_{E_{\text{shower}}}^{\infty} dE_\nu \frac{d\phi_{\nu}}{dE_\nu d\Omega} \frac{1}{E_\nu} \left( \frac{d\phi_{\mu} (\theta, E_\nu)}{dE_\nu d\Omega} P_{\nu_\mu \to \nu_\mu} (E_\nu, \theta) + \frac{d\phi_{\nu_e}}{dE_\nu d\Omega} P_{\nu_e \to \nu_\mu} (E_\nu, \theta) \right) \frac{d\sigma_{NC}}{dy} (y, E_\nu) \bigg|_{y = E_{\text{shower}}/E_\nu} + (\nu \to \bar{\nu}) .
\] (7.7)

These cascades also constitute a small part of the background cascades, 15% but become more important as a part of the background (\( > 30\% \)) as the energy increases and the relative proportion of atmospheric \( \nu_\mu \)'s grows as a percentage of the total neutrino flux.

7.6.4 Tau Neutrino Neutral Current Interaction Events

The tau-type neutrino NC interactions constitute the first part of the cascade events that we consider to be our signal. Due to the reduction in CC cross section imposed by the tau lepton production threshold effects, the number of NC events (about 80% of the signal at low energies) far out weighs the number of CC tau cascade events until energies increase past about 30 GeV incident neutrino energy
whereupon the proportions switch and the signal is dominated by the CC events. This also makes sense, when observing that the CC interaction cross section becomes larger than the NC cross section at this energy as well. For neutral current interactions, only a fraction $y$ of the initial neutrino interaction is transferred to the cascade, which will be hadronic in nature. The spectrum for NC tau neutrino interactions as a function of the shower energy is given by:

$$
\frac{dN_{NC,\nu_e}}{dE_{shwr}} = 2\pi n_T t \int d\cos \theta \int_{E_{shwr}}^{\infty} dE_{\nu} \frac{1}{E_{\nu}} \left( \frac{d\phi_{\nu_e}(\theta, E_{\nu})}{dE_{\nu}d\Omega} P_{\nu_e \rightarrow \nu_e}(E_{\nu}, \theta) + \frac{d\phi_{\nu_e}(\theta, E_{\nu})}{dE_{\nu}d\Omega} P_{\nu_e \rightarrow \nu_e}(E_{\nu}, \theta) \right)
$$

(7.8)

It should be noted that the $\nu_e \rightarrow \nu_e$ contribution is extremely small and can be ignored in practice, especially after about 20 GeV incident neutrino energy when the transition probability, $P_{\nu_e \rightarrow \nu_e}$, drops to almost zero.

### 7.6.5 Tau Neutrino Charged Current Interaction Events

For tau neutrinos, charged current (CC) interactions lead to a tau lepton which can provide two types of contributions to cascade events: electromagnetic and hadronic, depending on the tau decay mode. In the analysis, it was assumed that tau muonic decays led to a muon that could, in principle, be distinguished from the other decay modes because of the characteristic Cherenkov light track that such leptons leave when the muons possess energies above 5 GeV (25 m track length). In an extreme case of the analysis, track lengths of less than 75 m are considered indistinguishable, and the analysis still provides a statistically significant tau appearance.

#### 7.6.5.1 Tau Decay Modes

The tau lepton’s lifetime is very short, $2.91 \times 10^{-13}$ s, so taus would rather decay than lose energy. The tau decay modes are incorporated into the calculation of the
signal cascades by the inclusion of the decay distribution term, \( \frac{dn}{dz} \), where \( z \) can be formulated as either the energy retained from the tau lepton by the resulting tau neutrino or, equivalently, \( 1 - \frac{E_{\text{cascade}}}{E_\nu} \):

\[
\frac{dn}{dz} = \sum_i B_i \left( g_0^i + g_1^i \right),
\]

(7.9)

where the - (+) corresponds to neutrinos (antineutrinos), the \( B_i \) are the branching ratios for the decay mode \( i \) and the \( g_0 \) and \( g_1 \) are functions detailed in Table 7.1 in terms of \( z \) and \( r_i = \frac{m_i^2}{m_\tau^2} \) [79].

### Table 7.1. Functions used in calculating the tau lepton decay distributions. \( X \) represents hadrons \( \neq \pi, \rho, a_1 \). Equations and data from [79].

<table>
<thead>
<tr>
<th>Process</th>
<th>( B_i )</th>
<th>( g_0 )</th>
<th>( g_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau \to \nu_\tau \mu \nu_\mu )</td>
<td>0.18</td>
<td>( \frac{5}{3} - 3z^2 + \frac{4}{3}z^3 )</td>
<td>( \frac{1}{3} - 3z^2 + \frac{8}{3}z^3 )</td>
</tr>
<tr>
<td>( \tau \to \nu_\tau e \nu_e )</td>
<td>0.18</td>
<td>( \frac{5}{3} - 3z^2 + \frac{4}{3}z^3 )</td>
<td>( \frac{1}{3} - 3z^2 + \frac{8}{3}z^3 )</td>
</tr>
<tr>
<td>( \tau \to \nu_\tau \pi )</td>
<td>0.12</td>
<td>( \frac{1}{1-r_\pi} \theta(1 - r_\pi - z) )</td>
<td>( -\frac{2z-1+r_\pi}{(1-r_\pi)^2} \theta(1 - r_\pi - z) )</td>
</tr>
<tr>
<td>( \tau \to \nu_\tau \rho )</td>
<td>0.26</td>
<td>( \frac{1}{1-r_\rho} \theta(1 - r_\rho - z) )</td>
<td>( -\left( \frac{2z-1+r_\rho}{(1-r_\rho)^2} \right) \theta(1 - r_\rho - z) )</td>
</tr>
<tr>
<td>( \tau \to \nu_\tau a_1 )</td>
<td>0.13</td>
<td>( \frac{1}{1-r_{a_1}} \theta(1 - r_{a_1} - z) )</td>
<td>( -\left( \frac{2z-1+r_{a_1}}{(1-r_{a_1})} \right) \theta(1 - r_{a_1} - z) )</td>
</tr>
<tr>
<td>( \tau \to \nu_\tau X )</td>
<td>0.13</td>
<td>( \frac{1}{0.3} \theta(0.3 - z) )</td>
<td>0</td>
</tr>
</tbody>
</table>
7.6.5.2 Hadronic Decay Channel

The rate for hadronic events is given by:

\[
\frac{dN_{\text{had}}}{dE_{\text{shwr}}} = 2\pi n_T t \int d\cos \theta V \int_{E_{\text{shwr}}}^{\infty} dE_\nu \int_{E_{\text{shwr}}}^{E_\nu} dE_{\tau} \\
\left( \frac{d\phi_{\nu_\tau}(\theta, E_\nu)}{dE_\nu d\Omega} P_{\nu\mu \rightarrow \nu_\tau}(E_\nu, \theta) + \frac{d\phi_{\nu_\tau}(\theta, E_\nu)}{dE_\nu d\Omega} P_{\nu_\tau \rightarrow \nu_\tau}(E_\nu, \theta) \right) \\
\frac{d\sigma_{CC}^{\tau}}{dE_\tau}(E_\tau, E_\nu) \frac{dn_{\text{had}}}{dE_\tau}(E_\tau) \\
+ (\nu \rightarrow \bar{\nu}). \tag{7.10}
\]

In terms of dimensionless variables typically used for these cross-sections the number of events is evaluated as: \(d\sigma_{CC}^{\tau}/dE_\tau \sim 1/E_\nu d\sigma_{CC}^{\tau}/dy\) with \(y = 1 - E_\tau/E_\nu\) and \(dn_{\text{had}}/dE_\tau \sim E_{\text{shwr}}/E_\tau^2 dn_{\text{had}}/dz\) with \(z = 1 - E_{\text{shwr}}/E_\tau\). For the numerical calculations, the differential neutrino cross-sections as given by [22] with CTEQ6 parton distribution functions are used. It is important to note that for tau neutrinos, tau threshold suppression is still important in the lower energy range discussed here.

The threshold corrections following Ref. [80] were included as discussed above. The 3.5 GeV tau lepton production threshold energy has made the tau neutrino detection very difficult up to now, since the atmospheric neutrino flux for energies above tau lepton production threshold is very low for existing detectors. For instance, for the SK experiment, and assuming maximal mixing in the \(\nu_\mu - \nu_\tau\) sector, only one \(CC\) tau neutrino event is expected per kton-year of exposure. The ICDC experiment, which benefits from a much larger instrumented volume than the SK experiment, is in an unique position to detect tau neutrino interactions, as it has good sensitivity in an energy range where neutrino oscillations lead to a large number of tau neutrinos and, at the same time, is high enough to no longer be strongly affected by tau threshold suppression.
7.6.5.3 Electromagnetic Decay Channel

The rate for electromagnetic cascade events is given by an expression similar to Eq. (7.10), the only difference being in the decay rate $\frac{dn}{dE}$:

$$
\frac{dN^e}{dE_{\text{shwr}}} = 2\pi n_T t \int d \cos \theta V \int_{E_{\text{shwr}}}^{\infty} dE \int_{E_{\text{shwr}}}^{E} \frac{dE}{dE} \int_{E_{\text{shwr}}}^{E} dE \int_{E_{\text{shwr}}}^{E} dE \\
\left( \frac{d\phi_{\nu\mu}(\theta, E_{\nu})}{dE d\Omega} P_{\nu\mu \rightarrow \nu, \nu}(E_{\nu}, \theta) \right) + \frac{d\phi_{\nu\mu}(\theta, E_{\nu})}{dE d\Omega} P_{\nu\mu \rightarrow \nu, \nu}(E_{\nu}, \theta)
$$

As discussed above, in principle, electromagnetic cascades and hadronic cascades have different light yields and care must be taken when discussing their observable energy. It is also important to note that at these energies, the distinctions between the two different types of light may be extremely difficult. The showers themselves are only several meters long and for IC or ICDC with DOM spacing of 7 m, they will appear as point-like. Separate event rates are given to illustrate the relative rates for the different contributions to the signal and background.
Figure 7.3. Electron CC and all flavors NC events for one year ICDC exposure, see text for details in the calculation.

7.7 Results

Fig. 7.3 shows the cascade rates coming from $\nu_e$ charged current interactions and neutral current interactions of $\nu_e$ and $\nu_\mu$. These events are indistinguishable from the $\nu_\tau$ cascade events that are of interest to us, so they constitute an irreducible background for the $\nu_\tau$ search. Fig. 7.4 illustrates the cascade rates from charged current $\nu_\tau$ interactions followed by hadronic or electromagnetic tau decays, as well as $\nu_\tau$ neutral current interactions. While the background is significant, the number of events is very high. Neglecting systematics errors, the statistical significance could be defined as

$$S = \frac{n_s}{\sqrt{n_s + n_b}},$$

(7.12)

$n_s$ being the number of $\nu_\tau$ events and $n_b$ the number of background events (from $\nu_e$ charged current interactions and $\nu_{e,\mu}$ neutral current interactions). A statistically significant ($3\sigma$) $\nu_\tau$ appearance signal could be obtained in only a few months of observation. However, systematic uncertainties will limit the analysis. One of the biggest uncertainties is the energy threshold for cascade identification. At
Figure 7.4. Tau cascade events for one year of ICDC exposure.

the lowest energies considered here, muon track events would be very short \(^2\) and therefore, cascade events and muon track events would be indistinguishable. If cascade events can not be distinguished from muon tracks, the background to the \(\nu_\tau\) signal becomes significantly higher (by about an order of magnitude) due to the contribution from the muon event tracks. Even in the case where muon track events and cascade events can not be distinguished at low energies, the \(\nu_\tau\) signal can become statistically significant in a year of exposure. Good muon track reconstruction would be extremely useful for reducing this systematic uncertainty. The additional two strings in the center of the detector should help in this respect.

There are other systematic uncertainties affecting the analysis, as the knowledge of the interaction cross-sections, atmospheric neutrino fluxes, detector response, i.e., the effective volume, and other neutrino oscillation parameters. However, these systematic errors are expected to be under control in the next few years, exploiting data from future reactor and accelerator experiments as well as the muon track ICDC data and the atmospheric neutrino data from the ICDC experiment in different angular and energy ranges than the ones used for the \(\nu_\tau\) analysis.

\(^2\)A 5 GeV muon will have a track in ice of \(\sim 25\) m.
Appearance of tau neutrinos from oscillations of atmospheric $\nu_\mu$ is thus likely to be detected in the near future and can be used to further study $\nu_\tau$ interactions.
8.1 Future Work

The mechanism that leads to neutrino mass and neutrino oscillations constitutes physics beyond the standard model and this mechanism may also lead to other sub-dominant effects. These non-standard interactions (NSI) are an interesting topic being explored to determine if they may shed some light on a few experimental anomalies in addition to exploring new physics. Research has shown that oscillations involving $\nu_\tau$ are particularly sensitive to these new effects with the $\nu_\mu \to \nu_\tau$ channel showing promise [102], and the $\nu_e \to \nu_\tau$ even more so by allowing larger effects [103]. My collaborators and I have already begun work in this area considering ICDC and we are excited to continue our analysis. The effect of the NSI may be measurable as, for example, a deviation from the expected oscillation pattern and with event counts as high as those in ICDC, bounds on the size of the NSI are a possibility.

Model building involving the exchange of a new heavy particle and calculation of the NSI effects is one way to determine the size and effect of the NSI. One may also begin by including these effects through a phenomenological analysis where the NSI effects are parameterized by arbitrary $\epsilon$'s which are added in much the same way matter effects were added into oscillation analysis. It is interesting to note that the coherence requirement demands that the normal matter effects are only sensitive to interactions with background fermions that are flavor preserving. The new transition probabilities can be computed and then effects of the NSI can
be explored statistically in much the same way this dissertation explored precision measurements of normal oscillation parameters. The matter part of the Hamiltonian used to evolve the neutrino mass states in Chapter 2 can be updated as follows, for example:

\[
H_{\text{matter}} = \sqrt{2} G_F N_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \sqrt{2} G_F N_e \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu}^* & \epsilon_{e\tau}^* \\ \epsilon_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}^* \\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix}, \tag{8.1}
\]

where the 1 represents the normal matter effects discussed in Chapter 2 and the \(\epsilon\) represent the additional NSI effects that have been added.

The present bounds on these parameters are [104, 105]

\[
-0.9 < \epsilon_{ee} < 0.75, \\
|\epsilon_{e\mu}| < 3.8 \times 10^{-7}, \\
-0.05 < \epsilon_{\mu\mu} < 0.08, \\
|\epsilon_{e\tau}| < 0.25, \\
|\epsilon_{\mu\tau}| < 0.25, \\
|\epsilon_{\tau\tau}| < 0.4. \tag{8.2}
\]

A quick look at these numbers indicates that the \(\epsilon\)'s involving \(\tau\) have some of the weakest bounds. The tighter bounds on the other parameters are based on studies of \(\nu_\mu\) signals such as with MINOS and K2K data [103]. Our initial investigations show that ICDC may potentially use the high statistics signal and the \(\nu_\tau\) appearance to improve on the bounds involving \(\epsilon_{x\tau}\).

### 8.2 Unresolved Issues

There are many outstanding questions about neutrinos and neutrino mixing [12]. The experimental results discussed in Chapter 4 showed that great progress has been made in trying to understand neutrino masses and neutrino mixing. Of course there still remains a great deal to learn. Over the last decade, a large number of experiments of different types have provided strong evidence for neutrino
oscillations and thus for physics beyond the Standard Model, see Ref. [19] and references therein.

As discussed in Chapter 4, data is already being taken at some of the next reactor neutrino experiments to help improve by at least a factor of 10 the existing upper limit on the small neutrino mixing angle $\theta_{13}$. Future experiments will also explore the Dirac CP-violating phase determining the magnitude of CP-violation effects in neutrino oscillations. It is also important to explore the mechanism that gives neutrinos mass, beginning with the nature of massive neutrinos. Similarly, future experiments will need to explore the absolute mass scale of the neutrinos, and the order of the mass states. Of fundamental importance too is the mechanism that allows neutrino oscillations to violate individual lepton number conservation.

Establishing the validity of neutrino flavor oscillation as an explanation depends on two different types of observations, appearance observations in which a flux of a neutrino flavor that was not present in the source flux (or not present in the correct amount) appears during propagation, and disappearance observations in which the expected flux of neutrinos produced at a source disappears during propagation. This dissertation explored one of each of these situations in the IceCube DeepCore experiment. The results of this study showed that the IceCube DeepCore experiment may reveal a statistically significant $\nu$ appearance signal and provide even higher precision measurements of $\Delta m^2_{31}$ and $\theta_{23}$, the so-called atmospheric neutrino oscillation parameters. Future experiments will also need to explore the solar mixing parameters $\Delta m^2_{21}$, $\theta_{12}$, to greater precision.

8.3 Conclusion

The IceCube detector and its DeepCore array provide a great opportunity for studies of atmospheric neutrinos. Being the largest existing neutrino detector, it will accumulate a huge number of atmospheric neutrino events over an enormous energy range, thus allowing for detailed studies of oscillation physics, Earth density, atmospheric neutrino fluxes and new physics [98]. In order to extract all this information it is necessary to use energy and angular distribution information, as well as flavor composition, all possible to obtain with the IceCube detector. Qualitatively, there are three main energy intervals and three main angular regions.
which are sensitive to different types of physics.

At very high energies, above 10 TeV, neutrino interaction cross-sections become high enough that neutrinos going through the Earth start getting attenuated [99]. This effect is sensitive to neutrino interaction cross-sections and to the density profile of the Earth.

The “intermediate” energy region, between 50 GeV and 1 TeV can provide good information about the atmospheric neutrino flux, which can be used to improve the uncertainties in the simulated atmospheric neutrino fluxes [100].

In this dissertation, the concentration was on the “low” energy region, below about 100 GeV, where neutrino oscillation effects can be significant. Although the IceCube detector has higher energy thresholds, its low energy extension, the IceCube DeepCore array (ICDC), extends the IceCube neutrino detection capabilities in the low energy domain. It has been shown that tau neutrinos from oscillations of atmospheric muon neutrinos could be detected in the next few years with a high significance level. SK has provided a low significance analysis that shows that its data is statistically inconsistent with no $\nu_\mu \rightarrow \nu_\tau$ oscillations. More recently, OPERA, a LBL accelerator experiment, reported their first $\nu_\tau$ candidate event [101]. The large number of tau neutrinos expected from ICDC could allow for measurements of $\nu_\tau$ interaction cross-sections and studies of non-standard neutrino interactions, which are largely unconstrained in the tau sector. This data can be used for precise measurements of oscillation parameters like $\Delta m^2_{31}$ and $\theta_{23}$ and for resolving some of the neutrino oscillation parameter degeneracies.

A very large effort is directed toward the optimization and construction of a next generation of experiments that will address precision measurements and potential discovery of new phenomena in neutrino oscillations (see Refs. [16, 17, 18, 19] and references therein). While they will achieve extremely good sensitivities, these experiments face long construction and data-taking times. In the meantime, ICDC, which is already taking data, will acquire a high statistics atmospheric neutrino sample which can be very useful in extracting information about neutrino oscillation physics. Some of this information could be used for further optimization of the future accelerator experiments. Long baseline experiments can achieve very high precision, but have a fixed baseline and a limited energy coverage. Atmospheric neutrinos in ICDC cover a much more extended range of energies and many
baselines and thus provide potentially new, complementary information. Combining data from the two types of experiments would check the consistency of allowed neutrino oscillation parameter space over a large range of energies and propagation distances and can be very important for solving parameter degeneracies. If any new physics is present in the neutrino sector, its effects could be larger at high energies and long distances and would lead to additional effects in the atmospheric neutrino data.

In summary, ICDC offers a unique window toward a better understanding of neutrino properties due to its very high atmospheric neutrino statistics. Careful studies of the expected atmospheric neutrino oscillation signals in ICDC, as the one carried out here, are extremely important, since atmospheric neutrinos constitute an irreducible background to astrophysical neutrino searches and can offer information that is complementary to that obtained from other neutrino experiments.
Bibliography


[67] For a recent overview, see:


[74] Ty DeYoung, IceCube and DeepCore, Aspen Workshop on Neutrino Physics, 2009.


Vita
Gerardo Giordano

Gerardo Giordano was born in New Brunswick, NJ in 1980 and raised in a (very, very) small nearby town, Dunellen. While in high school, he acquired the nickname “G”, provided copious amounts of tutoring and met Diana Cuello who would (eventually) become his wife. He graduated from high school in 1998, the valedictorian in a class of about 50 people. After a brief stint as an electrical engineering technician, he began studying at both NJIT and Rutgers University in Newark, NJ in 2001. While riding on a train to his parents house during the first week of class, G witnessed the Twin Towers collapse and the subsequent chaos and panic at the mass transit stations. Diana and G were married in April of 2002 exactly 1371 days (which is the sum of the prime numbers less than and including 107) after they started dating. G graduated from university in 2005 with a degree in Physics from NJIT and a degree in Mathematics from Rutgers.

G then moved 3.5 hours almost exactly due west to State College, to begin graduate school in physics at the Pennsylvania State University. His wife, however, moved 6 hours almost exactly due west to Pittsburgh, to begin graduate school in psychology at Duquesne University thereby condemning each of their Toyota Corollas to 5 hours of driving every other weekend so they could see each other. It was in graduate school that G realized that he loved teaching physics. He has won three awards for his teaching and has a stack of positive student recommendations of which he is quite proud. In August of 2011 he will begin a faculty position at King’s College in Wilkes-Barre, PA.

Publications
